Available online at www.elixirpublishers.com (Elixir International Journal)

Advanced Mathematics

Elixir Adv. Math. 143 (2020) 54403-54407

ISSN: 2229-712X

Lagrangian Mechanics On (2, 0)-Jet Bundles

Ibrahim Yousif .I. Abad alrhman and Gebreel Mohammed khur Baba Gebreel
Department of Mathematics, Faculty of Education ,West Kordufan.University , Sudan.

ARTICLE INFO

Article history:
Received: 26 March 2020;
Received in revised form:
25 May 2020;
Accepted: 5 June 2020;

Keywords
Jet bundle,
Holomorphic bundle,
Complex ,
Lagrangian Dynamics.

ABSTRACT
In this study, we concluded the Lagrangian equations on $(J^{(2,0)}\mathcal{M}, \phi_L, \xi)$,being a model. Finally introduce, some geometrical and physical results on the related mechanic systems have been discussed.

© 2020 Elixir All rights reserved.

1. Introduction

Differential geometry is a branch of engineering concerned with the study of geometric shapes, in particular the curves, surfaces and envelopes of the families of curves and surfaces in the Euclidean and Chalcidice spheres. The first method of analysis is the differential calculus. The focus is especially on the differential characteristics of geometric shapes**.** which are the immutable characteristics of motion.

The emergence of differential geometry has been closely related to the emergence and evolution of the concept of coordinates, tangles, curves, and spaces. Here, the topology and the Li groups and the so-called geometrical structures have replaced the curves and surfaces that were the basic themes of classical differential geometry. Differential geometry is an important application in many branches of mathematical science And the basic and on top of classical mechanics (theory) and the theory of relativity and the theory of differential equations

Therefore, the equations of Eeller - Lagrange is one of the most important applications of classical mechanics. In this paper we will address the equations of Lagrangian Dynamics on (2,0)-jet bundles

2. The geometry of holomorphic $J^{(2,0)}\mathcal{M}$ bundles

Definition2.1 [1]

Let $\mathcal{M}$ be a complex manifold, $T_c\mathcal{M} = \acute{T}\mathcal{M} \oplus \acute{\acute{T}}\mathcal{M}$, the complexified tangent bundle of $(1,0)$- and of $(0,1)$-type vectors, respectively. If $(z^i)_{i=\overline{1;n}}$ are complex coordinates, then $\acute{T}_z\,\mathcal{M}$ is spanned by $\left\{\frac{\partial}{\partial z^i}\right\}_{i=\overline{1;n}}$ and $\acute{\acute{T}}_z\mathcal{M}$ is spanned by $\left\{\frac{\partial}{\partial \bar{z}^i}\right\}_{i=\overline{1;n}}$ moreover $\acute{T}\mathcal{M}$ is a holomorphic vector bundle

let $Z = (z^i, X^i = \eta^{i(1)} = \frac{dz^i}{d\theta}, Y^i = \eta^{i(2)} = \frac{d^2z^i}{d\theta^2})$ be local complex coordinates in the chart $(U;\psi)$ from $J^{(2,0)}\mathcal{M}$;

we shall the following notations

$$Z = \left(z^i, x^i = \eta^{i(1)}, y^i = \eta^{i(2)}\right) = (z^i, X^i, Y^i) \qquad (1)$$

Theorem 2.2 [1]

A local basis in $\acute{T}_z\left(J^{(2,0)}\mathcal{M}\right)$is $\left\{\frac{\partial}{\partial z^i}, \frac{\partial}{\partial x^i}, \frac{\partial}{\partial y^i}\right\}_{i=\overline{1;n}}$ and in $\acute{\acute{T}}_z\left(J^{(2,0)}\mathcal{M}\right)$theirs conjugates $\left\{\frac{\partial}{\partial \bar{z}^i}, \frac{\partial}{\partial \bar{x}^i}, \frac{\partial}{\partial \bar{y}^i}\right\}_{i=\overline{1;n}}$: Due to holomorphic changes on $J^{(2,0)}\mathcal{M}$, that is all of $\frac{\partial \acute{z}^i}{\partial \bar{z}^j}, \frac{\partial \acute{x}^i}{\partial \bar{z}^j}, \frac{\partial \acute{y}^i}{\partial \bar{z}^j}, \frac{\partial \acute{x}^i}{\partial \bar{x}^j}, \frac{\partial \acute{y}^i}{\partial \bar{x}^j}, \frac{\partial \acute{y}^i j}{\partial \bar{y}^j}$ are vanishing, and also theirs conjugates, it follows that local bases from $\acute{T}_z\left(J^{(2,0)}\mathcal{M}\right)$ change w.r.t. the transformations by the rules:

$$\frac{\partial}{\partial z^j} = \frac{\partial \acute{z}^i}{\partial z^j}\frac{\partial}{\partial \acute{z}^i} + \frac{\partial \acute{x}^i}{\partial z^j}\frac{\partial}{\partial \acute{x}^i} + \frac{\partial \acute{y}^i}{\partial z^j}\frac{\frac{\partial}{\partial \acute{y}^i}}{\Rightarrow}$$

$$\frac{\partial}{\partial x^j} = \frac{\partial \acute{x}^i}{\partial z^j}\frac{\partial}{\partial \acute{x}^i} + \frac{\partial \acute{y}^i}{\partial z^j}\frac{\partial}{\partial \acute{y}^i}$$

Tele:
E-mail address: iyibrahimi@gmail.com

© 2020 Elixir All rights reserved

$$\frac{\partial}{\partial x^j} = \frac{\partial \acute{y}^i}{\partial z^j}\frac{\partial}{\partial \acute{y}^i} \qquad (2)$$

Infer that $\frac{\partial \acute{z}^i}{\partial z^j} = \frac{\partial \acute{x}^i}{\partial x^j} = \frac{\partial \acute{y}^i}{\partial y^j}$ but in change $\frac{\partial \acute{z}^i}{\partial z^j} = \frac{\partial \acute{x}^i}{\partial x^j}$ contain the second order derivatives of $\acute{z}^i$. while $\frac{\partial \acute{x}^i}{\partial z^j}$contains even the 3-th derivatives of $\acute{z}^i$

Theorem 2.3

On $T_c\left(J^{(2,0)}\mathcal{M}\right)$ the natural complex structure $J^2 = -I$ acts as follows:

$$J\left(\frac{\partial}{\partial z^j}\right) = i\frac{\partial}{\partial z^j} \quad , \quad J\left(\frac{\partial}{\partial x^j}\right) = i\frac{\partial}{\partial x^j} \quad , \quad J\left(\frac{\partial}{\partial y^j}\right) = i\frac{\partial}{\partial y^j}$$

$$J\left(\frac{\partial}{\partial \bar{z}^j}\right) = -i\frac{\partial}{\partial z^j} \quad , \quad J\left(\frac{\partial}{\partial \bar{x}^j}\right) = -i\frac{\partial}{\partial x^j} \quad , \quad J\left(\frac{\partial}{\partial \bar{y}^j}\right) = -i\frac{\partial}{\partial y^j} \qquad (3)$$

Definition.**2. 4** [3] A Symplectic structure on an even dimensional manifold $\mathcal{M}$ is a 2-form ω on $\mathcal{M}$ satisfying

(i) $d\omega = 0$, i.e., ω is a closed form.

(ii) ω is non degenerate

$$Lemma\ 2.5[2]\ If\ \omega, \theta\ and\ k-form\ berespectively\ then$$

(i) $d\omega \wedge d\,\theta = -d\,\theta \wedge d\omega$

(ii) $d^2 = d \circ d = 0$

(iii) $d(\omega \wedge \psi) = d\omega \wedge \psi + (-1)^k d\psi \wedge \omega$

Lemma 2.6 [2]

Suppose $(\mathbf{U}, x_1,\ldots, x_n)$ is a chart on a manifold. Then $\left(\frac{\partial x^j}{\partial x^i}\right) = \delta^i_j = \begin{cases} 1 & , i = j \\ 0 & , i \neq j \end{cases} \qquad (4)$

Definition 2.7 An exterior differentiation or exterior derivative on a manifold $\mathcal{M}$ is an R-linear map

$$\mathbf{d} : \Omega^*(\mathcal{M}) \to \Omega^*(\mathcal{M})$$

Then the exterior derivative of ω is $(\mathbf{k}+\mathbf{1})-\mathbf{form}$ given by

$$d\omega = \sum_{i=1}^{m} d(\omega_{i_1,\ldots,i_k}) dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^{i_k} \quad Where\ k > 0 \qquad (5)$$

3. Lagrangian Dynamical Systems

In this section, we obtain complex Hamiltonian equations for classical mechanics structured on momentum space $T_c\left(J^{(2,0)}\mathcal{M}\right)$ that is 2m- dimensional tangent bundle of an m-dimensional configuration manifold $\mathcal{M}$

Definition 3.1

Let map $L: T\mathcal{M} \to \mathcal{M}$ such that

$$L = T - P \qquad (6)$$

The Lagrangian function ,where we find that

T= Kinetic energy , P = Potential energy

Definition 3.2 A given configuration manifold. If $\mathcal{M}$ is an m-dimensional configuration manifold and $\mathbf{L}: \mathbf{T}\mathcal{M} \longrightarrow \mathcal{M}$ is a regular Lagrangian function, then there is a unique vector field ξ_L on $\mathbf{T}\mathcal{M}$ such that dynamical equations

$$i_{\xi_L}\phi_L = dE_L \qquad (7)$$

where $\phi_L$is the symplectic form and E_L is the energy associated to $\mathbf{L}$

Let J be an almost complex structure on the $T_c\mathcal{M}$ and $(z^i, \bar{z}^i, x^i, \bar{x}^i, y^i, \bar{y}^i)$ its complex coordinates.. Assume to be semispray to the vector field ξ given as:

$$\xi = \xi_L = \xi^i\frac{\partial}{\partial z^i} + \bar{\xi}^i\frac{\partial}{\partial \bar{z}^i} + \eta^i\frac{\partial}{\partial x^i} + \bar{\eta}^i\frac{\partial}{\partial \bar{x}^i} + \zeta^i\frac{\partial}{\partial y^i} + \bar{\zeta}^i\frac{\partial}{\partial \bar{y}^i} \qquad (8)$$

$$\xi^i = \dot{z}^i = \bar{z}^i\ , \bar{\xi}^i = \dot{\xi}^i = \ddot{z}^i = \dot{\bar{z}}^i$$

$$\eta^i = \dot{x}^i = \bar{x}^i\ , \bar{\eta}^i = \dot{\eta}^i = \ddot{x}^i = \dot{\bar{x}}^i$$

$$\zeta^i = \dot{y}^i = \bar{y}^i\ , \bar{\zeta}^i = \dot{\zeta}^i = \ddot{y}^i = \dot{\bar{y}}^i$$

The vector field determined by

$$V = J\xi_L = J\left(\xi^i\frac{\partial}{\partial z^i} + \bar{\xi}^i\frac{\partial}{\partial \bar{z}^i} + \eta^i\frac{\partial}{\partial x^i} + \bar{\eta}^i\frac{\partial}{\partial \bar{x}^i} + \zeta^i\frac{\partial}{\partial y^i} + \bar{\zeta}^i\frac{\partial}{\partial \bar{y}^i}\right)$$

$$V = J\xi_L = J\xi^i\frac{\partial}{\partial z^i} + J\bar{\xi}^i\frac{\partial}{\partial \bar{z}^i} + J\eta^i\frac{\partial}{\partial x^i} + J\bar{\eta}^i\frac{\partial}{\partial \bar{x}^i} + J\zeta^i\frac{\partial}{\partial y^i} + J\bar{\zeta}^i\frac{\partial}{\partial \bar{y}^i}$$

$$V = J\xi_L = \xi^i J\left(\frac{\partial}{\partial z^i}\right) + \bar{\xi}^i J\left(\frac{\partial}{\partial \bar{z}^i}\right) + \eta^i J\left(\frac{\partial}{\partial x^i}\right) + \bar{\eta}^i J\left(\frac{\partial}{\partial \bar{x}^i}\right) + \zeta^i J\left(\frac{\partial}{\partial y^i}\right) + \bar{\zeta}^i J\left(\frac{\partial}{\partial \bar{y}^i}\right)$$

$$V = J\xi_L = i\xi^i\frac{\partial}{\partial z^i} - i\bar{\xi}^i\frac{\partial}{\partial \bar{z}^i} + i\eta^i\frac{\partial}{\partial x^i} - i\bar{\eta}^i\frac{\partial}{\partial \bar{x}^i} + i\zeta^i\frac{\partial}{\partial y^i} - i\bar{\zeta}^i\frac{\partial}{\partial \bar{y}^i} \qquad (9)$$

is called Liouville vector field on the complex manifold $J^{(2,0)}\mathcal{M}$. The closed 2-form given by $\phi_L = -dd_J L$ such that

$$d_J = i\frac{\partial}{\partial z^i} - i\frac{\partial}{\partial \bar{z}^i} + i\frac{\partial}{\partial x^i} - i\frac{\partial}{\partial \bar{x}^i} + i\frac{\partial}{\partial y^i} - i\frac{\partial}{\partial \bar{y}^i} : \mathcal{F}(J^{(2,0)}\mathcal{M}) \longrightarrow \wedge^1 T(J^{(2,0)}\mathcal{M}) \qquad (10)$$

Or

$$d_J L = \left(i\frac{\partial}{\partial z^i} - i\frac{\partial}{\partial \bar{z}^i} + i\frac{\partial}{\partial x^i} - i\frac{\partial}{\partial \bar{x}^i} + i\frac{\partial}{\partial y^i} - i\frac{\partial}{\partial \bar{y}^i}\right)L$$

$$d_J L = i\frac{\partial L}{\partial z^i} - i\frac{\partial L}{\partial \bar{z}^i} + i\frac{\partial L}{\partial x^i} - i\frac{\partial L}{\partial \bar{x}^i} + i\frac{\partial L}{\partial y^i} - i\frac{\partial L}{\partial \bar{y}^i} \tag{11}$$

$$-dd_J L = -d\left(i\frac{\partial L}{\partial z^i} - i\frac{\partial L}{\partial \bar{z}^i} + i\frac{\partial L}{\partial x^i} - i\frac{\partial L}{\partial \bar{x}^i} + i\frac{\partial L}{\partial y^i} - i\frac{\partial L}{\partial \bar{y}^i}\right)$$

$$\phi_L = -dd_J L = -d\left(i\frac{\partial L}{\partial z^i} - i\frac{\partial L}{\partial \bar{z}^i} + i\frac{\partial L}{\partial x^i} - i\frac{\partial L}{\partial \bar{x}^i} + i\frac{\partial L}{\partial y^i} - i\frac{\partial L}{\partial \bar{y}^i}\right)$$

$$\phi_L = -d\left(i\frac{\partial L}{\partial z^i} - i\frac{\partial L}{\partial \bar{z}^i}\right) - d\left(i\frac{\partial L}{\partial x^i} - i\frac{\partial L}{\partial \bar{x}^i}\right) - d\left(i\frac{\partial L}{\partial y^i} - i\frac{\partial L}{\partial \bar{y}^i}\right)$$

is found to be

$$\begin{aligned}\phi_L = {} & i\frac{\partial^2 L}{\partial z^j \partial z^i} dz^i \wedge dz^j + i\frac{\partial^2 L}{\partial \bar{z}^j \partial z^i} dz^i \wedge d\bar{z}^j + i\frac{\partial^2 L}{\partial z^j \partial \bar{z}^i} dz^j \wedge d\bar{z}^i + i\frac{\partial^2 L}{\partial \bar{z}^j \partial \bar{z}^i} d\bar{z}^j \wedge d\bar{z}^i \\ & + i\frac{\partial^2 L}{\partial x^j \partial x^i} dx^i \wedge dx^j + i\frac{\partial^2 L}{\partial \bar{x}^j \partial x^i} dx^i \wedge d\bar{x}^j + i\frac{\partial^2 L}{\partial x^j \partial \bar{x}^i} dx^j \wedge d\bar{x}^i + i\frac{\partial^2 L}{\partial \bar{x}^j \partial \bar{x}^i} d\bar{x}^j \wedge d\bar{x}^i \\ & + i\frac{\partial^2 L}{\partial y^j \partial y^i} dy^i \wedge dy^j + i\frac{\partial^2 L}{\partial \bar{y}^j \partial y^i} dy^i \wedge d\bar{y}^j + i\frac{\partial^2 L}{\partial y^j \partial \bar{y}^i} dy^j \wedge d\bar{y}^i + i\frac{\partial^2 L}{\partial \bar{y}^j \partial \bar{y}^i} d\bar{y}^j \wedge d\bar{y}^i \end{aligned} \tag{12}$$

$i_\lambda(\emptyset_L) = \emptyset_L(\lambda) = \emptyset_L(\xi)$

$$\begin{aligned} = {} & (i\frac{\partial^2 L}{\partial z^j \partial z^i} dz^i \wedge dz^j + i\frac{\partial^2 L}{\partial \bar{z}^j \partial z^i} dz^i \wedge d\bar{z}^j + i\frac{\partial^2 L}{\partial z^j \partial \bar{z}^i} dz^j \wedge d\bar{z}^i + i\frac{\partial^2 L}{\partial \bar{z}^j \partial \bar{z}^i} d\bar{z}^j \wedge d\bar{z}^i \\ & + i\frac{\partial^2 L}{\partial x^j \partial x^i} dx^i \wedge dx^j + i\frac{\partial^2 L}{\partial \bar{x}^j \partial x^i} dx^i \wedge d\bar{x}^j + i\frac{\partial^2 L}{\partial x^j \partial \bar{x}^i} dx^j \wedge d\bar{x}^i + i\frac{\partial^2 L}{\partial \bar{x}^j \partial \bar{x}^i} d\bar{x}^j \wedge d\bar{x}^i \\ & + i\frac{\partial^2 L}{\partial y^j \partial y^i} dy^i \wedge dy^j + i\frac{\partial^2 L}{\partial \bar{y}^j \partial y^i} dy^i \wedge d\bar{y}^j + i\frac{\partial^2 L}{\partial y^j \partial \bar{y}^i} dy^j \wedge d\bar{y}^i \\ & + i\frac{\partial^2 L}{\partial \bar{y}^j \partial \bar{y}^i} d\bar{y}^j \wedge d\bar{y}^i)\left(i\xi^i\frac{\partial}{\partial z^i} - i\bar{\xi}^i\frac{\partial}{\partial \bar{z}^i} + i\eta^i\frac{\partial}{\partial x^i} - i\bar{\eta}^i\frac{\partial}{\partial \bar{x}^i} + i\zeta^i\frac{\partial}{\partial y^i} - i\bar{\zeta}^i\frac{\partial}{\partial \bar{y}^i}\right)\end{aligned}$$

Let ξ be the semispray given by (7) and

$$\begin{aligned} i_\xi\phi_L = {} & i\xi^i\frac{\partial^2 L}{\partial z^j \partial z^i} dz^j - i\xi^i\frac{\partial^2 L}{\partial z^j \partial z^i}\delta_i^j dz^i + i\xi^i\frac{\partial^2 L}{\partial \bar{z}^j \partial z^i} d\bar{z}^j - i\bar{\xi}^i\frac{\partial^2 L}{\partial \bar{z}^j \partial z^i}\delta_i^j dz^i + i\xi^i\frac{\partial^2 L}{\partial z^j \partial \bar{z}^i}\delta_i^j d\bar{z}^i - i\bar{\xi}^i\frac{\partial^2 L}{\partial z^j \partial \bar{z}^i} dz^j \\ & + i\bar{\xi}^i\frac{\partial^2 L}{\partial \bar{z}^j \partial \bar{z}^i}\delta_i^j d\bar{z}^i - i\bar{\xi}^i\frac{\partial^2 L}{\partial \bar{z}^j \partial \bar{z}^i} d\bar{z}^j \\ & + i\eta^i\frac{\partial^2 L}{\partial x^j \partial x^i} dx^j - i\eta^i\frac{\partial^2 L}{\partial x^j \partial x^i}\delta_i^j dx^i + i\eta^i\frac{\partial^2 L}{\partial \bar{x}^j \partial x^i} d\bar{x}^j - i\bar{\eta}^i\frac{\partial^2 L}{\partial \bar{x}^j \partial x^i}\delta_i^j dx^i + i\eta^i\frac{\partial^2 L}{\partial x^j \partial \bar{x}^i}\delta_i^j d\bar{x}^i - i\bar{\eta}^i\frac{\partial^2 L}{\partial x^j \partial \bar{x}^i} dx^j \\ & + i\bar{\eta}^i\frac{\partial^2 L}{\partial \bar{x}^j \partial \bar{x}^i}\delta_i^j d\bar{x}^i - i\bar{\eta}^i\frac{\partial^2 L}{\partial \bar{x}^j \partial \bar{x}^i} d\bar{x}^j \\ & i\zeta^i\frac{\partial^2 L}{\partial y^j \partial y^i} dy^j - i\zeta^i\frac{\partial^2 L}{\partial y^j \partial y^i}\delta_i^j dy^i + i\zeta^i\frac{\partial^2 L}{\partial \bar{y}^j \partial y^i} d\bar{y}^j - i\bar{\zeta}^i\frac{\partial^2 L}{\partial \bar{y}^j \partial y^i}\delta_i^j dy^i + i\zeta^i\frac{\partial^2 L}{\partial y^j \partial \bar{y}^i}\delta_i^j d\bar{y}^i - i\bar{\zeta}^i\frac{\partial^2 L}{\partial y^j \partial \bar{y}^i} dy^j \\ & + i\bar{\zeta}^i\frac{\partial^2 L}{\partial \bar{y}^j \partial \bar{y}^i}\delta_i^j d\bar{y}^i - i\bar{\zeta}^i\frac{\partial^2 L}{\partial \bar{y}^j \partial \bar{y}^i} d\bar{y}^j \end{aligned} \tag{13}$$

Since the closed Kahlerian formϕ_L on $T\mathcal{M}$ is symplectic structure, it is obtained

$$E_L = \xi - L$$

$$E_L = i\xi^i\frac{\partial}{\partial z^i} - i\bar{\xi}^i\frac{\partial}{\partial \bar{z}^i} + i\eta^i\frac{\partial}{\partial x^i} - i\bar{\eta}^i\frac{\partial}{\partial \bar{x}^i} + i\zeta^i\frac{\partial}{\partial y^i} - i\bar{\zeta}^i\frac{\partial}{\partial \bar{y}^i} - L \tag{14}$$

Differential equation (14) we get

$$dE_L = d\left(i\xi^i\frac{\partial}{\partial z^i} - i\bar{\xi}^i\frac{\partial}{\partial \bar{z}^i} + i\eta^i\frac{\partial}{\partial x^i} - i\bar{\eta}^i\frac{\partial}{\partial \bar{x}^i} + i\zeta^i\frac{\partial}{\partial y^i} - i\bar{\zeta}^i\frac{\partial}{\partial \bar{y}^i} - L\right)$$

$$\begin{aligned} dE_L = {} & i\xi^i\frac{\partial^2 L}{\partial z^j \partial z^i} dz^j - i\bar{\xi}^i\frac{\partial^2 L}{\partial z^j \partial \bar{z}^i} dz^j - \frac{\partial L}{\partial z^j} dz^j + i\xi^i\frac{\partial^2 L}{\partial \bar{z}^j \partial z^i}\bar{z}^j - i\bar{\xi}^i\frac{\partial^2 L}{\partial z^j \partial \bar{z}^i} d\bar{z}^j - \frac{\partial L}{\partial \bar{z}^j} d\bar{z}^j + i\eta^i\frac{\partial^2 L}{\partial x^j \partial x^i} dx^j \\ & - i\bar{\eta}^i\frac{\partial^2 L}{\partial x^j \partial \bar{x}^i} dx^j - \frac{\partial L}{\partial x^j} dx^j + i\eta^i\frac{\partial^2 L}{\partial \bar{x}^j \partial x^i}\bar{z}^j - i\eta^i\frac{\partial^2 L}{\partial x^j \partial \bar{x}^i} d\bar{x}^j - \frac{\partial L}{\partial \bar{x}^j} d\bar{x}^j + i\zeta^i\frac{\partial^2 L}{\partial y^j \partial y^i} dy^j - i\bar{\zeta}^i\frac{\partial^2 L}{\partial y^j \partial \bar{y}^i} dy^j \\ & - \frac{\partial L}{\partial y^j} dy^j + i\zeta^i\frac{\partial^2 L}{\partial \bar{y}^j \partial y^i}\bar{y}^j - i\bar{\zeta}^i\frac{\partial^2 L}{\partial y^j \partial \bar{y}^i} d\bar{y}^j - \frac{\partial L}{\partial \bar{y}^j} d\bar{y}^j \end{aligned} \tag{15}$$

With respect to (1), if (15) and (13) is equalized, it is calculated as follows:

$$-i\xi^i\frac{\partial^2 L}{\partial z^j \partial z^i} dz^j - i\bar{\xi}^i\frac{\partial^2 L}{\partial \bar{z}^j \partial z^i} dz^j + \frac{\partial L}{\partial z^j} dz^j + i\xi^i\frac{\partial^2 L}{\partial z^j \partial \bar{z}^i}\bar{z}^j + i\bar{\xi}^i\frac{\partial^2 L}{\partial \bar{z}^j \partial \bar{z}^i} d\bar{z}^j + \frac{\partial L}{\partial \bar{z}^j} d\bar{z}^j$$

$$-i\eta^i \frac{\partial^2 L}{\partial x^j \partial x^i} dx^j - i\bar{\eta}^i \frac{\partial^2 L}{\partial \bar{x}^j \partial x^i} dx^j + \frac{\partial L}{\partial x^j} dx^j + i\eta^i \frac{\partial^2 L}{\partial x^j \partial \bar{x}^i} \bar{z}^j + i\bar{\eta}^i \frac{\partial^2 L}{\partial \bar{x}^j \partial \bar{x}^i} d\bar{x}^j + \frac{\partial L}{\partial \bar{x}^j} d\bar{x}^j$$

$$-i\zeta^i \frac{\partial^2 L}{\partial y^j \partial y^i} dy^j - i\bar{\zeta}^i \frac{\partial^2 L}{\partial \bar{y}^j \partial y^i} dy^j + \frac{\partial L}{\partial y^j} dy^j + i\zeta^i \frac{\partial^2 L}{\partial y^j \partial \bar{y}^i} \bar{y}^j + i\bar{\zeta}^i \frac{\partial^2 L}{\partial \bar{y}^j \partial \bar{y}^i} d\bar{y}^j + \frac{\partial L}{\partial \bar{y}^j} d\bar{y}^j = 0$$

Now, let the curve $\alpha : C \longrightarrow T\mathcal{M}$ be integral curve of ξ, which satisfies equations

$$-i\left[\xi^j \frac{\partial^2 L}{\partial z^j \partial z^i} + \dot{\xi}^i \frac{\partial^2 L}{\partial \dot{z}^j \partial z^i}\right] dz^j + \frac{\partial L}{\partial z^j} dz^j + i\left[\xi^i \frac{\partial^2 L}{\partial z^j \partial \dot{z}^i} + \dot{\xi}^i \frac{\partial^2 L}{\partial \dot{z}^j \partial \dot{z}^i}\right] d\dot{z}^j + \frac{\partial L}{\partial \dot{z}^j} d\dot{z}^j = 0$$

We infer the equations

$$\frac{\partial L}{\partial z^i} - i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial z^i}\right) = 0 \quad , \qquad \frac{\partial L}{\partial \dot{z}^i} + i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{z}^i}\right) = 0$$

Or

$$-i\left[\eta^j \frac{\partial^2 L}{\partial x^j \partial x^i} + \dot{\eta}^i \frac{\partial^2 L}{\partial \dot{x}^j \partial x^i}\right] dx^j + \frac{\partial L}{\partial x^j} dx^j + i\left[\eta^i \frac{\partial^2 L}{\partial x^j \partial \dot{x}^i} + \dot{\eta}^i \frac{\partial^2 L}{\partial \dot{x}^j \partial \dot{x}^i}\right] d\dot{x}^j + \frac{\partial L}{\partial \dot{x}^j} d\dot{x}^j = 0$$

we infer the equations

$$\frac{\partial L}{\partial x^i} - i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x^i}\right) = 0 \quad , \qquad \frac{\partial L}{\partial \dot{x}^i} + i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}^i}\right) = 0$$

Or

$$-i\left[\zeta^j \frac{\partial^2 L}{\partial y^j \partial y^i} + \dot{\zeta}^i \frac{\partial^2 L}{\partial \dot{y}^j \partial y^i}\right] dy^j + \frac{\partial L}{\partial y^j} dy^j + i\left[\zeta^i \frac{\partial^2 L}{\partial y^j \partial \dot{y}^i} + \dot{\zeta}^i \frac{\partial^2 L}{\partial \dot{y}^j \partial \dot{y}^i}\right] d\dot{x}^j + \frac{\partial L}{\partial \dot{y}^j} d\dot{y}^j = 0$$

we infer the equations

$$\frac{\partial L}{\partial y^i} - i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial y^i}\right) = 0 \quad , \qquad \frac{\partial L}{\partial \dot{y}^i} + i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{y}^i}\right) = 0$$

Thus

$$\begin{aligned} &\frac{\partial L}{\partial z^i} - i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial z^i}\right) = 0 \quad , \qquad \frac{\partial L}{\partial \dot{z}^i} + i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{z}^i}\right) = 0 \\ &\frac{\partial L}{\partial x^i} - i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x^i}\right) = 0 \quad , \qquad \frac{\partial L}{\partial \dot{x}^i} + i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}^i}\right) = 0 \\ &\frac{\partial L}{\partial y^i} - i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial y^i}\right) = 0 \quad , \qquad \frac{\partial L}{\partial \dot{y}^i} + i\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{y}^i}\right) = 0 \qquad (16) \end{aligned}$$

Thus, by complex Euler-Lagrange equations ,we may call the equations obtained in (16) on $J^{(2,0)}\mathcal{M}$. Then the quartet $(J^{(2,0)}\mathcal{M}, \phi_L, \xi)$ is named mechanical system with

4. Conclusions

The solutions of the Euler-Lagrange equations determined by (16) on the mechanical system $(J^{(2,0)}\mathcal{M}, \phi_L, \xi)$are the paths of vector field ξ on $J^{(2,0)}\mathcal{M}$.

References

[1] Violeta, Zalutchi, The geometry of (2; 0)-jet bundles , University "Transilvania of Brasov, of Brasov,Faculty of Mathematics and Informatics, 2010,,311-320.
[2] loring .W.Tu , S.Axler , K.A.Ribet ,An Introduction to Manifolds ,Springer
[3] Mehmet Tekkoyun -Lagrangian and Hamiltonian Dynamics on Para-Kahlerian Space Form-arXiv:0902.4522v1 [math.DS] 26 Feb 2009.
[4] A. Manea, A decomposition of the bundle of second order jets on a complex
manifold, Analle St. Univ. "Al. I. Cuza", Iasi, LVI (2010), 151-162.
[5] W. Stoll, P.-M. Wong, On holomorphic jet bunldes, preprint, arxiv:math/ 0003226v1/2000.
[6]http//syrianteacher.awardspace.com