Available online at www.elixirpublishers.com (Elixir International Journal)

Discrete Mathematics

Elixir Dis. Math. 133 (2019) 53553-53559

ISSN: 2229-712X

A Class of New Computational Methods for the Solutions of Parabolic Equations

Akinmoladun O. M.[1], Iluno C.[2] and Obiyemi D.A[3]

Department of Mathematics &Statistics, Lagos State Polytechnic, Ikorodu, Lagos, Department of Mathematics, Osun State Polytechnic, Iree, Nigeria.

ARTICLE INFO

Article history:

Received: 18 June 2019;
Received in revised form:
15 August 2019;
Accepted: 26 August 2019;

Keywords

Trigonometric Functions,
Taylor Series Expansion,
Time,
Finite & Space Difference.

ABSTRACT

In this work, a class of new computational methods for solution of variable coefficients partial differential equation was developed at step numbers j = 2; resulting into a Trapezoidal rule spectral based computational scheme as reported in Lambert (1973). The accuracy, consistency, stability and convergence properties of these methods were determined. The methods were implemented on some sampled problems that involve both constant and, variable coefficients parabolic partial differential equations; and evaluated by comparing them with some existing difference methods. The results obtained are found to be more rapidly converging as the step lengths h and k approaches zeros. This work provides better alternative numerical solutions to a class of dynamical problems having time dependent boundary conditions. Higher ordered parabolic partial differential equations with defined theoretical solutions to given boundary conditions can be solved directly using this method.

Introduction

1.0 Preambles

This study focus on the general second order linear non-homogenous partial differential equation of the form

$$a(x,t)U_{tt}+2b(x,t)U_{xt}+c(x,t)U_{xx}=d(x,t,u,U,U_x,U_t) \quad (1.01)$$

[as reported in [Igor (2005), Aregbesola (2007)]] where a, b, c, d are constants, as either characteristics or canonical; especially for a second order equation in which the derivatives of second order all occur linearly; with coefficients only depending on the independent variables.

The characteristic equation of (1.01) is

$$\frac{dy}{dt}=\frac{b\pm\sqrt{(b^2-ac)}}{2a};$$

$$\boldsymbol{equation}\ (1.01)\ \boldsymbol{is\ Parabolic};\ \boldsymbol{if}\ b^2-ac=0;\rightarrow \boldsymbol{one\ characteristic};;$$

$$\boldsymbol{equation}\ (1.01)\ \boldsymbol{is\ called\ Hyperbolic};\ \boldsymbol{if}\ b^2-ac>0;\rightarrow \boldsymbol{two\ characteristic};;$$

$$\boldsymbol{equation}\ (1.01)\ \boldsymbol{is\ called\ Elliptic};\ \boldsymbol{if}\ b^2-ac<0;\rightarrow \boldsymbol{two\ characteristic};$$

Equation (1.01) can be written in canonical form as follows;

$U_{xx}-U_t=0$ ***is Parabolic***; (***one dimensional heat equation***)

$U_{xx}-U_{tt}=0$ ***is Hyperbolic***; (***one dimensional wave equation***)

$U_{xx}+U_{tt}=0$ ***is Elliptic***; (***two − dimensional Laplace equation***)

[as reported in [Igor(2005)]]

2.0 Methodology

In this work, the spectral based theoretical solution $U_{(x_{m+i},t_{n+j})}=aCos(x_{m+i},t_{n+j})+bSin(x_{m+i},t_{n+j}),\ i,j=0,1,2,3.$ is partly differentiated at constant time t but varying on the space (x), and substituted into a general partial differential equation of the form; $a(x,t)U_{tt}+2b(x,t)U_{xt}+c(x,t)U_{xx}=d(x,t,u,U,U_x,U_t)$leading to a general parabolic partial differential equation of the form $c(x,t)U_{xx}=d(x,t,u,U,U_x,U_t)$ and; by adopting the spectral based theoretical solution$U_{(x_{m+i},t_{n+j})}=aCos(x_{m+i},t_{n+j})+bSin(x_{m+i},t_{n+j}),\ i,j=0,1,2,3.$ to aforementioned general parabolic equation and further simplifications were performed resulting into development of a class of spectral based computational methods for solution of partial differential equations at step numbers j = 2, we obtained three distinct spectral based computational schemes as stated below;

Methods for step number j = 2

$$(i)\quad U_{(m,n+2)}=U_{(m,n)}+\frac{k}{2}[f_{(m,n+2)}+2f_{(m,n+1)}+f_{(m,n)}]\ \boldsymbol{Scheme\ 1}$$

Tele: 08035792692
E-mail address: obiyemida@gmail.com

Equation (i) is a spectral Trapezoidal Rule to the method with step number two [Lambert, 1973].

$$(ii)U_{(m+2,,n)} = 2U_{(m+1,n)} - U_{(m,n)} + \frac{h^2}{2^2}\left(g_{(n+2,t)} + 2g_{(n+1,t)} + g_{n,t}\right) \; Scheme\ 2$$

$$(iii)\ U_{\ (m,n+2)} = U_{m,n} + k[f_{x,(n+1)} + f_{x,n}] + \frac{h^2}{4}\left(g_{(n+2),t} + 2g_{(n+1),t} + g_{n,t}\right) Scheme\ 3$$

Equate (i) and (iii) to obtain a generalized spectral based method for cases in step number two as

(a) $f_{m,n+2} - f_{m,n} = \frac{h^2}{2k}\left(g_{m+2,t} + 2g_{m+1,t} + g_{m,t}\right)$

Equation (1.01) was re-defined based on the behavior of its constant coefficients as reiterated by (Ioannis P Stavroulakis et.al (2004)) as follows; taking partial derivatives of equation (1.01) with respect to x at time t constant then $a = b = 0, and\ c \neq 0;\ then;\ b^2 - ac = 0\ will\ be\ satisfied.$

Equation (1.01) becomes a general linear second order parabolic partial differential equation of the form

$$c(x,t)U_{xx} = d(x,t,u,U,U_x,U_t) \qquad (1.1)$$

$$where;\ U_t = U_t(x,t,u), U_{xx} = U_{xx}(x,t,u),$$

$$wit\square initialcondition\ 0 < x < L;\ \forall\ t \geq 0$$

$$andboundaryconditions\ , U(0,t) = U(L,t) = 0$$

$$theregionxt - planeinwhichthesolutionissoughtisdescribedbyinequality$$

$$0 \leq x \leq L;\ andt \geq 0\ and, \quad c = f(x)\forall x\epsilon(q,r), q, r, Laresetofintegers,$$

$$cisavariablecoefficient;$$

[Cheney et. al (2004), Ancona (2002), Jain (1992), William et. al (1992)]

Various authors like Akinfenwa et. al.(2011), Aregbesola (2007), Richard et. al.(2004), William et. al(1992), Jain(1992), Cheney et. Al (2004), to mention but a few; reconsidered equation (1.1) at c = f(x) = 1; as x = 1; to suit the development of their schemes before generalizing the resulting difference methods. Thus, equation (1.1) can be written as a diffusion equation of the form

$$U_t = U_{xx} \qquad (1.2)$$

$$where; U_t = U_t(x,t,u), U_{xx} = U_{xx}(x,t,u),$$

$$withnitialcondition\ \ 0 < x < L;\ \forall\ t \geq 0$$

$$andboundaryconditions, U(0,t) = U(L,t) = 0,$$

$$theregionxt - planeinwhichthesolutionissoughtisdescribedbyinequality$$

$$0 \leq x \leq L;\ andt \geq 0\ and, \quad p = f(x)\forall x\epsilon(q,r);\ q, r, Laresetofintegers,$$

$pisavariablecoefficient.$ The grid points (x_m, t_n) where $x_m = x_0 + m\square, t_n = t_0 + nk,\ \square = \frac{r-q}{m},\ r, q, m, n \in I$; a set of integers, k and h are constants step size; t is the time, and x is the distance coordinate. [Cheney et. al (2004), Ancona(2002), Jain(1992), William et. al(1992)]

3.0 Properties of the Scheme in equation (1.33)

Definition 1

In the spirits of Lambert (1991), Jain et. al (1984) and Awoyemi (2001) as emphasized in Akinmoladun et. al. (2013) that a linear multistep method of the type in equation (1.33) is said to be consistence if and only if it satisfies the following conditions $(i)\ \ the\ order\ p \geq 1 (ii) \sum_{j=0}^{k} \alpha_j = 0 (iii)\ \ \rho(r) = \rho'(r) = 0 (iv)\ \ \rho^n(1) = n!\,\partial(1);$

with the principal root r ≤ 1, $|r_s| : s = 1, 2, ,, k\ \ \forall\ |r_s| < |r_1|$, [Lambert 1991]

Consistency Test on Scheme in equation (1.33)

The order and error constant of the scheme in equation (1.33) are estimated as follows;

$$U_{\ (m,n+2)} = U_{(m,n)} + \frac{k}{2}[f_{(m,n+2)} + 2f_{(m,n+1)} + f_{(m,n)}$$

$$U_{\ (m,n+2)} - U_{(m,n)} = \frac{k}{2}[f_{(m,n+2)} + 2f_{(m,n+1)} + f_{(m,n)}]$$

$$U_{m,n+2} = \frac{(2h)^0}{0!}y_n + \frac{(2h)^1}{1!}y_n^1 + \frac{(2h)^2}{2!}y_n^2 + \frac{(2h)^3}{3!}y_n^3 + \frac{(2h)^4}{4!}y_n^4 + \frac{(2h)^5}{5!}y_n^5 + ..$$

$$U_{m,n} = y_n$$

$$\frac{1}{2}f_{\ m,n+2,} = \frac{1}{2}[\frac{(2h)^0}{0!}y_n^1 + \frac{(2h)^1}{1!}y_n^2 + \frac{(2h)^2}{2!}y_n^3 + \frac{(2h)^3}{3!}y_n^4 + \frac{(2h)^4}{4!}y_n^5 + \frac{(2h)^5}{5!}y_n^6 + \cdots]$$

$$\frac{1}{2}(2)f_{\ m,n+1,} = [\frac{(h)^0}{0!}y_n^1 + \frac{(h)^1}{1!}y_n^2 + \frac{(h)^2}{2!}y_n^3 + \frac{(h)^3}{3!}y_n^4 + \frac{(h)^4}{4!}y_n^5 + \frac{(h)^5}{5!}y_n^6 + \cdots]$$

$$\frac{1}{2}f_{\ m,n+1,,} = \frac{1}{2}y_n^1$$

By collecting the coefficients (C_is) of equal powers of y_n and simplifying; we obtain

$$C_0 = 1 - 1 = 0 C_1 = 2 - \frac{1}{2} - 1 - \frac{1}{2} = 0 C_2 = 2 - 1 - 1 = 0$$

$C_3 = \frac{4}{3} - 1 - 1 = \frac{-2}{3} = -0.66666667 \neq 0$ The order of the method is 2

$$Error\ constant\ of\ Scheme(1) = -0.66666667$$

The first and second characteristic polynomials of scheme (1) are $\rho(r)\ and\ \sigma(r)$ respectively

$\rho(r) = (r)^2 - 1$ and $\partial(r) = \frac{1}{2}(r^2 + 2r + 1)$

$for\ r = 1,\quad (1)^2 - 1 = \rho(1) = 0$(zero stability)

$$\sum_{j=0}^{k} j\alpha_j = \sum_{j=0}^{k} \beta_j\, j\alpha_j = 2\alpha_2 = 2 = \beta_j = \frac{1}{2}(1+2+1) = 2$$

$\rho'(r) = 2r$, $\rho''(r) = 2$,

$\partial(r) = \frac{1}{4}\left(r^2 + 2r + 1\right)$; $\rho''(1) = 2 = 2!\,\partial(1)$

Definition 2

A linear multistep method is convergent if and only if it is consistent and zero – stable. [Fatunla(1992)]

The Scheme in equation (1.33) is consistence, zero stable and convergent.

To affirm the stability criteria of the scheme equation (1.33); using Crank –Nicolson with matrix difference method (as reported in Cheney et.al(2004)).

$$U_{m,n+2} - U_{m,n} = \frac{k}{2}\left(f_{m,n+2,} + 2f_{m,n+1,} + f_{m,n}\right)$$

$$U(x,t+2k) - U(x,t) = \frac{k}{2}[U(x,t+2\mathrm{k}) + 2\mathrm{U}(\mathrm{x},\mathrm{t}+\mathrm{k}) + U(x,t)]$$

$$\frac{2}{k}[U(x,t+2k) - U(x,t)] = [U(x,t+2\mathrm{k}) + 2\mathrm{U}(\mathrm{x},\mathrm{t}+\mathrm{k}) + U(x,t)]$$

$Let\ S = \frac{2}{k}$; then, $S[U(x,t+2k) - U(x,t)] = [U(x,t+2\mathrm{k}) + 2\mathrm{U}(\mathrm{x},\mathrm{t}+\mathrm{k}) + U(x,t)]$

$$[SU(x,t+2k) - SU(x,t) = [U(x,t+2\mathrm{k}) + 2\mathrm{U}(\mathrm{x},\mathrm{t}+\mathrm{k}) + U(x,t)]$$

$$U(x,t+2\mathrm{k}) + 2\mathrm{U}(\mathrm{x},\mathrm{t}+\mathrm{k}) + U(x,t) + SU(x,t) = SU(x,t+2k)$$

$$U(x,t+2\mathrm{k}) + 2\mathrm{U}(\mathrm{x},\mathrm{t}+\mathrm{k}) + (S+1)U(x,t) = SU(x,t+2k),$$

$$assuming\ r = S+1, therefore,$$

$U(x,t+2\mathrm{k}) + 2\mathrm{U}(\mathrm{x},\mathrm{t}+\mathrm{k}) + rU(x,t) = SU(x,t+2k)$ (**)

On the t level, U is known but on t+2k, U is unknown. Therefore, the unknown parameter can be introduced as $U_i = U(x,\ ik) and\ known\ quantity\ to\ be\ b_i = SU(ih, t+2k)$

Equation (**) can be transform into tri - diagonal matrix difference of the form

$$\begin{pmatrix} 1, & 2 & \cdots & \cdots & r \\ 0 & 1 & 2 & & r-1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_{n-2} \\ U_{n-1} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-2} \\ b_{n-1} \end{pmatrix}$$

$AU_i = b_i$, A is upper triangular matrix $therefore,\ A^{-1}b_i = U_i\ i = 1,2,3 \ldots, 21$

k = 0.1; $S = \frac{2}{k} = \frac{2}{0.1} = 20r = S + 1 = 20 + 1 = 21$

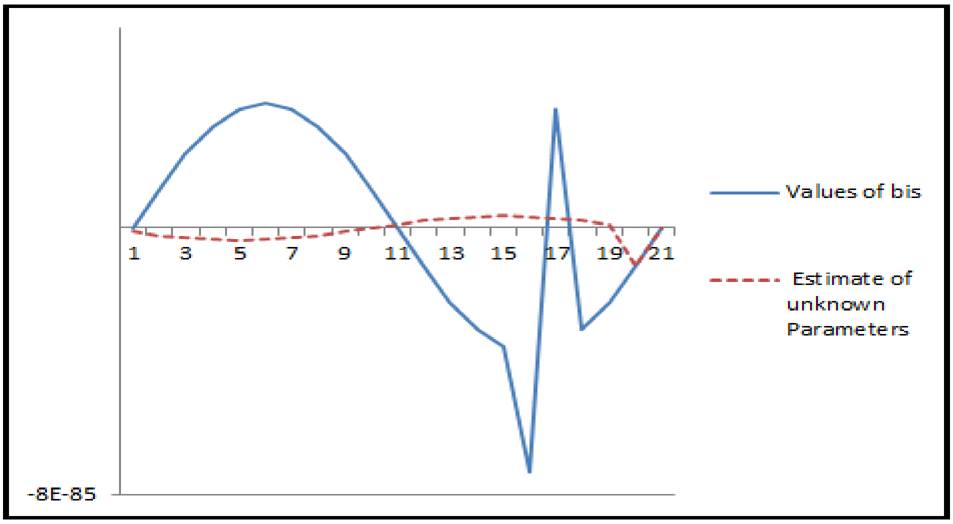
These result into 21 x 21 upper triangular matrix A as shown below.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	0	0	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	0	0	0	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	0	0	0	0	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	7	8	9	10	11	12
A=0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	`4	5	6	7	8
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

A^{-1} =

1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Values of bis	Estimate of unknown Parameters
0	-1.14E-86
1.16E-85	-2.16E-86
2.21E-85	-2.97E-86
3.04E-85	-3.50E-86
3.57E-85	-3.68E-86
3.76E-85	-3.50E-86
3.57E-85	-2.97E-86
3.04E-85	-2.16E-86
2.21E-85	-1.14E-86
1.16E-85	-1.71E-100
4.60E-101	1.14E-86
-1.16E-85	2.16E-86
-2.21E-85	2.97E-86
-3.04E-85	3.50E-86
-3.57E-85	3.68E-86
-7.33E-85	3.50E-86
3.57E-85	2.97E-86
-3.04E-85	2.16E-86
-2.21E-85	1.14E-86
-1.16E-85	-1.16E-85
-9.20E-101	-9.20E-101



The graph of r against U_i s and b_i

The points of interception of the two graphs indicate the point which satisfies the conditions for equilibrium and quasi – equilibrium of the parameters of the method in scheme (1).

The spectral based method in scheme 1a is tridiagonal and diagonal dominant because

$|r| = 1 + \frac{2k}{h^2} > 1$. This shows that the method is stable.

4.0 Implementation of the scheme on some sampled problems

Numerical Example

Consider the equation of the form

$$U(x,t)_t = U(x,t)_{xx}, \quad 0 < x < 1, \qquad 0 < t$$

With the boundary conditions $u(0,t) = u(1,t), \quad 0 < t$

And initial conditions $u(x,0) = \sin(\pi x), \quad 0 \le x \le 1$

Theoretical solution to this problem is $u(x,t) = e^{-\pi^2 t}\sin(\pi x)$

[Richard et. al. (2004), Cheney et. al.(2004) and Akinfenwa et. al (2011)]

Table 1. Comparison of the Absolute Errors of Schemes 1,2 3 with that of Richard et al(2004) and the methods proposed by Akinfenwa, Jato & Yao(2011); evaluated at zero of Chebyshev polynomial (CHMC), & a Point Chosen Symmetrically (CHMS) at h = 0.1 and k = 0.01

x_i	**Scheme 1 (Absolute Error)**	**Scheme 2 (Absolute Error)**	**Scheme 3 (Absolute Error)**	**(Akinfenwa et al (Absolute Errors) CHMC (2011)**	**Akinfenwa et al (Absolute Errors) CHMS (2011)**	**Richard et al (2004) (Absolute Error**
0.1	8.62368E-06	1.36352E-06	1.36E-06	9.35042E-04	9.35942E-04	6.756E-03
0.2	1.64082E-05	3.08959E-06	3.09E-06	6.99825E-04	7.00501E-04	1.285E-03
0.3	2.25922E-05	4.50761E-06	4.51E-06	3.92835E-04	3.93216E-04	1.769E-03
0.4	2.65563E-05	5.49278E-06	5.49E-06	1.96012E-04	1.96203E-04	2.079E-03
0.5	2.7917E-05	5.94419E-06	5.94E-06	9.16911E-04	9.17807E-04	2.186E-03
0.6	2.65563E-05	5.80241E-06	5.83E-06	4.11761E-04	4.12166E-04	2.079E-03
0.7	2.25922E-05	5.09657E-06	5.14E-06	1.79776E-04	1.79953E-04	1.769E-03
0.8	1.64082E-05	3.90022E-06	3.91E-06	7.68894E-04	7.69654E-04	1.285E-03
0.9	8.62368E-06	2.31647E-06	2.32E-06	3.23715E-04	3.23812E-04	6.756E-04
1	0	-	-	1.34607E-04	1.34647E-04	-

Discussion

Schemes 1, 2, and 3 absolute errors are of higher order as indicated in table 4 and, it implies that the results in the proposed schemes are more rapidly converging compared to that of Richard et al (2004) and that of Akinfenwa et al (2011).

Table 2. The exact solution of Problem 1 at h = 0.01, k = 0.01 and collocated at some sample points of x and t.

x\t	**0.92**	**0.93**	**0.94**	**0.95**	**0.96**	**0.97**	**0.98**	**0.99**	**1**
0.01	3.58E-06	3.24E-06	2.94E-06	2.66E-06	2.41E-06	2.18E-06	1.98E-06	1.79E-06	1.62E-06
0.05	1.78E-05	1.61E-05	1.46E-05	1.33E-05	1.20E-05	1.09E-05	9.86E-06	8.93E-06	8.09E-06
0.1	3.52E-05	3.19E-05	2.89E-05	2.62E-05	2.37E-05	2.15E-05	1.95E-05	1.76E-05	1.60E-05
0.15	5.17E-05	4.69E-05	4.25E-05	3.85E-05	3.48E-05	3.16E-05	2.86E-05	2.59E-05	2.35E-05
0.2	6.70E-05	6.07E-05	5.50E-05	4.98E-05	4.51E-05	4.09E-05	3.70E-05	3.36E-05	3.04E-05
0.25	8.06E-05	7.30E-05	6.61E-05	5.99E-05	5.43E-05	4.92E-05	4.46E-05	4.04E-05	3.66E-05
0.3	9.22E-05	8.35E-05	7.57E-05	6.85E-05	6.21E-05	5.63E-05	5.10E-05	4.62E-05	4.18E-05
0.35	0.000102	9.20E-05	8.33E-05	7.55E-05	6.84E-05	6.20E-05	5.61E-05	5.09E-05	4.61E-05
0.4	0.000108	9.82E-05	8.89E-05	8.06E-05	7.30E-05	6.61E-05	5.99E-05	5.43E-05	4.92E-05
0.45	0.000113	0.000102	9.24E-05	8.37E-05	7.58E-05	6.87E-05	6.22E-05	5.64E-05	5.11E-05
0.5	0.000114	0.000103	9.35E-05	8.47E-05	7.68E-05	6.95E-05	6.30E-05	5.71E-05	5.17E-05
0.55	0.000113	0.000102	9.24E-05	8.37E-05	7.58E-05	6.87E-05	6.22E-05	5.64E-05	5.11E-05
0.6	0.000108	9.82E-05	8.89E-05	8.06E-05	7.30E-05	6.61E-05	5.99E-05	5.43E-05	4.92E-05
0.65	0.000102	9.20E-05	8.33E-05	7.55E-05	6.84E-05	6.20E-05	5.61E-05	5.09E-05	4.61E-05
0.7	9.22E-05	8.35E-05	7.57E-05	6.85E-05	6.21E-05	5.63E-05	5.10E-05	4.62E-05	4.18E-05
0.75	8.06E-05	7.30E-05	6.61E-05	5.99E-05	5.43E-05	4.92E-05	4.46E-05	4.04E-05	3.66E-05
0.8	6.70E-05	6.07E-05	5.50E-05	4.98E-05	4.51E-05	4.09E-05	3.70E-05	3.36E-05	3.04E-05
0.85	5.17E-05	4.69E-05	4.25E-05	3.85E-05	3.48E-05	3.16E-05	2.86E-05	2.59E-05	2.35E-05
0.9	3.52E-05	3.19E-05	2.89E-05	2.62E-05	2.37E-05	2.15E-05	1.95E-05	1.76E-05	1.60E-05
0.91	3.18E-05	2.88E-05	2.61E-05	2.36E-05	2.14E-05	1.94E-05	1.76E-05	1.59E-05	1.44E-05
0.95	1.78E-05	1.61E-05	1.46E-05	1.33E-05	1.20E-05	1.09E-05	9.86E-06	8.93E-06	8.09E-06
0.96	1.43E-05	1.29E-05	1.17E-05	1.06E-05	9.62E-06	8.72E-06	7.90E-06	7.16E-06	6.48E-06
0.97	1.07E-05	9.71E-06	8.80E-06	7.97E-06	7.22E-06	6.54E-06	5.93E-06	5.37E-06	4.87E-06
0.98	7.15E-06	6.48E-06	5.87E-06	5.32E-06	4.82E-06	4.37E-06	3.96E-06	3.58E-06	3.25E-06
0.99	3.58E-06	3.24E-06	2.94E-06	2.66E-06	2.41E-06	2.18E-06	1.98E-06	1.79E-06	1.62E-06

Table 3.The numerical solutions of Scheme 1 on Problem 1 at h = 0.01, k = 0.01 and collocated at some sample points of x and t.

x\t	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1
0.01	3.56E-06	3.23E-06	2.93E-06	2.65E-06	2.40E-06	2.18E-06	1.97E-06	1.79E-06	1.62E-06
0.05	1.78E-05	1.61E-05	1.46E-05	1.32E-05	1.20E-05	1.08E-05	9.82E-06	8.90E-06	8.09E-06
0.1	3.51E-05	3.18E-05	2.88E-05	2.61E-05	2.36E-05	2.14E-05	1.94E-05	1.76E-05	1.60E-05
0.15	5.15E-05	4.67E-05	4.23E-05	3.83E-05	3.47E-05	3.15E-05	2.85E-05	2.58E-05	2.35E-05
0.2	6.67E-05	6.04E-05	5.48E-05	4.96E-05	4.49E-05	4.07E-05	3.69E-05	3.34E-05	3.04E-05
0.25	8.02E-05	7.27E-05	6.59E-05	5.97E-05	5.41E-05	4.90E-05	4.44E-05	4.02E-05	3.66E-05
0.3	9.18E-05	8.32E-05	7.54E-05	6.83E-05	6.19E-05	5.60E-05	5.08E-05	4.60E-05	4.18E-05
0.35	0.000101	9.16E-05	8.30E-05	7.52E-05	6.81E-05	6.17E-05	5.59E-05	5.07E-05	4.61E-05
0.4	0.000108	9.78E-05	8.86E-05	8.03E-05	7.27E-05	6.59E-05	5.97E-05	5.41E-05	4.92E-05
0.45	0.000112	0.000102	9.20E-05	8.34E-05	7.55E-05	6.84E-05	6.20E-05	5.62E-05	5.11E-05
0.5	0.000113	0.000103	9.31E-05	8.44E-05	7.65E-05	6.93E-05	6.28E-05	5.69E-05	5.17E-05
0.55	0.000112	0.000102	9.20E-05	8.34E-05	7.55E-05	6.84E-05	6.20E-05	5.62E-05	5.11E-05
0.6	0.000108	9.78E-05	8.86E-05	8.03E-05	7.27E-05	6.59E-05	5.97E-05	5.41E-05	4.92E-05
0.65	0.000101	9.16E-05	8.30E-05	7.52E-05	6.81E-05	6.17E-05	5.59E-05	5.07E-05	4.61E-05
0.7	9.18E-05	8.32E-05	7.54E-05	6.83E-05	6.19E-05	5.60E-05	5.08E-05	4.60E-05	4.18E-05
0.75	8.02E-05	7.27E-05	6.59E-05	5.97E-05	5.41E-05	4.90E-05	4.44E-05	4.02E-05	3.66E-05
0.8	6.67E-05	6.04E-05	5.48E-05	4.96E-05	4.49E-05	4.07E-05	3.69E-05	3.34E-05	3.04E-05
0.85	5.15E-05	4.67E-05	4.23E-05	3.83E-05	3.47E-05	3.15E-05	2.85E-05	2.58E-05	2.35E-05
0.9	3.51E-05	3.18E-05	2.88E-05	2.61E-05	2.36E-05	2.14E-05	1.94E-05	1.76E-05	1.60E-05
0.91	3.17E-05	2.87E-05	2.60E-05	2.35E-05	2.13E-05	1.93E-05	1.75E-05	1.59E-05	1.44E-05
0.95	1.78E-05	1.61E-05	1.46E-05	1.32E-05	1.20E-05	1.08E-05	9.82E-06	8.90E-06	8.09E-06
0.96	1.42E-05	1.29E-05	1.17E-05	1.06E-05	9.58E-06	8.68E-06	7.87E-06	7.13E-06	6.48E-06
0.97	1.07E-05	9.68E-06	8.77E-06	7.94E-06	7.20E-06	6.52E-06	5.91E-06	5.35E-06	4.87E-06
0.98	7.13E-06	6.46E-06	5.85E-06	5.30E-06	4.80E-06	4.35E-06	3.94E-06	3.57E-06	3.25E-06
0.99	3.56E-06	3.23E-06	2.93E-06	2.65E-06	2.40E-06	2.18E-06	1.97E-06	1.79E-06	1.62E-06

Table 4.The computed absolute errors of Scheme 1 on Problem 1 at h = 0.01, k = 0.01 and collocated at some sample points of x and t.

x\t	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1
0.01	1.39E-08	1.26E-08	1.14E-08	1.03E-08	9.36E-09	8.48E-09	7.68E-09	6.96E-09	6.31E-09
0.05	6.92E-08	6.27E-08	5.68E-08	5.15E-08	4.66E-08	4.22E-08	3.83E-08	3.47E-08	3.14E-08
0.1	1.37E-07	1.24E-07	1.12E-07	1.02E-07	9.21E-08	8.34E-08	7.56E-08	6.85E-08	6.21E-08
0.15	2.01E-07	1.82E-07	1.65E-07	1.49E-07	1.35E-07	1.23E-07	1.11E-07	1.01E-07	9.12E-08
0.2	2.60E-07	2.36E-07	2.13E-07	1.93E-07	1.75E-07	1.59E-07	1.44E-07	1.30E-07	1.18E-07
0.25	3.13E-07	2.83E-07	2.57E-07	2.33E-07	2.11E-07	1.91E-07	1.73E-07	1.57E-07	1.42E-07
0.3	3.58E-07	3.24E-07	2.94E-07	2.66E-07	2.41E-07	2.18E-07	1.98E-07	1.79E-07	1.62E-07
0.35	3.94E-07	3.57E-07	3.23E-07	2.93E-07	2.66E-07	2.41E-07	2.18E-07	1.97E-07	1.79E-07
0.4	4.21E-07	3.81E-07	3.45E-07	3.13E-07	2.83E-07	2.57E-07	2.33E-07	2.11E-07	1.91E-07
0.45	4.37E-07	3.96E-07	3.59E-07	3.25E-07	2.94E-07	2.67E-07	2.42E-07	2.19E-07	1.98E-07
0.5	4.42E-07	4.01E-07	3.63E-07	3.29E-07	2.98E-07	2.70E-07	2.45E-07	2.22E-07	2.01E-07
0.55	4.37E-07	3.96E-07	3.59E-07	3.25E-07	2.94E-07	2.67E-07	2.42E-07	2.19E-07	1.98E-07
0.6	4.21E-07	3.81E-07	3.45E-07	3.13E-07	2.83E-07	2.57E-07	2.33E-07	2.11E-07	1.91E-07
0.65	3.94E-07	3.57E-07	3.23E-07	2.93E-07	2.66E-07	2.41E-07	2.18E-07	1.97E-07	1.79E-07
0.7	3.58E-07	3.24E-07	2.94E-07	2.66E-07	2.41E-07	2.18E-07	1.98E-07	1.79E-07	1.62E-07
0.75	3.13E-07	2.83E-07	2.57E-07	2.33E-07	2.11E-07	1.91E-07	1.73E-07	1.57E-07	1.42E-07
0.8	2.60E-07	2.36E-07	2.13E-07	1.93E-07	1.75E-07	1.59E-07	1.44E-07	1.30E-07	1.18E-07
0.85	2.01E-07	1.82E-07	1.65E-07	1.49E-07	1.35E-07	1.23E-07	1.11E-07	1.01E-07	9.12E-08
0.9	1.37E-07	1.24E-07	1.12E-07	1.02E-07	9.21E-08	8.34E-08	7.56E-08	6.85E-08	6.21E-08
0.91	1.23E-07	1.12E-07	1.01E-07	9.18E-08	8.31E-08	7.53E-08	6.82E-08	6.18E-08	5.60E-08
0.95	6.92E-08	6.27E-08	5.68E-08	5.15E-08	4.66E-08	4.22E-08	3.83E-08	3.47E-08	3.14E-08
0.96	5.54E-08	5.02E-08	4.55E-08	4.12E-08	3.73E-08	3.38E-08	3.07E-08	2.78E-08	2.52E-08
0.97	4.16E-08	3.77E-08	3.42E-08	3.10E-08	2.80E-08	2.54E-08	2.30E-08	2.09E-08	1.89E-08
0.98	2.78E-08	2.52E-08	2.28E-08	2.07E-08	1.87E-08	1.70E-08	1.54E-08	1.39E-08	1.26E-08
0.99	1.39E-08	1.26E-08	1.14E-08	1.03E-08	9.36E-09	8.48E-09	7.68E-09	6.96E-09	6.31E-09

5.0 Conclusion

The tables depicted above shows that the results obtained are found to be more rapidly converging as the step lengths h and k approaches zeros.

6.0 Recommendation

This work will provide better numerical solutions to a class of dynamical problems having time dependent boundary conditions.

Matlab Code

```
a=0;        %% initial value
b=0.9;        %% a < x < b
c=0.05;        %% a < t < c
h = 0.1;     %% interval along x
```

```
m=10;   %% number of points along x
k=0.01;       %% interval long t
n=6;  %% number of points along t
for j = 2:n;
for i = 2:m;
          x(i)= a + (i-1) * h;
          t(j)= a + (j-1) * k;
          u(i,j) =((6/(factorial(7)))*(x(i))^7)*cos(t(j));
          f(i,j)  =  -1*((6/(factorial(7)))*(x(i))^7)*sin(t(j));
          g(i,j)= ((6/(factorial(6)))*(x(i))^6)*cos(t(j));
          g1(i,j)= ((6/(factorial(5)))*(x(i))^5)*cos(t(j));
          g2(i,j)= ((6/(factorial(4)))*(x(i))^4)*cos(t(j));
          g3(i,j)= ((6/(factorial(3)))*(x(i))^3)*cos(t(j));
          g4(i,j)= ((6/(factorial(2)))*(x(i))^2)*cos(t(j));
          f1(i,j)  =  -1*((6/(factorial(7)))*(x(i))^7)*cos(t(j));
          f2(i,j)  =  ((6/(factorial(7)))*(x(i))^7)*sin(t(j));
          f3(i,j)  =  ((6/(factorial(7)))*(x(i))^7)*cos(t(j));
          f3(i,j)  =  -1*((6/(factorial(7)))*(x(i))^7)*sin(t(j));
          u1(i,j) = u(i,j) + (((k/2)*(f2(i,j)+ 2*f1(i,j)+ f(i,j))))/10^4;
          error(i,j) = abs(u(i,j)-u1(i,j));
end
end
plot(x(2:m), u(2:m,2),'-', x(2:m), u1(2:m,2),':'); xlabel('x');
ylabel('U(x,t)'); title('Plot of ((6/7!)*x^7)*cos(t) at t = 0.01')
```

References

1. Ademiluyi R. A. and Kayode S. J., ‘Maximum Order Second Derivative Hybrid Multistep Methods for Integration of IVP in Ordinary Differential Equations. ’’Journal of Nigerian Association of Mathematical Physics Vol. 5, pp 254-262, 2001.

2. Akinmoladun O. M., ‘Numerical Methods for the Solutions of IVP with oscillatory solutions’’ M. Tech. Thesis (Industrial Mathematics) FUTA, Nigeria. (Unpublished), 2011.

3. Akinmoladun O. M. , Ademiluyi R.A., Abdrasid A. A. and D.A. Farinde , “Solutions of Second Order ODE with periodic solutions” published in International Journal for Science & Engineering Reseach (IJSER), USA. Vol. 4 issue 9, Sept. 2013, ISSN 2229 – 5518; pp 2604-2612.

4. Akinmoladun, O. M. Ademiluyi R. A. and Kayode S. J., ‘Spectral Computational Based Methods For Solution of Second Order Partial Differential Equations’’ published in International Journal of Science & Technology,(IJST), UK PUBLICATION, LONDON, Vol. 4, No. 4, April, 2015. ISSN 2049 -7318. pp 178-189.

5. Akinfenwa O. A, Jato S. N. and Yao N. M.,‘A Continuous Hybrid Method for Solving Parabolic Partial Differential Equation’ AMSE Journal, July,, 2011.

6. Ancona M. G., ‘’ Computational Method For Applied Science & Engineering: An Interactive Approach’’ Rinto Press Inc, USA, 2002, pp 299 - 351

7. Aregbesola Y.A.S.,‘Partial Differential Equation’ (Lecture note on MTH 702), Postgraduate Student Programme, Lautech, Ogbomoso (Unpublished), 2009.

8. Cheney Ward and David Kincaid,‘Numerical Mathematics and Computing’ Fifth edition,, published by Brooks/Cole-Thumson Learning, USA, pp 618 – 643,2004.

9. Fatunla S. O., ‘Parallel Methods for second order ODE’s’’ Computational Ordinary Differential Equations proceedings of comp. conference (eds) pp 87-99, 1992.

10. Ioannis P. Stavroulakis and Stepan A. Tersian, ‘ Partial Differential Equations’ (Second Edition) – An Introduction with Mathematica and Maple – World Scientific – Printed in Singapore. ISBN 981 – 238 – 815x, pp 50 – 52, 2004.

11. Igor Yanousky, ‘Partial Differential Equations: Graduate Level Problems and Solutions’. (Copy on Net) pp 14 – 19, 2005.

12. Jain et al., ‘Numerical Solutions of Differential Equations’, WILEY EASTERN LIMITED, New Delhi, pp 215, 1984.

13. Jain M. K., ‘Numerical Solutions of Differential Equations’, pp 251- 352, 1992.

14. Lambert J.D., ‘Computational Methods in Ordinary Differential Equation’ John-Wiley and SONS Inc. New York, pp249, 1973.

15. Trefethen, L. N., ‘Spectra Method in MATLAB’’ [SIAM, Philadelphia],2000.

16. Richard L, Burden J. and Douglass Faires : ‘Numerical Analysis’.Third Edition, pp563 – 570, 2004.