Available online at www.elixirpublishers.com (Elixir International Journal)

Statistics

Elixir Statistics 126 (2019) 52481-52484

ISSN: 2229-712X

Simulation Study on Reliability Estimates of a Repairable System with Lethal and Non-Lethal Common Cause Shock Failures

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ARTICLE INFO

Article history:

Received: 05 December 2018;
Received in revised form:
01 January 2019;
Accepted: 10 January 2019;

Keywords

Reliability,
Availability,
Common Cause Shock Failure,
Maximum Likelihood,
Estimation,
Simulation.

ABSTRACT

In order to solve the reliability assessment of repairable systems, this article, based on two-component system, provides the maximum likelihood estimation. The system can be restored through proper repairing even from Common Cause Shock (CCS) failure. We derived M L estimates of availability for series and parallel systems. The approach used is empirical one with Monte Carlo simulation.

1. Introduction

Reliability can be expressed as the probability of a component/system completing its expected function during an interval of time, which gives an assessment of the overall performance of a system. The theory of reliability was formally recognized by statistical and modeling of the components of the manufacturing systems. System analysis was used to develop various reliability measures that are important to assess the system performance [5]. An accurate reliability measure for a system guarantees its functionality, efficiency and safety [8]. However, reliability can be influenced by various factors. Conventionally the reliability analysts assumed that the components in the system will fail individually by inherent incapability and randomly. This type of failure is called intrinsic failure in the reliability analysis. In addition to intrinsic failures, the researchers encountered another type of failure known as Common Cause Shock (CCS) failures.

The study of CCS failures started in the mid-1980 [5], since then lots of research have been carried out in this area. The event may be outside of the component/system, and CCS failures were classified into two categories depending on the intensity of the shock [9], one is lethal Common Cause Shock (LCCS) failures, which is the occurrence of simultaneous outage of all components in the system and the other is Non-Lethal Common Cause Shock (NCCS) failures, which is the occurrence of random number of components to simultaneous outage, follows probability distribution, viz., Binomial distribution. Billinton and Allan [3], discussed the basic concept and method of reliability evaluations in the presence of CCS failures. Atwood and Steverson [1, 2], studied the role of CCS failures and identified their occurrence with high intensity in nuclear power plants. Chari et al. [6], considered lethal and non-lethal CCS failures and developed reliability measures of identical component system.

Sagar G Y [10, 11, 12], developed the system reliability measures to two unit and three unit systems by considering the CCS failures and Human errors. He also considered M L approach to develop frequency of failures of two unit identical system. Sreedhar et al. [4], proposed M L estimation approach for estimating the reliability measures of two unit system with LCCS and NCCS failures.

The aim of this investigation is to incorporate repairs in Chari et al. [6] model for the system state when both components are down due to individual or CCS failures. If the repair of down system is not considered, our model coincides with that of Chari et al. [6]. Also this paper for the first time proposes M L estimation to derive the reliability estimates such as availability for the series and parallel systems of the present repair model.

2. Assumptions

Consider a system with two identical components:

1.The component fails individually and also simultaneously due to lethal or non-lethal CCS failures in Poisson manner.

2.Individual, LCCS and NCCS failures are independent to each other.

3.The failed components are repaired singly and service times follow an exponential distribution.

3. Notations

λ : Failure rate of individual component

ω : LCCS failure

β : NCCS failure

μ_1, μ_2: Service rates of first and second components respectively

μ_c : Service rate when both components fail simultaneously

$p(q)$: The probability of simultaneous failures of the components due to NCCS (LCCS)

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$A_{LNS}(t)$: Availability of the series system
$A_{LNP}(t)$: Availability of the parallel system
$A_{LNS}(\infty)$: Steady-state availability of the series system
$A_{LNP}(\infty)$: Steady-state availability of the parallel system
$\widehat{A}_{LNS}(\infty)$: M L Estimate of steady-state availability of series system
$\widehat{A}_{LNP}(\infty)$: M L Estimate of steady-state availability of parallel system
$\overline{x}$, $\overline{y}$&$\overline{w}$: Sample means of the occurrence of individual, NCCS and LCCS failures respectively.
$\overline{z}$: Sample mean of service time of the components
$\hat{\hat{x}}$, $\hat{\hat{y}}$&$\hat{\hat{w}}$: Sample estimates of individual, NCCS and LCCS failure rates respectively.
$\hat{\hat{z}}$: Sample estimate of service time of the components
n : Sample size
N: Number of simulated samples
M S E: Mean square error

4. Markov model for state transition

The Markov diagram is shown in Fig.1. The numerical in Fig.1 denote the system state.

The probability equations associated with the system states are given by

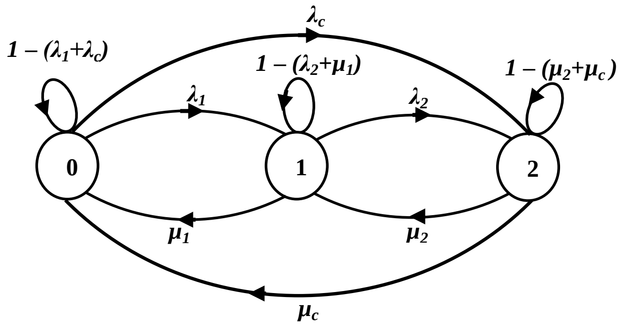

Fig. 1 Markov graph

$$\left.\begin{aligned} p_0'(t) &= -(\lambda_1+\lambda_c)p_0(t)+\mu_1 p_1(t)+\mu_c p_2(t) \\ p_1'(t) &= \lambda_1 p_0(t)-(\lambda_2+\mu_1)p_1(t)+\mu_2 p_2(t) \\ p_2'(t) &= \lambda_c p_0(t)+\lambda_2 p_1(t)-(\mu_2+\mu_c)p_2(t) \end{aligned}\right\} \quad (1)$$

Where $\lambda_1 = 2(\lambda+\beta pq)$

$\lambda_2 = (\lambda+\beta p)$

$\lambda_c = \beta p^2+\omega$

By using the Laplace transformation technique, the set of equations (1) can be solved with the help of the initial conditions, at $t=0$, $p_0(t)=1$, $p_i(t)=0$; $i = 1,2$ we obtain

$$p_0(t) = \frac{r_1^2+r_1 l_1+m_1}{r_1(r_1-r_2)}\exp(r_1 t) - \frac{r_2^2+r_2 l_1+m_1}{r_2(r_1-r_2)}\exp(r_2 t) + \frac{m_1}{r_1 r_2} \quad (2)$$

$$p_1(t) = \frac{r_1 l_2+m_2}{r_1(r_1-r_2)}\exp(r_1 t) - \frac{r_2 l_2+m_2}{r_2(r_1-r_2)}\exp(r_2 t) + \frac{m_2}{r_1 r_2} \quad (3)$$

$$p_2(t) = \frac{r_1 l_3+m_3}{r_1(r_1-r_2)}\exp(r_1 t) - \frac{r_2 l_3+m_3}{r_2(r_1-r_2)}\exp(r_2 t) + \frac{m_3}{r_1 r_2} \quad (4)$$

Where

$l_1 = \lambda_2+\mu_1+\mu_2+\mu_c$; $l_2 = \lambda_1$; $l_3 = \lambda_c$
$m_1 = \lambda_2\mu_c+\mu_1\mu_c+\mu_1\mu_2$;
$m_2 = \lambda_1\mu_c+\lambda_1\mu_2+\lambda_c\mu_2$;
$m_3 = \lambda_2\mu_c+\lambda_c\mu_1+\lambda_1\lambda_2$

$$r_1, r_2 = -(\lambda_1+\lambda_2+\lambda_c+\mu_1+\mu_2+\mu_c) \pm SQRT[(\lambda_1+\lambda_2+\lambda_c+\mu_1+\mu_2+\mu_c)^2 - 4(\mu_2(\lambda_1+\lambda_c+\mu_1) + \mu_c(\lambda_1+\lambda_2+\mu_1)+\lambda_2(\lambda_1+\lambda_c)+\lambda_c+\mu_1)]$$

We can verify that $\sum_{i=0}^{2} p_i(t) = 1$, $i = 0,1,2$

5. Maximum Likelihood Estimation (MLE)

MLE is used to estimate the values of model parameters. In this section, we shall derive explicit results for some performance measures of two-component series and parallel systems in the presence of NCCS and LCCS failures.

Let $x_1, x_2 \dots\dots\dots x_n$ be a sample of size 'n' number of times between individual failures which will obey exponential law.

Let $y_1, y_2 \dots\dots\dots y_n$ be sample of size 'n' representing time between NCCS failures which follow exponential as well.

Let $w_1, w_2 \dots\dots\dots w_n$ be sample of size 'n' representing times between LCCS failures which will obey exponential population.

Let $z_{11}, z_{12} \dots\dots\dots z_{1n}$; $z_{21}, z_{22} \dots\dots\dots z_{2n}$ & $z_{31}, z_{32} \dots\dots\dots z_{3n}$ be 'n' number of times between repairs of the components with exponential population.

Where, $\hat{\hat{x}}$, $\hat{\hat{y}}$, $\hat{\hat{w}}$, $\hat{\hat{z}}_1$, $\hat{\hat{z}}_2$ & $\hat{\hat{z}}_3$ are the M L estimates of individual failure rates (λ), NCCS failure rate (β), LCCS failure rate (ω) and repair rates of (μ_1, μ_2, μ_c) of the system respectively. Where,

$$\hat{\hat{x}} = \frac{1}{\overline{x}};\ \hat{\hat{y}} = \frac{1}{\overline{y}};\ \hat{\hat{w}} = \frac{1}{\overline{w}};\ \hat{\hat{z}}_1 = \frac{1}{\overline{z}_1};\ \hat{\hat{z}}_2 = \frac{1}{\overline{z}_2};\ \hat{\hat{z}}_3 = \frac{1}{\overline{z}_3}$$

$$\overline{x} = \frac{\sum x_i}{n};\ \overline{y} = \frac{\sum y_i}{n};\ \overline{w} = \frac{\sum w_i}{n};\ \overline{z}_1 = \frac{\sum z_{1i}}{n};$$

$$\overline{z}_2 = \frac{\sum z_{2i}}{n};\ \overline{z}_3 = \frac{\sum z_{3i}}{n}$$

5.1 Series System

For series system, the availability is given by

$$A_{LNS}(t) = p_0(t) \quad (5)$$

In limiting case when $t \to \infty$, the availability is

$$A_{LNS}(\infty) = \frac{\lambda_2\mu_c+\mu_1\mu_c+\mu_1\mu_2}{\lambda_2(\lambda_1+\lambda_c)+\mu_2(\lambda_1+\lambda_c+\mu_1)+\mu_c(\lambda_1+\lambda_2+\mu_1)+\lambda_c\mu_1} \quad (6)$$

$$\widehat{A}_{LNS}(\infty) = \frac{(\hat{\hat{y}}p^2+\hat{\hat{w}})\hat{\hat{z}}_3+\hat{\hat{z}}_1\hat{\hat{z}}_3+\hat{\hat{z}}_1\hat{\hat{z}}_2}{\{(\hat{\hat{y}}p^2+\hat{\hat{w}})(2(\hat{\hat{x}}+\hat{\hat{y}}pq)+\hat{\hat{x}}+\hat{\hat{y}}p)+\hat{\hat{z}}_2(2(\hat{\hat{x}}+\hat{\hat{y}}pq)+\hat{\hat{x}}+\hat{\hat{y}}p+\hat{\hat{z}}_1) + \hat{\hat{z}}_3(2(\hat{\hat{x}}+\hat{\hat{y}}pq)+\hat{\hat{y}}p^2+\hat{\hat{w}}+\hat{\hat{z}}_1)+(\hat{\hat{x}}+\hat{\hat{y}}p)\hat{\hat{z}}_1\}} \quad (7)$$

5.2 Parallel System

In this case, the availability is given by

$$A_{LNP}(t) = p_0(t)+p_1(t)$$

$$= \frac{r_1^2+r_1(l_1+l_2)+(m_1+m_2)}{r_1(r_1-r_2)}\exp(r_1 t) - \frac{r_2^2+r_2(l_1+l_2)+(m_1+m_2)}{r_2(r_1-r_2)}\exp(r_2 t) + \frac{m_1+m_2}{r_1 r_2} \quad (8)$$

The limiting availability is

$$A_{LNP}(\infty) = \left[1 - \frac{\lambda_2(\lambda_1+\lambda_c)+\lambda_c\mu_1}{\mu_2(\lambda_1+\lambda_c+\mu_1)+\mu_c(\lambda_1+\lambda_2+\mu_1)}\right] \quad (9)$$

Thus, the M L estimate of limiting availability is

$$\hat{A}_{LNP}(\infty) = \left[1 - \frac{\hat{y}p^2+\hat{w}(2(\hat{x}+\hat{y}pq)+\hat{x}+\hat{y}p)+\hat{x}+\hat{y}\hat{z}_1 p}{\hat{z}_2(2(\hat{x}+\hat{y}pq)+\hat{x}+\hat{y}p+\hat{z}_1)+\hat{z}_3(2(\hat{x}+\hat{y}pq)+\hat{y}p^2+\hat{w}+\hat{z}_1)}\right] \quad (10)$$

6. Simulation

The M L estimates of system availability functions were not identified with exact or analytical form of probability density function since they are complex functions of the sample information. As such, the probability density function of the estimates is not identified analytically. Hence an attempt is made to establish the validity and to find precision of the estimates of our model; Monte-Carlo simulation is used. For a range of specified values of the rates of individual (λ), NCCS failures (β), LCCS failures (ω) and service rate (μ_1, μ_2, μ_c) for the samples of sizes n=5(5)30 were simulated in each case with N=10,000(20,000)90,000 in order to evolve mean square error (MSE) in each case. It is interesting to note that estimation gives a very close estimate in the case of very small samples of size n=5. This shows that M L approach and estimators are quite useful in estimating reliability and availability measures. In particular case when repair rate is constant and repair of down system is not considered in the model our results match with those of Chari et al. [6]. As evidence, Table 1 and Table 2 show availability of the system before considering repair of down system and after considering repair of down system (present model) and also given the point estimates of the present model.

Table 1. Series system: β = 0.1, ω = 0.01, μ_1 = 1, μ_2 = 1.2, μ_c = 2, p = 0.6

n	λ	Availability		Estimates				
		Before	After	N				
				10000	30000	50000	70000	90000
5	0.1	0.729111	0.788912	0.765019	0.765016	0.766583	0.765989	0.766793
10	0.1	0.729111	0.788912	0.777662	0.777513	0.777271	0.778055	0.778043
15	0.1	0.729111	0.788912	0.780926	0.781558	0.782219	0.782005	0.781919
20	0.1	0.729111	0.788912	0.783715	0.783501	0.783555	0.783486	0.783356
25	0.1	0.729111	0.788912	0.784633	0.784362	0.784510	0.784525	0.784633
30	0.1	0.729111	0.788912	0.785961	0.785387	0.785434	0.785170	0.785224
5	0.2	0.609992	0.691034	0.672188	0.672097	0.674107	0.673389	0.674316
10	0.2	0.609992	0.691034	0.682333	0.681980	0.681656	0.682709	0.682717
15	0.2	0.609992	0.691034	0.684449	0.685296	0.684840	0.685887	0.685732
20	0.2	0.609992	0.691034	0.687106	0.686818	0.686847	0.686803	0.686644
25	0.2	0.609992	0.691034	0.687602	0.687308	0.687535	0.687573	0.687715
30	0.2	0.609992	0.691034	0.689013	0.688359	0.688341	0.688089	0.688094

Table 2. Parallel system: β = 0.1, ω = 0.01, μ_1 = 1, μ_2 = 1.2, μ_c = 2, p = 0.6

n	λ	Availability		Estimates				
		Before	After	N				
				10000	30000	50000	70000	90000
5	0.1	0.940082	0.978704	0.972880	0.972747	0.972938	0.972859	0.972932
10	0.1	0.940082	0.978704	0.976313	0.976280	0.976258	0.976302	0.976280
15	0.1	0.940082	0.978704	0.977045	0.977215	0.977149	0.977211	0.977187
20	0.1	0.940082	0.978704	0.977598	0.977619	0.977595	0.977562	0.977584
25	0.1	0.940082	0.978704	0.977918	0.977803	0.977818	0.977825	0.977837
30	0.1	0.940082	0.978704	0.978062	0.977990	0.977997	0.977961	0.977997
5	0.2	0.902700	0.966511	0.956402	0.956206	0.956585	0.956449	0.956595
10	0.2	0.902700	0.966511	0.962375	0.962216	0.962198	0.962339	0.962292
15	0.2	0.902700	0.966511	0.963579	0.963895	0.963757	0.963917	0.963864
20	0.2	0.902700	0.966511	0.964575	0.964593	0.964543	0.964507	0.964563
25	0.2	0.902700	0.966511	0.965065	0.964912	0.964969	0.964967	0.964976
30	0.2	0.902700	0.966511	0.965376	0.965286	0.965267	0.965211	0.965272

Table 3. Simulation results for steady-state availability function for series system with λ = 0.5, β = 0.1, ω = 0.01, μ_1 = 1, μ_2 = 1.2, μ_c = 2, p = 0.6

Sample size (n = 5)			
N	$A_{LNS}(\infty)$	$\hat{A}_{LNS}(\infty)$	M S E
10000	0.515056	0.512599	0.016932
30000	0.515056	0.512192	0.017089
50000	0.515056	0.514471	0.017037
70000	0.515056	0.513687	0.017118
90000	0.515056	0.514600	0.017041

Sample size (n = 10)			
N	$A_{LNS}(\infty)$	$\hat{A}_{LNS}(\infty)$	M S E
10000	0.515056	0.514793	0.008678
30000	0.515056	0.514270	0.008823
50000	0.515056	0.513838	0.008708
70000	0.515056	0.515099	0.008756
90000	0.515056	0.515124	0.008799

Sample size (n = 15)			
N	$A_{LNS}(\infty)$	$\hat{A}_{LNS}(\infty)$	M S E
10000	0.515056	0.513871	0.005871
30000	0.515056	0.514976	0.005904
50000	0.515056	0.514424	0.005878
70000	0.515056	0.515550	0.005834
90000	0.515056	0.515348	0.005891

Sample size (n = 20)			
N	$A_{LNS}(\infty)$	$\hat{A}_{LNS}(\infty)$	M S E
10000	0.515056	0.515339	0.004506
30000	0.515056	0.515058	0.004441
50000	0.515056	0.515034	0.004456
70000	0.515056	0.515012	0.004440
90000	0.515056	0.514868	0.004378

Sample size (n = 25)			
N	$A_{LNS}(\infty)$	$\hat{A}_{LNS}(\infty)$	M S E
10000	0.515056	0.514889	0.003561
30000	0.515056	0.514601	0.003553
50000	0.515056	0.514842	0.003499
70000	0.515056	0.514938	0.003530
90000	0.515056	0.515062	0.003522

Sample size (n = 30)			
N	$A_{LNS}(\infty)$	$\hat{A}_{LNS}(\infty)$	M S E
10000	0.515056	0.515886	0.003014
30000	0.515056	0.515222	0.002914
50000	0.515056	0.515159	0.002922
70000	0.515056	0.514931	0.002963
90000	0.515056	0.514887	0.002909

Table. 4 Simulations results for steady-state availability function for parallel system with $\lambda = 0.5$, $\beta = 0.1$, $\omega = 0.01$, $\mu_1 = 1$, $\mu_2 = 1.2$, $\mu c = 2$, $p = 0.6$.

Sample size (n = 5)			
N	$A_{LNP}(\infty)$	$\hat{A}_{LNP}(\infty)$	MSE
10000	0.914781	0.892234	0.009668
30000	0.914781	0.891679	0.009792
50000	0.914781	0.892715	0.009806
70000	0.914781	0.892410	0.009916
90000	0.914781	0.892774	0.009606

Sample size (n = 10)			
N	$A_{LNP}(\infty)$	$\hat{A}_{LNP}(\infty)$	MSE
10000	0.914781	0.905643	0.002704
30000	0.914781	0.905057	0.002788
50000	0.914781	0.904999	0.002714
70000	0.914781	0.905480	0.002666
90000	0.914781	0.905340	0.002737

Sample size (n = 15)			
N	$A_{LNP}(\infty)$	$\hat{A}_{LNP}(\infty)$	M S E
10000	0.914781	0.908077	0.001584
30000	0.914781	0.908926	0.001514
50000	0.914781	0.908543	0.001540
70000	0.914781	0.909069	0.001490
90000	0.914781	0.908852	0.001530

Sample size (n = 20)			
N	$A_{LNP}(\infty)$	$\hat{A}_{LNP}(\infty)$	MSE
10000	0.914781	0.910437	0.001037
30000	0.914781	0.910463	0.001054
50000	0.914781	0.910295	0.001064
70000	0.914781	0.910278	0.001049
90000	0.914781	0.910429	0.001036

Sample size (n = 25)			
N	$A_{LNP}(\infty)$	$\hat{A}_{LNP}(\infty)$	MS E
10000	0.914781	0.911426	0.000794
30000	0.914781	0.911102	0.000799
50000	0.914781	0.911301	0.000783
70000	0.914781	0.911300	0.000794
90000	0.914781	0.911338	0.000796

Sample size (n = 30)			
N	$A_{LNP}(\infty)$	$\hat{A}_{LNP}(\infty)$	MSE
10000	0.914781	0.912246	0.000644
30000	0.914781	0.912119	0.000627
50000	0.914781	0.911979	0.000633
70000	0.914781	0.911904	0.000643
90000	0.914781	0.911981	0.000631

7. Conclusion

This paper established a new reliability analysis model considering the repair of down system. The model considered two kinds of CCS failures, namely lethal and non-lethal. The estimates of steady-state availability function were developed in the absence of analytical approach for both series and parallel systems. From the simulation results, we observed that the point estimates become more accurate when the sample size is large and each of MSE decreases with increasing the sample size. Also we observed that availability of the system in parallel is greater than that of system in series which agrees with physical situations. However, the simulation study had shown excellent performance of the proposed estimates and its superiority when compared to non-repair of down system.

References

[1]. Atwood, C. L, Estimates for the Binomial failure rate Common cause model, U.S, *Nuclear Regulatory Commission Report*, NUREG/CR-1401, ECG-EA-5112, 1980.

[2]. Atwood C. L, Steverson J. A, Common cause failure rates, *Journal of US Nuclear Regulatory Commission Report*, NUREG CR-2770, EGG-Ea-5485, 1983.

[3]. Billinton, R and Allan, R. N, Reliability evaluation of engineering systems; concepts and techniques, (Plenum Press, 1983, New York).

[4]. Sreedhar B. R, Sagar G Y, Pushpanjali K and Reddy Y R, M L Estimation of the reliability measures of a two unit system in the presence of two kinds of CCS failures, *ARPN Journal of Engineering and Applied Sciences, 7(8), 2012, 980 – 986.*

[5]. Chari A. A, *A Markovian approach to system reliability and Availability measures with CCS failures*, Ph. D Thesis, *Sri Venkateswara University*, Tirupathi, 1988.

[6]. Chari A. A, M. P Sastry and MadhusudhanaVerma S, *Micro-Electronics and reliability, 31, 15 -19, 1991.*

[7]. Dhillon B. S, On common cause failures, Bibliography, *Microelectronics and Reliability, 18(6), 533-534.*

[8]. Liyun Xu, Yanhao Chen, Florent Briand, Fengxu Zhou and Moroni Givanni, Reliability measurement for multistate manufacturing systems with failure interaction, *Elsevier B. V, Procedia CIRP*, 63 (2017) 242 -247.

[9]. Reddy Y. R, Reliability analysis for two unit non-identical system with CCS failures, Ph. D thesis, *S. K. University*, Anantapur, 2003.

[10]. Sagar G. Y, *Markovian approach to system reliability measures with common cause shock failures and human errors*, Ph. D theis, *S. K. University*, Anantapur, 2007.

[11]. Sagar G Y, Frequency of Failures of a System and Confidence-interval, *Elixir International Journal, 70(2014), 23818 -23821.*

[12]. Sagar G. Y, Reliability estimation and confidence interval for two-unit system in the presence of common cause failures and human errors, *International Journal of Statistics and Applications, 2012, 2(6), 114 - 119.*