Prime Labeling Of Snow Graph S_{n,1,m}

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ABSTRACT

A graph $G = (V, E)$ with n vertices is said to be admitted prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the label of each pair of adjacent vertices are relatively prime. Some previous work on prime labeling includes prime labeling of bipartite graphs and ladder graphs. In this work, a new graph called Snow graph which is a union of wheel and star graphs is introduced. A theorem is given to obtain a prime labeling for this graph and an algorithm for prime labeling is given.

Introduction

Graph labeling has many applications in Computer Science and Engineering. Prime labeling is a special case of graph labeling which is very useful for research in graph theory. In our previous researches prime labeling of known graphs, Roach graph and Scorpion graph have been done. For the present work, we have defined a new graph with 3 parameters called *Snow graph*, $S_{n,1,m}$ which is a shape of snowflake and it is a union of wheel and star graphs.

The concept of prime labeling was introduced by Roger Entringer around 1980's in the setting of *trees* where he stated the conjecture; *every tree is prime*. The theory was developed to the current setting by A. Tout et al. in 1982 [3].Prime labeling can be applied for various types of graphs. In this paper, we have given prime labeling of Snow Graphs. Since the Snow Graph represents a structure of snowflake, it can be used to model as a computer network system structure or as a part of it. In our work, we have introduced an Algorithm for prime labeling of Snow graphs. Some most important definitions in this research are given below.

Definition 1 (Cycle graph; C_n)

A connected graph that is regular of degree 2 is a *Cycle graph* and it is denoted by C_n *for* n *ver*tices.[7]

Definition 2 (Wheel graph; W_n)

The graph obtained from C_{n-1} by joining each vertex to a newly introduce vertex at its center is the Wheel on 1 *vertices.*) [7]

Definition 3 (Star *graph*; $K_{1,m}$)

A complete bipartite graph of the form $K_{1,m}$ is a Star graph with $m +$ 1 vertices where a single vertex belongs to one set and all the remaining vertices belong to the other set and each edge joins single vertex to all the other vertices.

Definition4

(Snow graph; $S_{n,1,m} = W_n \cup K_{1,m} \cup K_{1,m} \cup ... \cup K_{1,m}$, a newly defined graph)

A graph obtained from the wheel graph W_n by adjoining one vertex of each of n star graph $K_{1,m}$ to the outer cycle C_{n-1} of W_n is called a *Snow graph.*

Number of vertices of the Snow graph $= mn + n - m$
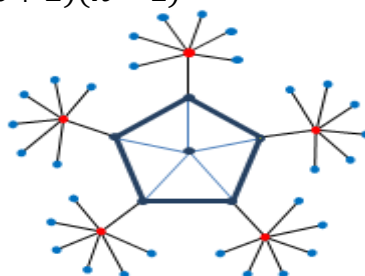
Number of edges of the Snow graph $= (m + 2)(n - 1)$

Figure 1. The Snow graph of the form $S_{6,1,7}$.

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Definition 5 (Prime Labeling)

Let G
$= G(V, E)$ *be a simple graph where V and E denotes the set of vertices and edges respectively. A Prime labeling o*
$\cdots, n\}$ *in such a way that the labels of any two adjacent vertices are relatively prime. Such a graph is called a Prin*

Definition 6 (Greatest Common Divisor)

Let a and b be integer that are not both zero. Then, the greatest common divisor (gcd) of a and b is the largest po:
Two integers a and b are relatively prime if $gcd(a, b) = 1$. [5]

Materials and Methods

Theorem: *The Snow graph of the form $S_{6,1,m} = W_6 \cup K_{1,m} \cup K_{1,m} \cup K_{1,m} \cup K_{1,m} \cup K_{1,m}$ with $5m +$*
6 vertices has a prime labeling.

Algorithm

Data: $S_{6,1,m}$, $5(m + 1) + 1 = P$; P is a prime.

Result : $S_{6,1,m}$ has a prime labeling.

Step 1

Count the number of vertices in the graph $S_{6,1,m}$.

Total number of vertices of Snow graph $S_{n,1,m} = mn + n - m$.

For $S_{6,1,m}$, the number of vertices $= 5(m + 1) + 1 = P$; P is a prime number.
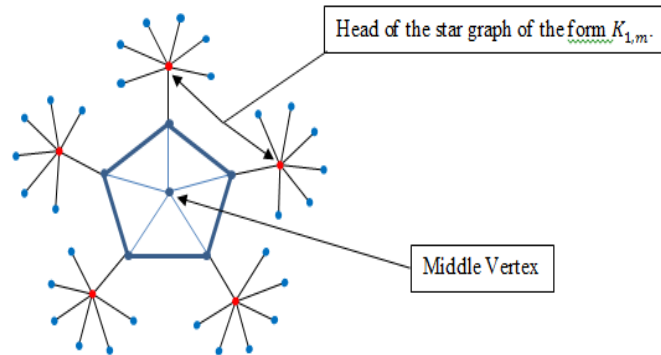
Step 2

Take the largest **10** prime numbers in the interval $[1, P - 1]$.

Denote those prime numbers as $P_1, P_2, P_3, \ldots, P_{10}$ and $P = P_{11}$

Here, $P_1 < P_2 < P_3 < \cdots < P_9 < P_{10} < P_{11} = P$ and define a set L such that

$L = \{P_1, P_2, P_3, \ldots, P_9, P_{10}, P_{11}\}$

Step 3

Head of the star graph of the form $K_{1,m}$.

Middle Vertex

- Label the middle vertex of the wheel using $min\{P_1, P_2, P_3, \ldots, P_9, P_{10}, P_{11}\}$
- Next, label the other vertices of the wheel graph such that P_2, P_3, P_4, P_5, P_6 $(P_2 < P_3 < P_4 < P_5 < P_6)$
- Label the head of the star graph $P_7, P_8, P_9, P_{10}, P_{11} = P (P_7 < P_8 < P_9 < P_{10} < P_{11})$

Step 4

Divide other integers in $[1, P]$ into five groups.

Define those five group as F_1, F_2, F_3, F_4, F_5 such that

$$F_i = \left\{x \in [1, P] \backslash \left\{L \cup \sum_{j=1}^{i-1} F_j\right\} \bigg/ x < 2P_{i+6}; gcd(x, P_{i+6}) = 1; |F_i| = m - 1; i = \overline{1, 5}\right\}$$

Note: $|F_1| = |F_2| = |F_3| = |F_4| = |F_5| = m - 1$

Step 5

Label other vertices in the $S_{6,1,m}$ using corresponding F_i to P_{i+6}; $i = \overline{1, 5}$ such that

$$F_1 \longrightarrow P_7$$
$$F_2 \longrightarrow P_8$$
$$F_3 \longrightarrow P_9$$
$$F_4 \longrightarrow P_{10}$$
$$F_5 \longrightarrow P_{11}$$

Now, consider the each step carefully.

It is shown that the graphs $S_{6,1,m}$ have prime labeling.

Results and Discussion

We divide the labeling into two parts as follows:

i. A part of wheel graph and a part of star graph($K_{1,m}$) of head vertices.
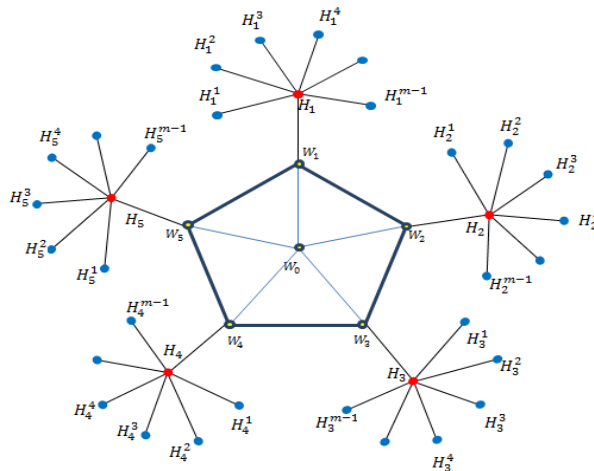
ii. Other vertices in $K_{1,m}$ graph.

Figure 2. The Snow graph of the form $S_{6,1,m}$.

Let $A = \{W_0, W_1, W_2, W_3, W_4, W_5\} \cup \{H_1, H_2, H_3, H_4, H_5\}$; $|A| = 11$ and

$B = \{H_j^i \in V(K_{1,m}) | j = \overline{1,5}; i = \overline{1, m-1}\}$; $|B| = 5(m-1)$

In step 3, labeling can be done using $P_1, P_2, P_3, \ldots, P_{10}, P_{11} = P$ which are prime.

So, it is clear that those adjacent pairs are relatively prime.

Therefore, $\gcd(W_i, W_j) = 1; i \neq j; i, j = 0, 1, 2, 3, 4, 5$

Furthermore,

$\gcd(W_i, H_i) = 1; i = 1, 2, 3, 4, 5$

Now, to label set B,

Defined set F_i which is corresponding to prime number $H_i = P_{i+6}; i = 1, 2, 3, 4, 5$ and

$|F_i| = m - 1; i = 1, 2, 3, 4, 5$

Clearly, $\forall\, x \in F_i; \gcd(x, P_{i+6}) = 1$, since P_{i+6} is prime and $x < 2P_{i+6}$.

Therefore, a labeling is a prime labeling.

Conclusions

We have obtained a theorem for prime labeling of a new graph, which is known as Snow graph. Prime labeling can be obtained by the constructed algorithm for Snow graphs. As a future work, planning to write a computer program for this algorithm and generalize this theorem for different n integer values such that $(m+1)(n-1) + 1$ is a prime.

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