Magnetic Response on Wave Propagation of Double Layered Nanoplate Embedded in an Elastic Medium

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ABSTRACT

The magnetic response of transverse wave propagation of double layered nanoplate embedded in an elastic medium is studied employing nonlocal continuum theory. The displacement equation of nanoplate with magnetic and elastic medium is devised with the help of Lorentz force and Winkler elastic foundation. The frequency equations are acquired for nanoplate with simply supported edges. The numerical result reveals that the magnetic strength and elastic medium increases the frequencies of double layered nanoplates. The perfection of the present frequency value is noted by comparing it with the existing literature.

1. Introduction

The carbon nanoplates are semiconducting material and the study of vibration of nanoplates has applications in the design of nanodevices. Based on Galerkin method and Hamilton's principle Barati [1] analyzed the vibration of double layered nanoplate with the effect of temperature and elastic medium using nonlocal stress strain theory. Karimi et al. [2] used differential quadrature method and investigated the shear vibration of double-layer orthotropic nanoplate. In addition the surface and non-local effects are considered using Gurtin – Murdoch's theory. Considering the initial stress and gradient theory Nami and Janghorban [3] studied the wave propagation of nanoplates. Zhang et al. [4] developed the surface layer model and examined the characteristics of vibration of nanoplates with nonlocal elastic theory. The surface and nonlocal effect decreases the magnitude of phase velocity.

Wang et al. [5] established non-linear vibration equations of double-layered nanoplates with various boundary conditions. Ebrahimi and Dabbagh [6] investigated the wave dispersion of rotating magneto-electro-elastic nanoplates. Later the same author [7] analyzed the vibration of sandwich nanoplates under the effect of thermo-magnetic field. Based on strain gradient theory Arani and Shokravi [8] discussed the wave propagation of coupled double layered nanoplate with magnetic effect embedded in an elastic medium. The result demonstrated that the magnetic field effect increases the vibration of nanoplates. Aranasov et al. [9] studied the nonlocal forced vibration of an orthotropic nanoplate subjected to magnetic field and elastic medium. Using nonlocal shear deformable theory Kiani [10] analyzed the free vibration of nanoplates in the presence of magnetic field. Ponnusamy [11] investigated the dispersion behavior of generalized thermo elastic plate of polygonal cross sections using Fourier expansion collocation method. Later Ponnusamy and Amuthalakshmi [12, 13] respectively studied the vibration of carbon nanotubes and nanoplates embedded in an elastic medium.

This paper explores the impact of magnetic field on double layered nanoplate embedded in an elastic medium. A nonlocal elastic theory is proposed to investigate the small scale effects of nanoplate made of graphene sheets. The vibration equation of nanoplates is obtained by considering Lorentz force and elastic foundation. The dispersion relations of nanoplates with simply supported edges are procured with the help of periodic solution. The numerical values of natural frequencies are computed and the characteristics of the dispersion curves are analyzed. The value of natural frequencies of nanoplate by neglecting magnetic and elastic medium is computed and is compared with the results of Kitipornchai et al. [14] and it reveals the exactness of the present result.

2. Formulation of the problem

A nanoplate of thickness h along z direction and length l_a and l_b along x and y direction is considered and is shown in Figure 1. The displacement equation of nanoplate is formulated [15, 16] as

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + q = \rho h \frac{\partial^2 w}{\partial t^2} + kw, \tag{1}$$

where w, ρ, t, q and k denotes the nanoplate deflection, mass density, time, pressure acting on nanoplate and Winkler foundation modulus.

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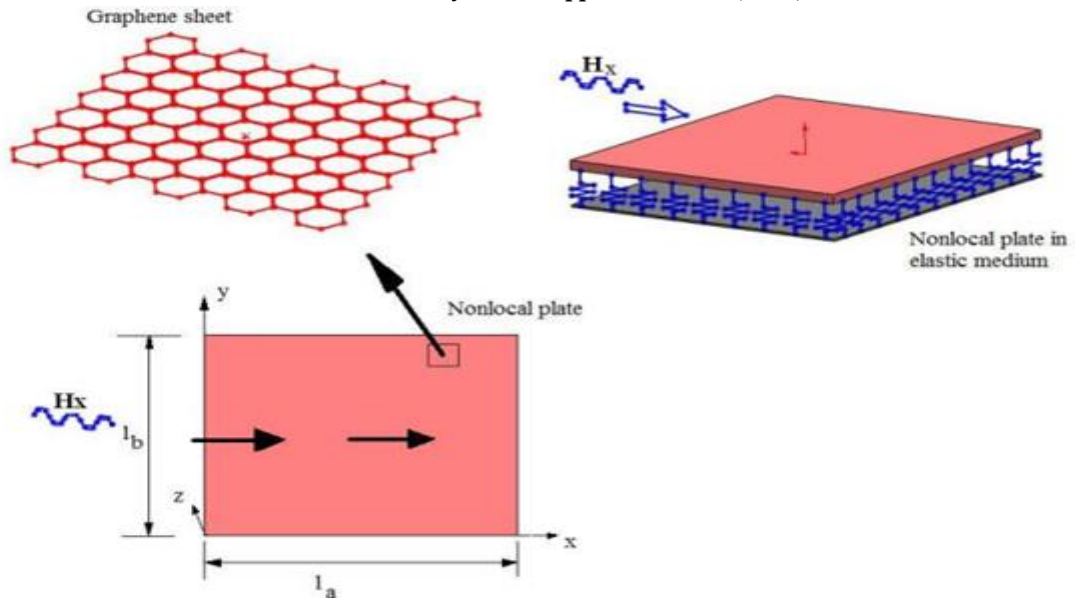
Figure 1. Model of nanoplate with magnetic effect embedded in an elastic medium.

Assuming a magnetic field vector $\overline{H}(H_x, 0, 0)$ acts on a nanoplate with magnetic field permeability η, it produces a Lorentz force on nanoplate. Therefore the pressure on the nanoplate due to magnetic force considered from Maxwell equation is expressed as [17]

$$q = \int_{-h/2}^{h/2} f_z dz = h f_z = \eta h H_x^2 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right). \tag{2}$$

Considering the nonlocal continuum theory, the normal and shearing stresses based on generalized Hooke's law are given by

$$\sigma_x - (e_0 a)^2 \nabla^2 \sigma_x = \frac{E}{1 - v^2} \left(\varepsilon_x + v \varepsilon_y \right), \tag{3a}$$

$$\sigma_y - (e_0 a)^2 \nabla^2 \sigma_y = \frac{E}{1 - v^2} \left(\varepsilon_y + v \varepsilon_x \right), \tag{3b}$$

$$\tau_{xy} - (e_0 a)^2 \nabla^2 \tau_{xy} = G \gamma_{xy}, \tag{3c}$$

where $e_0 a$ represents nonlocal parameter that denotes effect of small scale. Also $\varepsilon_x, \varepsilon_y$ and γ_{xy} are normal and shearing strain given as

$$\varepsilon_x = \frac{\partial u}{\partial x}, \ \varepsilon_y = \frac{\partial v}{\partial y}, \ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \tag{4}$$

in which u and v denote components of displacement along x and y direction given by

$$u = -z \frac{\partial w}{\partial x}, \quad v = -z \frac{\partial w}{\partial y} \tag{5}$$

where z is the distance of the lamina below the mid plane of the plate.

The expression of bending moments M_x, M_y and M_{xy} are defined as

$$M_x = \int_{-h/2}^{-h/2} z \sigma_x dx,$$

$$M_y = \int_{-h/2}^{-h/2} z \sigma_y dz,$$

$$M_{xy} = \int_{-h/2}^{h/2} z \tau_{xy} dz, \tag{6}$$

Substituting Equations (3) – (5) in Equation (6) and evaluating Equation (6), the bending moment expression takes the form

$$M_x - (e_0 a)^2 \left(\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_x}{\partial y^2} \right) = -D \left(\frac{\partial^2 w}{\partial x^2} + v \frac{\partial^2 w}{\partial y^2} \right), \tag{7a}$$

$$M_y - (e_0 a)^2 \left(\frac{\partial^2 M_y}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} \right) = -D \left(\frac{\partial^2 w}{\partial y^2} + v \frac{\partial^2 w}{\partial x^2} \right), \tag{7b}$$

$$M_{xy} - (e_0 a)^2 \left(\frac{\partial^2 M_{xy}}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial y^2} \right) = -D (1-v) \frac{\partial^2 w}{\partial x \partial y}, \tag{7c}$$

where $D = Eh^3/12(1-v^2)$ denote nanoplate bending stiffness.

Using Equations (3) and (7) in Equation (1), the displacement equation of magnetic effect on nanoplate embedded in an elastic medium can be obtained as follows
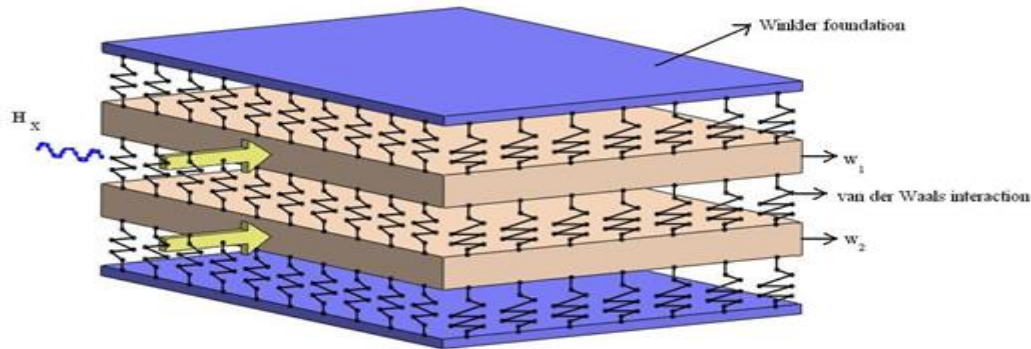
$$\left[D + (e_0 a)^2 \eta h H_x^2\right]\nabla^4 w - (e_0 a)^2\left(\rho h \frac{\partial^2}{\partial t^2}\nabla^2 w + k\nabla^2 w\right) - \eta h H_x^2\nabla^2 w + \rho h \frac{\partial^2 w}{\partial t^2} + kw = 0, \tag{8}$$

where ∇^2 and ∇^4 are Laplacian and biharmonic operator given by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \ \nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}.$$

3. Double layered nanoplate

A nanoplate of double layer with magnetic effect embedded in an elastic medium is considered as shown in Figure 2. Also $w_i (i = 1, 2)$ represents the nanoplate deflection of i – th layer.

Figure 2. Model of magnetic effect on double layered nanoplate embedded in an elastic medium.

The displacement equation of nanoplate under magnetic effect embedded in an elastic medium takes the form

$$\left[D + (e_0 a)^2 \eta h H_x^2\right]\nabla^4 w_1 + (e_0 a)^2\left[\nabla^2 p_1 - \rho h \frac{\partial^2}{\partial t^2}\nabla^2 w_1 - k\nabla^2 w_1\right] - \eta h H_x^2\nabla^2 w_1 + \rho h \frac{\partial^2 w_1}{\partial t^2} + kw_1 = p_1, \tag{9a}$$

$$\left[D + (e_0 a)^2 \eta h H_x^2\right]\nabla^4 w_2 + (e_0 a)^2\left[\nabla^2 p_2 - \rho h \frac{\partial^2}{\partial t^2}\nabla^2 w_2 - k\nabla^2 w_2\right] - \eta h H_x^2\nabla^2 w_2 + \rho h \frac{\partial^2 w_2}{\partial t^2} + kw_2 = p_2, \tag{9b}$$

where p_i is the pressure produced on i – th layer because of van der Waals (vdW) interaction and is written as

$$p_i = \sum_{j=1}^{2} c_{ij}\left(w_i - w_j\right), \tag{10}$$

where c_{ij} is interaction coefficient of vdW with $c_{ij} = c_{ji}(i \neq j)$. Here the pressure is proportional to the deflection difference between the nanoplate and is due to the negligible interaction between the plates.

Using Equation (10) in Equations (9), the governing displacement equation takes the form

$$\left[D + (e_0 a)^2 \eta h H_x^2\right]\nabla^4 w_1 + (e_0 a)^2\left[c_{12}\nabla^2\left(w_2 - w_1\right) - \rho h \frac{\partial^2}{\partial t^2}\nabla^2 w_1 - k\nabla^2 w_1\right] - \eta h H_x^2\nabla^2 w_1 + \rho h \frac{\partial^2 w_1}{\partial t^2} \tag{11a}$$

$$+ kw_1 = c_{12}\left(w_2 - w_1\right),$$

$$\left[D + (e_0 a)^2 \eta h H_x^2\right]\nabla^4 w_2 + (e_0 a)^2\left[c_{12}\nabla^2\left(w_1 - w_2\right) - \rho h \frac{\partial^2}{\partial t^2}\nabla^2 w_2 - k\nabla^2 w_2\right] - \eta h H_x^2\nabla^2 w_2 + \rho h \frac{\partial^2 w_2}{\partial t^2} \tag{11b}$$

$$+ kw_2 = c_{12}\left(w_1 - w_2\right),$$

Equations (11a) and (11b) are coupled differential equation for the vibration of double-layered nanoplate.

Consider the nanoplate edges as simply supported, therefore the nanoplate deflection in terms of periodic solution can be written as

$$w_k(x, y, t) = W_k \sin\frac{m\pi x}{l_a}\sin\frac{n\pi y}{l_b}\exp(i\omega t), \tag{12}$$

where $i = \sqrt{-1}$, ω represents circular frequency, m and n denotes half wave numbers for the x and y direction, $W_k (k = 1, 2)$ indicates amplitudes.

Substituting the above solution in Equation (11), the differential equation which is coupled takes the form

$$\left\{\left[(e_0a)^2\left(\rho h\omega^2-c_{12}-k\right)-\eta hH_x^2\right]\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)-\left[D+(e_0a)^2\eta hH_x^2\right]\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2\right.$$ (13a)

$$\left.+\rho h\omega^2-c_{12}-k\right\}W_1+c_{12}\left[(e_0a)^2\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)+1\right]W_2=0,$$

$$\left\{\left[(e_0a)^2\left(\rho h\omega^2-c_{12}-k\right)-\eta hH_x^2\right]\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)-\left[D+(e_0a)^2\eta hH_x^2\right]\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2\right.$$ (13b)

$$\left.+\rho h\omega^2-c_{12}-k\right\}W_2+c_{12}\left[(e_0a)^2\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)+1\right]W_1=0,$$

Equations (13a) and (13b) are coupled homogeneous equation, therefore solving it a trivial solution is obtained. In order to get non-trivial solution, the coefficient matrix determinant of Equation (13) is equated to zero as follows

$$\begin{vmatrix} a_1\rho h\omega^2-a_1c_{12}-a_1k+a_2 & a_1c_{12} \\ a_1c_{12} & a_1\rho h\omega^2-a_1c_{12}-a_1k+a_2 \end{vmatrix}=0,$$ (14)

where the notation a_1 and a_2 are given as follows

$$a_1=\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1,$$ (15a)

$$a_2=-\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\eta hH_x^2-D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2,$$ (15b)

Solving the determinant given in Equation (14), a numerical expression in terms of a polynomial of degree four is obtained as follows

$$b_1\omega^4+b_2\omega^2+b_3=0,$$ (16)

where,

$$b_1=(a_1\rho h)^2,$$

$$b_2=2a_1\rho h(a_2-a_1c_{12}-a_1k),$$

$$b_3=a_2^2+a_1^2k^2-2a_1a_2(c_{12}+k)+2a_1^2kc_{12}.$$

Solving Equation (16), the dispersion relation of both lower (mode 1) and higher (mode 2) occur in the form

$$\omega_1=\sqrt{\frac{D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2+\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\eta hH_x^2}{\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}\rho h}+\frac{k}{\rho h}},$$ (17a)

$$\omega_2=\sqrt{\frac{D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2+\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\eta hH_x^2}{\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}\rho h}+\frac{(2c_{12}+k)}{\rho h}},$$ (17b)

It is examined from Equations (17), that on neglecting the nonlocal, magnetic effect and elastic medium the equation matches with the frequency equation of Kitipornchai et al. [14] and it reveals the precision of the present result.

Further we deliberate about the frequency ratio of nonlocal and magnetic effect of nanoplate with double layer. The Nonlocal Frequency Ratio (NFR) of both modes are given as follows:

NFR of mode 1 (lower mode)

$$=\sqrt{\frac{D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2+\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}\left[k+\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]\eta hH_x^2}{\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right]\left\{D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2+\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\eta hH_x^2+k\right\}}},$$ (18a)

NFR of mode 2 (higher mode)

(18b)

$$= \sqrt{\frac{D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2+\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}\left[2c_{12}+k+\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\eta h H_x^2\right]}{\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right]\left\{D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2+\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\eta h H_x^2+2c_{12}+k\right\}}}.$$

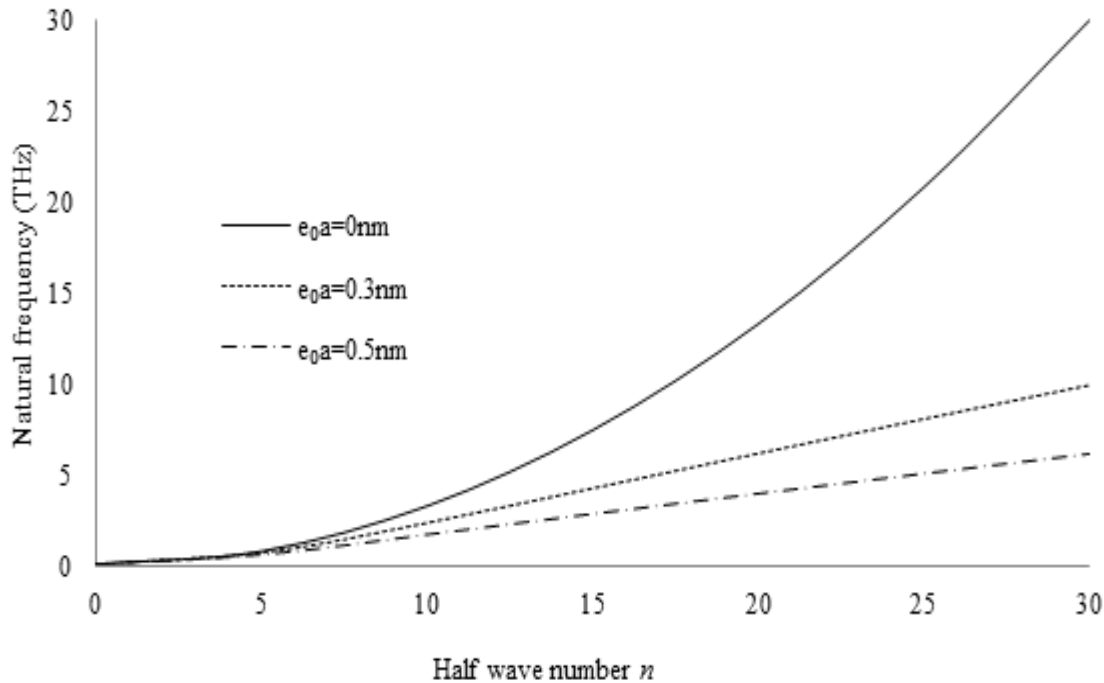The Magnetic Frequency Ratio (MFR) of both modes is given as follows:

MFR of mode 1 (lower mode)

(19a)

$$= \sqrt{\frac{D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2+\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}\left[k+\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\eta h H_x^2\right]}{D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2+\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}k}},$$

MFR of mode 2 (higher mode)

(19b)

$$= \sqrt{\frac{D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2+\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}\left[2c_{12}+k+\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\eta h H_x^2\right]}{D\left[\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)\right]^2+\left\{\pi^2\left(\frac{m^2}{l_a^2}+\frac{n^2}{l_b^2}\right)(e_0a)^2+1\right\}(2c_{12}+k)}}.$$
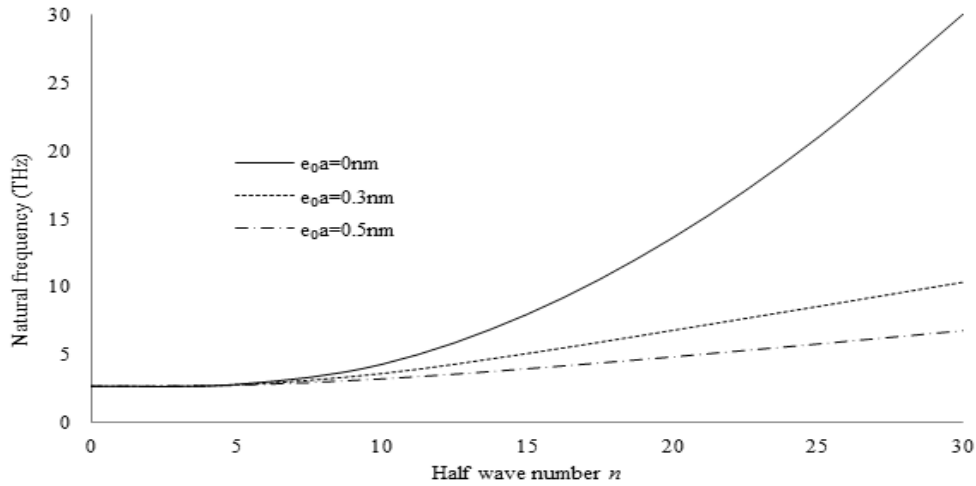
4. Numerical results

Figure 3. Dispersion curves of double layered nanoplate under the impact of magnetic and elastic medium for lower mode of vibration with distinct nonlocal parameter.

This paper investigates magnetic response on wave propagation of nanoplate with double layer embedded in an elastic medium. The material constants are considered from Kitipornchai et al. [14] as $E = 1.02 TPa$ and Poisson's ratio $\nu = 0.25$. Further the mass density, vdW coefficient of interaction and Winkler modulus are taken from Wang et al. [15] as $\rho = 2250 kg/m^3$, $c_{ij} = 108 GPa/nm$ and $k = 1.13 \times 10^{18}\ Pa/m$. The width along the x and y direction are $l_a = l_b = 10 nm$. The value of half wave number m along x direction is fixed as $m = 1$. Further the magnetic constants are considered from Li et al. [18] as $H_x = 10^7\ A/m$ and $\eta = 4\pi \times 10^{-7}\ A/m$. The dispersion curves between the half wave number n and natural frequency is studied.

The dispersion curves between the half wave number n and natural frequency of double layered nanoplate with the impact of magnetic and elastic medium for both modes of vibration are drawn and are shown in Figures 3 and 4. From Figures 3 and 4, it is perceived that as the wave number increases the natural frequencies also increases. Due to the interaction of nonlocal carbon atoms the natural frequencies decrease with the increase in nonlocal parameter for both modes of vibration. It reveals that the nonlocal elastic theory has lower estimation than the continuum elastic theory. The numerical value of natural frequencies of lower mode is less than the higher modes of vibration.
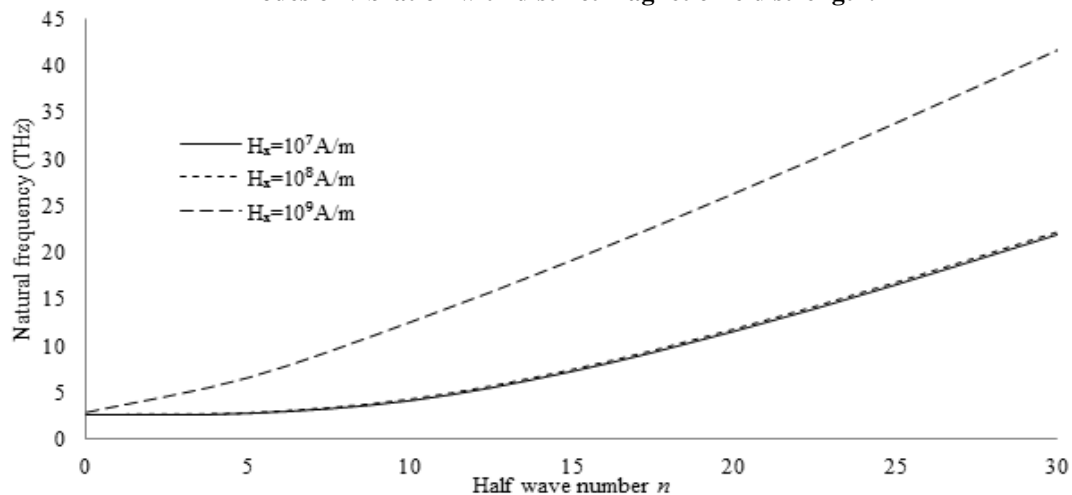
Figure 4. Dispersion curves of double layered nanoplate under the impact of magnetic and elastic medium for higher mode of vibration with distinct nonlocal parameter.

The dispersion curves between half wave number n along y direction and natural frequencies of double layered nanoplate with distinct strength of magnetic field for lower and higher modes of vibration are drawn and are shown in Figures 5 and 6. From Figures 5 and 6, it is found that the magnetic field strength increases the vibration of double layered nanoplates. This fact is due to the impact of coupling between the nanoplate and magnetic field. The powerful magnetic field strength in nanoplate is used for resonators with high frequency.
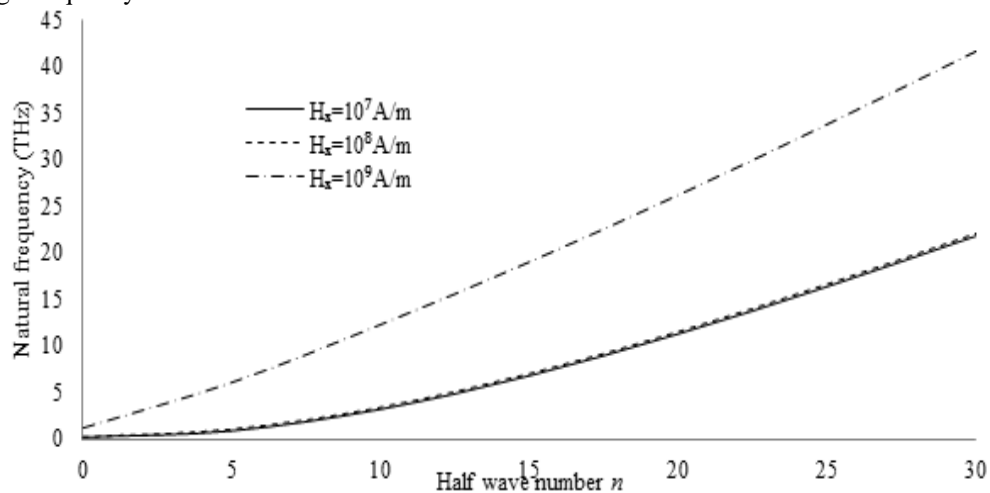
Figure 5. Dispersion curves of a double layered nanoplate under the impact of magnetic and elastic medium for lower modes of vibration with distinct magnetic field strength.

Figure 6. Dispersion curves of a double layered nanoplate under the impact of magnetic and elastic medium for higher modes of vibration with distinct magnetic field strength.

The dispersion curves between the wave number and natural frequency of double layered nanoplate under magnetic response both in the presence and absence of elastic medium is drawn and is shown in Figure 7. Figure 7 perceive that the presence of elastic medium increases the vibration of layered nanoplate than in the absence of elastic medium. Graph is drawn between the nonlocal parameter and nonlocal frequency ratio of double layered nanoplate with distinct magnetic field strength and is shown in Figure 8. From Figure 8, it is perceived that as the nonlocal parameter increases the nonlocal frequency ratio decreases. Further it is observed that the magnetic field strength increases the values of nonlocal frequency ratio.
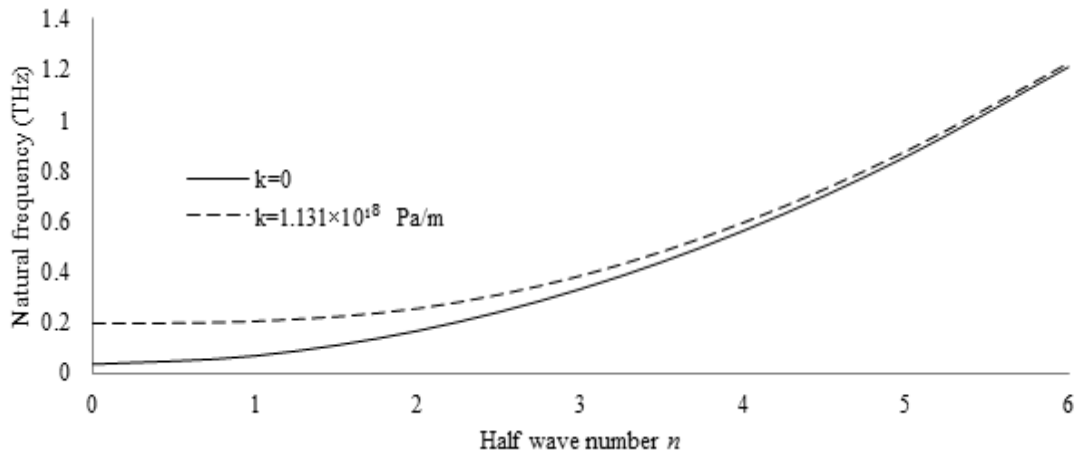
Figure 7. Dispersion curves of magnetic response on double layered nanoplate with and without Winkler elastic foundation.
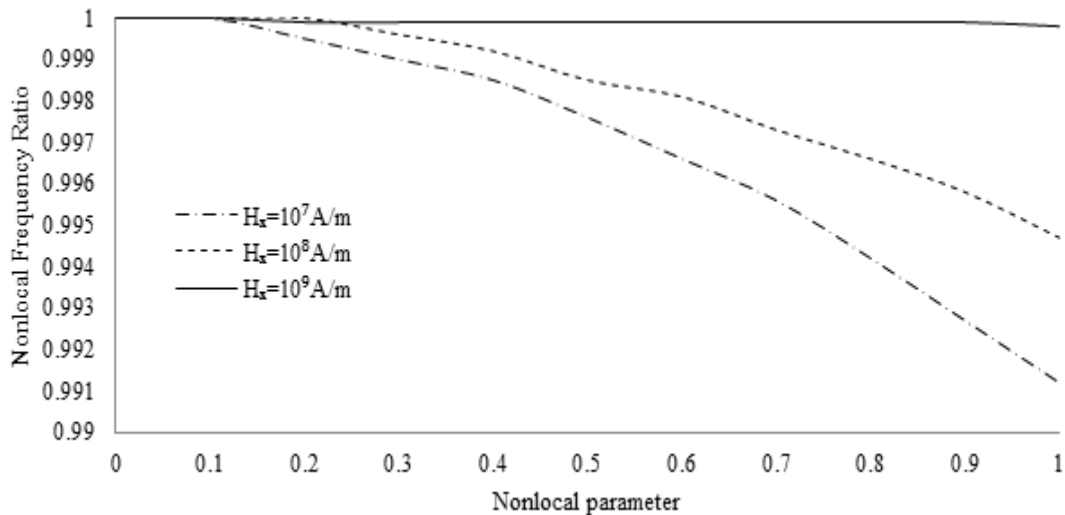

Figure 8. Variation of nonlocal frequency ratio versus nonlocal parameter of a double layered nanoplate with distinct magnetic field strength.

In order to prove the accuracy of the present numerical result with the existing literature, the natural frequencies of double layered nanoplate in the absence of magnetic and elastic medium is compared with the numerical values of Kitiporanchai et al. [14]. The natural frequencies (THz) of double layered nanoplated are computed and are shown in Table 1. From Table 1, it is perceived that the frequencies matches with the frequencies of Kitipornchai et al. [14] and the variation is in second or third decimal position. It is analyzed that as the mode number increases the natural frequencies also increases.

Table 1.

Natural frequencies (THz)		*m=1*			*m=2*			*m=3*		
ω_1	Author	0.067	0.166	0.333	0.166	0.266	0.432	0.333	0.432	0.599
	Kitipornchai et al. [14]	0.069	0.173	0.346	0.173	0.276	0.449	0.346	0.449	0.622
ω_2	Author	2.675	2.680	2.695	2.680	2.688	2.709	2.695	2.709	2.741
	Kitipornchai et al. [14]	2.683	2.688	2.705	2.688	2.697	2.720	2.705	2.720	2.754

5. Conclusions

The magnetic response on wave propagation of double layered nanoplate embedded in an elastic medium is analyzed using nonlocal continuum theory. The dispersion equations are obtained for double layered nanoplates in the presence of magnetic and elastic medium and their properties of dispersion are investigated. The numerical result reveals that the nonlocal continuum theory has lower estimation than classical plate theory. The impact of magnetic and elastic medium increases the vibration of double layered nanoplates. Further the natural frequencies acquired by neglecting magnetic and elastic medium matches with the natural frequencies existing in the available literature.

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