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Construction of Magic Squares of Orders q^n ,Where q is Odd and $n \in \mathbb{N}$

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ABSTRACT

Present paper is an important study for constructing magic squares of odd orders.This magic squares have been formed using the properties of Latin squares. Here, reduced Latin square is used and it is formulated by using the cyclic shifting method with the entries $(1,2,\dots,n),(n+1,n+2,\dots,2n),\dots(n(n-1),\dots,n^2)$.This work has been generalized for magic square of order q^n, when q is odd and $n \in \mathbb{N}$, by using the mathematical formula $x \equiv y(mod\ q^n)$: $y = 1,2,\dots,q^n$. The method is tested for $3, 3^2$ and 5.

Introduction

The name "Magic Square" was inspired by the ancient Chinese literature where it was named as the Lo Shu square. It was told to have been painted on the shell of a sea turtle and was a special type of Latin square. For centuries, math enthusiasts and mathematicians alike have found the recreation mathematical topic of magic squares in nature to be an interesting and entertaining subject such as rhythm in music and very popular puzzle Soduku. [1,6]

Definition (Latin Squares)

A *Latin square* is an $n \times n$ array filled with n different symbols, each occurring exactly once in each row and exactly once in each column. An example of a 3x3 Latin square is: (Fig. 1)

1	3	2
3	2	1
2	1	3

Figure 1. Latin square of order 3.

Definition (Reduced Latin Squares)

It is said to be *reduced* (also, normalized or in standard form) if both its first row and its first column are in their natural order. (Fig. 2)

1	2	3
2	3	1
3	1	2

Figure 2.Reduced Latin square of order 3.

Definition (Magic Squares)

A *natural magic square* of order n is a square array of numbers consisting of distinct positive integers $1,2,3,\dots,n^2$ arranged such that each cell contains a different integer and the sum of the integers in each row, column, and diagonal is always the same number known as the magic constant or magic sum. The magic constant of a normal magic square with numbers depends only on n and equal to $\frac{n(n^2+1)}{2}$. A line of a magic square is any row, any column or either of the two main diagonals of the square. A magic square of order n thus has $2n+2$ such lines. [2]

Throughout our work, we concern magic squares when n is odd. Then middle cell is $\frac{(n^2+1)}{2}$. It is called as the pivot element. (Fig. 3)

4	9	2
3	5	7
8	1	6

Figure 3.Natural magic square of order 3.

Pivot element $= \frac{(3^2+1)}{2} = 5$ and Magic constant $= \frac{3(3^2+1)}{2} = 15$

Material and Methods

There is no specific method or limited algorithm to build all types of magic squares. The present work seeks for the method of creating magic squares of order q^n, when q is odd and $n \in \mathbb{N}$. There is something about the symmetry and patterns contained in such squares that carry great appeal. We shall prove the following simple and pleasing properties which are exhibited by any odd-order magic square. [4, 5]

Case I: The construction of magic square of odd orders by using basic Latin square can be expressed in the following steps:

Step1: First, arrange the reduced Latin square by using the cyclic shifting method [3] with the entries $(1,2,\dots,n),(n+1,n+2,\dots,2n),\dots(n(n-1),\dots,n^2)$. (Fig. 4) & (Fig. 5)

1	2	3	...	...	....	n-2	n-1	n
2	3	4	...	...	...	n-1	n	1
3	4	5	...	...	...	n	1	2
.	.	.			.	.	.	.
.	.	.		.		.	.	.
.	.	.	.			.	.	.
n-2	n-1	n	...	...	...	n-5	n-4	n-3
n-1	n	1	...	...	...	n-4	n-3	n-2
n	1	2	...	...	...	n-3	n-2	n-1

Figure 4. Reduced Latin square with entries 1, 2 ,...,n.

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1	2	3	...	...	...	n-2	n-1	n
n+2	n+3	n+4	...	...	...	2n-1	2n	n+1
2n+3	2n+4	2n+5	...	...	...	3n	2n+1	2n+2
.	.	.	.	.	.	.	.	.
$(n^2+1)/2$	$(n^2+3)/2$	$(n^2+5)/2$	...	$(n^2+n)/2$	...	$(n^2-5)/2$	$(n^2-3)/2$	$(n^2-1)/2$
.	.	.	.	.	.	.	.	.
n^2-2n-2	n^2-2n-1	n^2-2n	...	...	...	n^2-2n-5	n^2-2n-4	n^2-2n-3
n^2-n-1	n^2-n	n^2-2n+1	...	...	...	n^2-n-4	n^2-n-3	n^2-n-2
n^2	n^2-n+1	n^2-n+2	...	...	...	n^2-3	n^2-2	n^2-1

Figure 5.Reduced Latin square with entries 1, 2 ,...,n^2 .

$(n^2+n+4)/2$	...	n^2-1	1	n+3	...	$(n^2+n)/2$
$(n^2+3n+6)/2$	...	n	n+2	2n+4	...	$(n^2+n+2)/2$
$(n^2+5n+8)/2$	...	n+1	2n+3	3n+5	...	$(n^2+3n+4)/2$
.		.	.	.		.
.	.	.	.	$(n^2+3)/2$	.	.
.		.	$(n^2+1)/2$	.		.
.	.	$(n^2-1)/2$	.	.	.	.
.		.	.	.		.
$(n^2-3n-2)/2$	...	n^2-3n-4	n^2-2n-2	n^2-n	...	$(n^2-5n-6)/2$
$(n^2-n)/2$	...	n^2-2n-3	n^2-n-1	n^2-n+1	...	(n-3n-4)/2
$(n^2-n+2)/2$	...	n^2-n-2	n^2	2	...	$(n^2-n-2)/2$

Figure 6.Magic square with entries 1, 2,...,n^2.

Step 2: Determine the pivot element and consider the central line (The line with the pivot element)

Pivot element: $\frac{(n^2+1)}{2}$ and the central line: $\left(\left(\frac{n^2+1}{2}\right),\left(\frac{n^2+3}{2}\right),\dots,\left(\frac{n^2-3}{2}\right),\left(\frac{n^2-1}{2}\right)\right)$

Step 3: Assign this central diagonal elements by fixing the pivot element in the middle and arrange other elements in an orderly manner to give the desired magic square. (Fig.6)

Results and Discussion

In this work, we shall prove two simple results about 3×3 and 9×9 magic squares.

1. When $q=3$ and $n=1$

Reduced Latin square has been arranged by using cyclic shifting method whose entries are (1,2,3), (4,5,6), (7,8,9).

1	2	3
5	6	4
9	7	8

Here magic sum is 15, pivot element is 5 and central line is (5, 6, 4). By assigning the elements in the central line as diagonal elements and fixing the pivot element in the middle and arranging other elements in an orderly manner gives the required magic square.

8	1	6
3	5	7
4	9	2

2. When and

Consider the formulae

y	Congruent solutions for x								
1	1	10	19	28	37	46	55	64	73
2	2	11	20	29	38	47	56	65	74
3	3	12	21	30	39	48	57	66	75
4	4	13	22	31	40	49	58	67	76
5	5	14	23	32	41	50	59	68	77
6	6	15	24	33	42	51	60	69	78
7	7	16	25	34	43	52	61	70	79
8	8	17	26	35	44	53	62	71	80
9	9	18	27	36	45	54	63	72	81
	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓
	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9

Here the magic sum is 369 and the pivot element is 41.

Applying the recursive algorithm for T_i' separately, and then, applying the method taking those blocks as the entries of the Latin square, the following magic square of order $3^2\times 3^2$ can be obtained.

T_8	T_1	T_6
T_3	T_5	T_7
T_4	T_9	T_2

									369
71	64	69	8	1	6	53	46	51	369
66	68	70	3	5	7	48	50	52	369
67	72	65	4	9	2	49	54	47	369
26	19	24	44	37	42	62	55	60	369
21	23	25	39	41	43	57	59	61	369
22	27	20	40	45	38	58	63	56	369
35	28	33	80	73	78	17	10	15	369
30	32	34	75	77	79	12	14	16	369
31	36	29	76	81	74	13	18	11	369
369	369	369	369	369	369	369	369	369	369

3. When $q=5$ and $n=1$

Reduced Latin square has been arranged by using cyclic shifting method whose entries are (1,2,3,4,5), (6,7,8,9,10), (11,12,13,14,15), (16,17,18,19,20), (21,22,23,24,25).

Here magic sum is 65, pivot element is 13 and central line is (13, 14, 15, 11, 12). By assigning the elements in the central line as diagonal elements and fixing the pivot element in the middle and arranging other elements in an orderly manner gives the required magic square.

1	2	3	4	5
7	8	9	10	6
13	14	15	11	12
19	20	16	17	18
25	21	22	23	24

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Conclusion

In this work, magic squares of odd orders have been constructed using the properties of Latin squares. This work has been generalized for magic square of order q^n, when q is odd and $n\in\mathbb{N}$, by using the mathematical formula $x \equiv y(mod\ q^n): y = 1,2,\ldots,q^n$. As a future work, we are planning to automate this work so that magic squares of higher order can be constructed easily.

References

[1] Jared Weed, Applications of AI for Magic Squares

[2] Jayesh Jain, A Study of Magic Squares, 5thNational Conference on Role of Engineers in Nation Building, 2017

[3] Nishadi W.V., An Algorithm to Construct Symmetric Latin Squares of Order q n forq ≥ 2 andn ≥ 1, American Journal of Engineering Research, Volume-6, Issue-2, pp-42-50, 2017

[4]Tomba I, A Technique for Constructing Odd-order Magic Squares Using Basic Latin Squares, International Journal of Scientific and Research Publications, Volume 2, Issue 5, 2012

[5]Tomba Ia and Shibiraj Nb, Improved Technique for Constructing Doubly-even Magic Squares using Basic Latin Squares, International Journal of Scientific and Research Publications, Volume 3, Issue 6, 2013

[6]Peter Loly, Scientific Studies of Magic Squares, Department of Physics, The University of Manitoba, Winnipeg, Manitoba Canada, 2003