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Study of Unsteady Gravity-Driven Convective Flow and Heat Transfer of Optically Thick Nanofluid Past an Oscillating Vertical Plate in Presence of Magnetic Field

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ABSTRACT

The unsteady gravity-driven convective flow and heat transfer of optically thick nanofluid past an oscillating, vertical plate is considered. The flow is confined to y > 0 where y is the coordinate measure in the normal direction to the plate. The fluid is assumed to be electrically conducting with an uniform magnetic field applied in a direction perpendicular to the plate. The resulting equation is solved by making use of Laplace transform method. Velocity profile and temperature profile are drawn for different values of radiation parameter and magnetic parameter.

1. Introduction

Nanofluids can be considered as the future of heat transfer fluids in various heat transfer applications. They are expected to give better thermal performance than conventional fluids due to the presence of suspended nanoparticles which have high thermal conductivity. About 1% by volume of copper nanoparticles or carbon nanotubes dispersed in ethylene glycol or oil increases the thermal conductivity by 40% (copper) and 150% (carbon nanotubes). Similar enhancements using ordinary particles would require more than 10% by volume. The nanofluids are also much more stable than suspensions of conventional particles.

Choi and Eastman[1] for the first time used nanoparticles to intensify thermal conductivity of fluids and heat transfer rate. Suspensions of nanoparticles in a base fluid are known as nanofluids and are used to increase the heat transfer rate of microelectronics, microchips in computers, fuel cells, transportation, biomedicine, food processing, solid state lightening and manufacturing. Nadeem [4] studied the magnetohydrodynamic problem with convective boundary layer flow and heat transfer over a vertical plate. The magnetohydrodynamic boundary layer flow over a vertical stretching/shrinking sheet in a nanofluid was investigated by Makinde [2],[3]. Nadeem [4],[5] investigated magnetohydrodynamic flow of different types of nano fluids over a convective surface.

The gravity- driven convective heat transfer is an important phenomenon in the cooling mechanism of many engineering systems like the electronic industry, solar collectors and cooling systems for nuclear reactors because of its minimum cost, low noise, smaller size and reliability. To understand the behaviour of the performance of fluid motion in several applications It is essential to study Magneto hydrodynamic gravity-driven convection through an optically thick fluid past an infinite vertical plate. The problem of gravity-driven convection in a regular fluid past a vertical plate is a classical problem solved by Ostrach[6]. Seigel[7] was the first to study the transient free-convective flow past a semi-infinite vertical plate by integral method. Convective flows with radiation are also encountered in many industrial processes such as heating and cooling of chambers, energy processes, evaporation from large reservoirs, solar power technology.

In this paper we study the magneto hydromagnetic gravity-driven convective boundary layer flow of nanofluids past an oscillating vertical plate in the presence of an uniform transverse magnetic field and thermal radiation. In the fluid considered water is base fluid with suspensions of nanosized copper (Cu) and silver (Ag) particles. The governing equations are solved analytically and presented in closed form.

2. Flow Ananlysis

The unsteady gravity-driven convective flow and heat transfer of optically thick nano fluid past an oscillating vertical plate in the presence of magnetic field is considered. At time t = 0 the plate is at rest with the constant basic temperature T_0. At time t > 0, the plate begins to oscillate in its own plane according to $u_0 = \sin \omega t$ where u_0 is amplitude of the plate oscillations and the temperature of the plate is maintained at constant temperature T_w. We use Rosseland approximation for radiative flux and it is assumed that the radiative heat flux is applied in the normal direction to the plate. In an optically thick fluid the radiation exchange

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takes place only among neighbouring volume elements. In the fluid considered the base fluid is water and suspended nanoparticles are copper and silver and it is further assumed that the base fluid and the suspended nanoparticles are in thermal equilibrium and density is linearly dependent on temperature buoyancy forces.

Thermo-physical properties of the base fluid and different nano particles are given [8]

Physical properties	Water	Cu	Ag
Cp(J/kg K)	4179	385	235
ρ(kg/m^3)	997.1	8893	10500
K(W/mK)	0.613	401	429
βx10^5(K^{-1})	21	1.67	1.89

The basic equations in the presence of thermal radiation and magnetic field past an oscillating vertical plate are

$$\rho_{nf}\frac{\partial u}{\partial t}=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g\left(\rho\beta\right)_{nf}\left(T-T_0\right)-\sigma_{nf}B^2u \tag{1}$$

$$\left(\rho c_p\right)_{nf}\frac{\partial T}{\partial t}=k_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{2}$$

where

$$\rho_{nf}=(-\phi)\rho_f+\phi\rho_s,\quad \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},$$

$$\sigma_{nf}=\sigma_f\left[1+\frac{3(\sigma-1)}{(\sigma+2)-(\sigma-1)\phi}\right],$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s,$$

$$k_{nf}=k_f\left[1-\frac{3\phi\left(k_f-k_s\right)}{2k_f+k_s+\phi\left(k_f-k_s\right)}\right]$$

$$\left(\rho c_p\right)_{nf}=(1-\phi)\left(\rho c_p\right)_f+\phi\left(\rho c_p\right)_s,$$

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{3}$$

Considering temperature differences to be sufficiently small within the flow using Taylor series and neglecting higher order terms, radiation heat flux q_r becomes

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial\left(4T_0^3T-3T_0^4\right)}{\partial y} \tag{4}$$

Using equation (4) in equation (2)

$$\left(\rho c_p\right)_{nf}\frac{\partial T}{\partial t}=\left(K_{nf}+\frac{16\sigma^*T_0^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} \tag{5}$$

The initial and boundary conditions are

$$t=0\quad u=0\quad T=T_0\quad \text{for } y\geq 0$$

$$t>0\quad u=u_0\sin\omega t\quad T=T_\omega\quad \text{for } y=0$$

$$t>0\quad u\to 0\quad T\to T_0\quad \text{for } y\to\infty$$

Using non-dimensional variables

$$y^*=\frac{u_0 y}{\upsilon_f},t^*=\frac{u}{u_0},\theta=\frac{T-T_0}{T_w-T_0},\omega^*=\frac{\upsilon_f\omega}{u_0^2}$$

Equation (1) and equation (5) becomes

$$\frac{\partial u}{\partial t}=a_1\frac{\partial^2 u}{\partial y^2}+Gr\,a_2\theta-M^2a_3u \tag{6}$$

$$\frac{\partial\theta}{\partial t}=a_4\frac{\partial^2\theta}{\partial y^2} \tag{7}$$

where $b_0=1-\phi$, $b_1=\left(b_0+\phi\frac{\rho_s}{\rho_f}\right)$,

$$b_2=\left(b_0+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right),\quad b_3=\left(b_0+\phi\frac{\left(\rho c_p\right)_s}{\left(\rho c_p\right)_f}\right),$$

$$b_4=\frac{k_{nf}}{k_f},\quad b_5=\frac{\sigma_{nf}}{\sigma_f},\quad b_6=\frac{b_4}{b_3},\quad a_1=\frac{1}{b_0^{2.5}b_1}$$

$$a_1=\frac{1}{b_0^{2.5}b_1},\quad a_2=\frac{b_2}{b_1},\quad a_3=\frac{b_5}{b_1},$$

$$a_4=\frac{b_4+Nr}{b_3\,\mathrm{Pr}} \tag{8}$$

Magnetic parameter $M^2=\dfrac{\sigma_f B^2U_f}{\rho_f u_0^2}$,

Radiation parameter $Nr=\dfrac{16\sigma^*T_0^3}{3k^*k_f}$,

Prandtl number $Pr=\dfrac{\mu_f c_p}{k_f}$, Grashof number $Gr=\dfrac{g\beta_f\left(T_w-T_0\right)\upsilon_f}{u_0^3}$, k is thermal conductivity, k^* is Rosseland mean absorption coefficient, ω is amplitude of plate oscillation. f is the fluid phase, nf is nano-fluid

The corresponding initial and boundary conditions are

$$t=0 \quad u=0 \quad \theta=0 \quad \text{for } y\ge 0$$
$$t>0 \quad u=\sin\omega t \quad \theta=1 \quad \text{for } y=0$$
$$t>0 \quad u\to 0 \quad \theta\to 0 \quad \text{for } y\to\infty \tag{9}$$

Taking Laplace transform of equation (7) and equation (9) we get

$$\bar{\theta}=\frac{1}{s}e^{-ys\sqrt{a_4}} \tag{10}$$

Taking Laplace transform of equation (6) we get

$$\bar{u}=\frac{i}{2}F_1(y,s)-\frac{i}{2}F_2(y,s)-c_6F_3(y,s)+ c_6F_4(y,s)+c_6F_5(y,s)-c_6F_6(y,s) \tag{11}$$

$$F_1(y,s)=\frac{e^{-y\sqrt{\frac{s+c_1}{a_1}}}}{s+i\omega},\quad F_2(y,s)=\frac{e^{-y\sqrt{\frac{s+c_1}{a_1}}}}{s-i\omega},$$

$$F_3(y,s)=\frac{e^{-y\sqrt{\frac{s+c_1}{a_1}}}}{s},\quad F_4(y,s)=\frac{e^{-y\sqrt{\frac{s+c_1}{a_1}}}}{s-c_4},$$

$$F_5(y,s)=\frac{e^{-y\sqrt{\frac{s}{a_4}}}}{s},\quad F_6(y,s)=\frac{e^{-y\sqrt{\frac{s}{a_4}}}}{s-c_4},$$

$$c_1=M^2a_3,\ c_2=Gr\,a_2\quad c_3=\frac{a_1}{a_4}-1,$$

$$c_4=\frac{c_1}{c_3},\ c_5=\frac{c_2}{c_3},\ c_6=\frac{c_5}{c_4}$$

Taking Inverse Laplace transform of equation (10) and equation (11) we get

$$\theta(y,t)=erfc\left(\frac{y\sqrt{\frac{1}{a_4}}}{2\sqrt{t}}\right) \tag{12}$$

$$u(y,t)=\frac{i}{2}f_1(y,t)-\frac{i}{2}f_2(y,t)-c_6 f_3(y,t)+ c_6 f_4(y,t)+c_6 f_5(y,t)-c_6 f_6(y,t) \tag{13}$$

From equation (10) Nusselt number Nu can be written as

$$Nu=-\left(\frac{\partial\theta}{\partial y}\right)_{y=0}=-\sqrt{\frac{1}{a_4\pi t}} \tag{14}$$

From equation (9) skin friction τ can be written as

$$\tau^*(y,t)=-\tau$$

$$\tau=\left.\frac{\partial\theta}{\partial y}\right|_{y=o}=\frac{i}{2}h_1(t)-\frac{i}{2}h_2(t)-c_6h_3(t)+ c_6h_4(t)+c_6h_5(t)-c_6h_6(t)$$

where $m_1=\frac{y}{2\sqrt{a_1t}}-\sqrt{(c_1-i\omega)t}$,

$$m_2=\frac{y}{2\sqrt{a_1t}}-\sqrt{(c_1+i\omega)t}$$

$$f_1(y,t)=\left[e^{-y\sqrt{\frac{1}{a_1}(c_1-i\omega)}}erfc(m_1(t))+ e^{y\sqrt{\frac{1}{a_1}(c_1-i\omega)}}erfc(m_1(t))\right]\left(\frac{e^{-i\omega t}}{2}\right)$$

$$f_2(y,t)=\left[e^{-y\sqrt{\frac{1}{a_1}(c_1+i\omega)}}erfc(m_2(t))+ e^{y\sqrt{\frac{1}{a_1}(c_1+i\omega)}}erfc(m_2(t))\right]\left(\frac{e^{i\omega t}}{2}\right)$$

$$f_3(y,t)=\frac{1}{2}\left[e^{-y\sqrt{\frac{c_1}{a_1}}}erfc\left(\frac{y}{2\sqrt{a_1t}}-\sqrt{c_1t}\right)+ e^{-y\sqrt{\frac{c_1}{a_1}}}erfc\left(\frac{y}{2\sqrt{a_1t}}+\sqrt{c_1t}\right)\right]$$

$$f_4(y,t)=\left[e^{-y\sqrt{\frac{1}{a_4}(c_1+c_4)}}erfc\left(\frac{y}{2\sqrt{a_1t}}-\sqrt{(c_1+c_4)t}\right)+\right.$$

$$e^{y\sqrt{\frac{1}{a_4}(c_1+c_4)}}erfc\left(\frac{y}{2\sqrt{a_1t}}+\sqrt{(c_1+c_4)t}\right)\frac{e^{c_4t}}{2}$$

$$f_5(y,t)=erfc\left(\frac{y}{2\sqrt{a_4t}}\right)$$

$$f_6(y,t)=\left[e^{-y\sqrt{\frac{c_4}{a_4}}}erfc\left(\frac{y}{2\sqrt{a_4t}}-\sqrt{c_4t}\right)+\right.$$

$$\left.+e^{y\sqrt{\frac{c_4}{a_4}}}erfc\left(\frac{y}{2\sqrt{a_4t}}+\sqrt{c_4t}\right)\right]\frac{e^{c_4t}}{2}$$

$$h_1(t)=e^{-i\omega t}\sqrt{\frac{c_1-i\omega}{a_1}}erfc\left(\sqrt{(c_1-i\omega)t}\right)+\frac{e^{-c_1t}}{\sqrt{\pi a_1t}}$$

$$h_2(t)=e^{i\omega t}\sqrt{\frac{c_1+i\omega}{a_1}}erfc\left(\sqrt{(c_1+i\omega)t}\right)+\frac{e^{-c_1t}}{\sqrt{\pi a_1t}}$$

$$h_3(t)=-\sqrt{\frac{c_1}{a_1}}erfc\left(\sqrt{c_1t}\right)+\frac{e^{-c_1t}}{\sqrt{\pi a_1t}}$$

$$h_4(t)=e^{c_1t}\sqrt{\frac{c_4+c_1}{a_1}}erfc\left(\sqrt{(c_4+c_1)t}\right)+\frac{e^{-c_1t}}{\sqrt{\pi a_1t}}$$

3. Results and Discussion

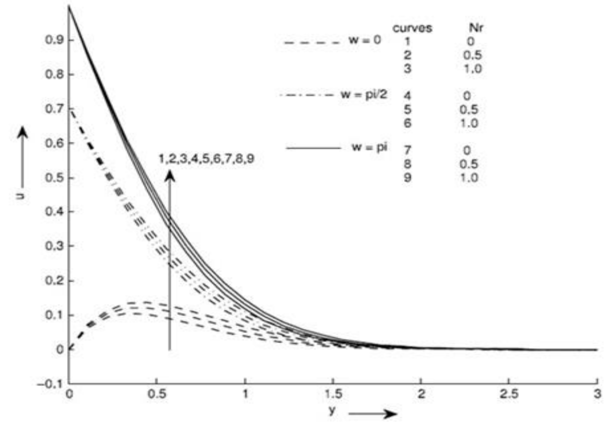

Fig. 1. Velocity profile for different values of y and Nr at $t = 05$ $M^2 = 3$ $G_r = 5$ $\emptyset = 0.15$ $p_r = 6.2$.

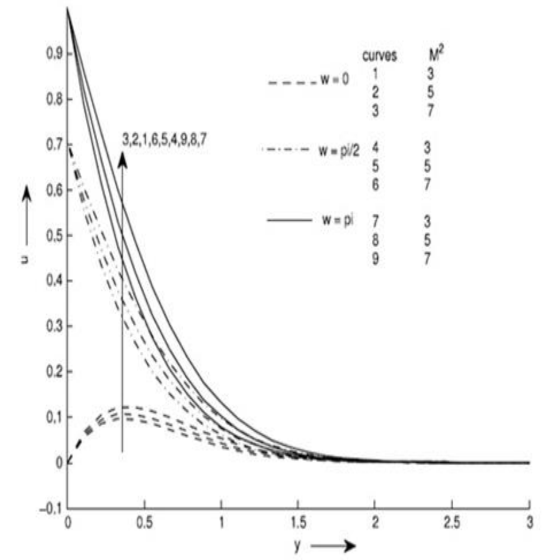

Fig. 2. Velocity profile for different values of y and M^2 at $t = 05$ $G_r = 5$ $\emptyset = 0.15$ $p_r = 6.2$ $N_r = 0.5$.

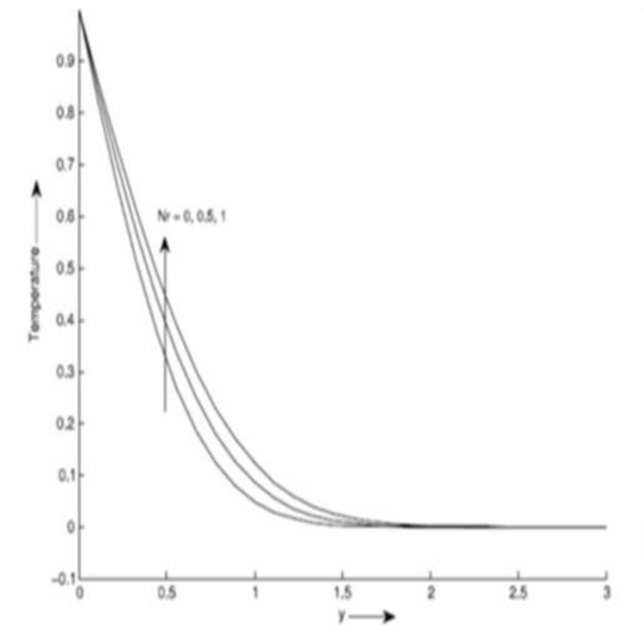

Fig3 . Temperature profile for different values of y and N_r at $M^2 = 3$ $t = 0.5$ $\emptyset = 0.15$ $G_r = 5$ $p_r = 6.2$.

Fig 1 represents the velocity profile for different values of Nr, radiation parameter and w, frequency of oscillation of the plate. The fluid velocity increases steeply near the surface of the plate. Fig.2 shows the effect of magnetic parameter M on the velocity. The amplitude of velocity as well as the boundary layer thickness decreases when M is increased. This is due to the fact that the effect of transverse magnetic field which results in Lorentz force is similar to the drag force. Fig.3 shows the effect of radiation parameter Nr on the temperature profiles. Temperature decreases with increase in radiation parameter as release of heat energy from the flow region is also increased and it results in decrease in fluid temperature.

References

1. S.U.S. Choi and J.A.Eastman, Enhancing thermal conductivity of fluids with nano particles, Mater Aci,2,199-105(1995).

2. O. D. Makinde, Computational Modelling of nanofluids flow over a convectively heated unsteady stretching sheet, Current Nano science, 9(5), 673-678(2013).

3. O. D. Makinde, W. A. Khan, ZH Khan, Buoyancy effects on MHD stagnation point flow and heat transfer of a nano fluid past a convectively heated stretching/ shrinking sheet, International Journal of Heat and Mass Transfer 62, 526-533(2013).
4. S. Nadeem, Rashid Mehmood, Noreen Sher Akbar, Partial slip effect on non-aligned stagnation point nanofluid over a stretching convective surface, Chin Phys B, 24(1)(2015).
5. S. Nadeem, Rashid Mehmood, Noreen Sher Akbar, Oblique stagnation point flow of CNT based fluid over a convective surface, J.Comput Theor Nanosci,12, 1-8(2015).
6. S.Ostrach, An analysis of laminar free-convection flow and heat transfer about a flat plate parallel to the direction of the generating body force, NACA Rep, 1111, 63-79 (1953).
7. R.Seigel, Transient free convection from a vertical flat plate, Trans Amer Soc Mech Eng, 80,347-359(1958).
8. H.F.Oztop and E.Abu-Nada, Numerical study of natural convection in partially heated rectangular enclosures filled with nano fluids,Int J Heat fluid flow,29, 1326-1336(2008).