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Review on the Black Hole Theory

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ABSTRACT

In this study a quick summary for the back-hole theory was presented. It was presented earlier in from of graduation project at university of Bahri, college of applied and industrial sciences department of physics, Khartoum, Sudan by the second author and under supervision of the first author Black holes were once a star shining for years, before eventually collapse on their core forming the BH. There are three cases for a star non stable end is either a white dwarf, a neutron star or a black hole which is completely depend on the mass of the previous star which are called a compact star due to their highly condensation states. In this paper a quick summary for the theory was presented.

Introduction

Theory of Black Holes

Subrahmanyam Chandrasekhar pointed out, way back in 1934, that the life-history of a star of small mass must be essentially different from that of a star of large mass. Stars like the Sun, when they run out of their internal nuclear fuel, stabilize into smaller entities of about a thousand kilometers in size, called white dwarfs. On the other hand, a star of larger mass, beyond about 1.4 solar masses, cannot pass into this stage, and as Chandrasekhar pointed out, 'one is left speculating on other possibilities. For decades, the question as to what is the final fate of a massive star has remained unanswered. In recent years much research has focused on this issue, which has become one of the most important unresolved problems in astrophysics and cosmology today. Large-mass stars, when they run out of their internal nuclear fuel, undergo a continual gravitational collapse, which is a catastrophic shrinkage of the star's size under the pull of its own gravity. In this case, exciting outcomes for the collapsing star are predicted by the general theory of relativity. These have profound implications for fundamental physics and cosmology. We will now introduce the central objects of interest, namely, black holes. These arise from gravitational collapse of an object with some mass which is compressed into a small region of space-time. It is characterized by a curvature singularity at the origin which is 'screened' to outside observers by a coordinate singularity at finite radial distance. This coordinate singularity is known as the *event horizon*, which will be seen to exhibit deep connections with thermodynamic systems [1,2,3,4].

Schwarzschild Solution

The metric of a star of mass M which we derived last time is the large r, small M limit of the Schwarzschild solution, which we consider here. We look for an exact solution of $R_{\mu v} - \frac{1}{2} R g_{\mu v} = 8\pi G T_{\mu v} + \frac{1}{2} \Lambda g_{\mu v}$ under the following assumptions.

i. Vacuum solution: $T_{\mu v} = 0$

ii. Spherical symmetry: $\partial_{\emptyset} g_{\mu v} = 0$

iii. Asymptotic flatness: $\Lambda = 0$, where Λ is the cosmological constant

The solution which satisfies these assumptions was found by Schwarzschild only a few months after Einstein published his theory of general relativity. It reads

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + \underbrace{r^2(d\theta^2 + \sin^2\theta d\varphi^2)}_{r^2 d\Omega^2}$$

(1)

This is known as the Schwarzschild metric. We have one parameter, mass M, while electric charge Q and angular momentum L are zero. Birkhoff's theorem then tells us that the Schwarzschild black hole is the only spherically symmetric solution of Einstein's equations. We see that for large we can expand $\left(1 - \frac{2M}{r}\right)^{-1} \approx 1 + \frac{2M}{r}$, which gives the metric for a star derived last time. For $r \to \infty$as well as $M \to 0$, we retrieve Minkowski space.

We now compute the curvature of this geometry. In this metric, we have $R_{\mu v} = 0$, so that this is a so-called *Ricci flat* space. Hence, we consider another quantity than the Ricci tensor, namely, the *Kretschmann scalar*, which, for the Schwarzschild metric, is given by

$$R_{\mu v\rho\sigma} R^{\mu v\rho\sigma} = \frac{48M^2}{r^2} \xrightarrow{r \to 0} \infty \quad (2)$$

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The $r \rightarrow 0$ behavior of the Kretschmann scalar expresses the fact that $r = 0$ is the locus of a *space-time singularity*, namely, a singularity which cannot be resolved by a coordinate transformation. This is different from the *coordinate singularity* at $r = 2M$, where the Kretschmann scalar is finite. This coordinate transformation can be removed by going to a different coordinate system, as we will see later.

Event Horizon

Einstein's theory was first used for understanding the final fate of massive collapsing stars by *Oppenheimer* and *Snyder*, and independently by *Datt* in the late 1930s. OSD studied the continual gravitational collapse of a pressure less, homogeneous uniform density matter cloud using the general theory of relativity. The most interesting result was that the collapse, as it evolves in time, leads to the formation of what was later called an event horizon. This is a one-way surface formed in spacetime which causes a region of space and time to be invisible and non-communicable to observers far away in the Universe. This hidden region is called a black hole, though the terms black hole and event horizon were actually coined much later, in 1969, by *John Wheeler*. Thus, the continual collapse of such a massive matter cloud in the OSD model creates a black hole as the collapse end-state.To understand the formation of a black hole during collapse, the concept of an *escape cone* is helpful. Consider the collapsing star and an observer located on the surface of the star who keeps emitting beams of light as the collapse proceeds. As shown in Fig. 3.3, as long as the star has not entered the event horizon, the light emitted by the observer can escape and reach a faraway observer. Once the observer reaches the horizon, all the light rays that he emits fall into the singularity at the center, except one radial ray that just stays at the horizon, at a constant distance from the singularity. Further to this, once the observer enters the horizon, all the rays emitted fall into the singularity, and there is no escape possible. Eventually the observer falls into the singularity, to be crushed out of existence. The event horizon has the topology of $\boldsymbol{S^2}$ and is located at $r = 2M$. Here, the metric becomes divergent. However, all components of $\boldsymbol{R^{\lambda}_{\mu\nu\rho}}$are finite. Hence, we see (as stated before) that this is not a true (space-time) singularity but merely a coordinate singularity. However, in spite of the fact that this singularity is removable, a lot of interesting physics takes place here. We distinguish between three different parts of a black hoe space-time

- For $r > 2M$, called region *I* or the *exterior* of the black hole, the metric has signature *(−,+,+,+)*.
- For $r < 2M$, called region *II* or the *interior* of the black hole, the metric has signature *(+,−,+,+)*.
- At $r = 2M$, we have the event horizon, which is a *null surface* since it has signature *(*0, 0,+,+*)*.

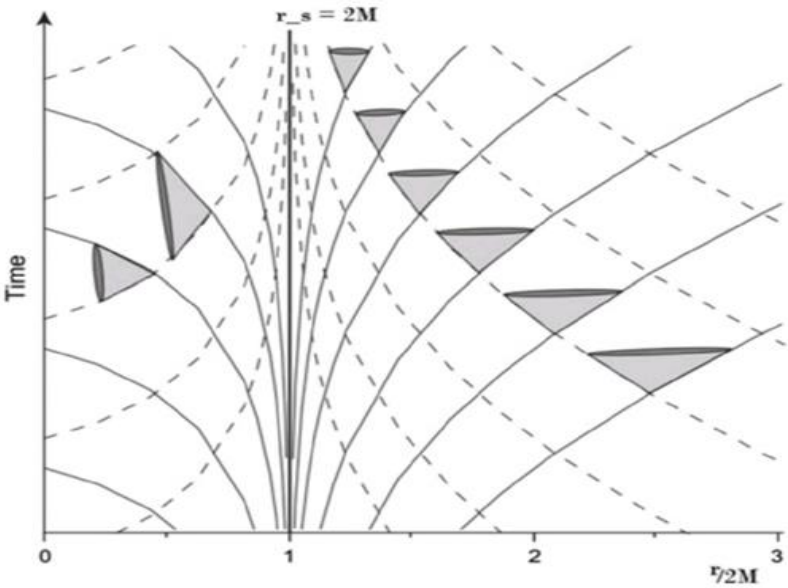


Fig 1. Space-time diagram of the near-horizon region of the Schwarzschild metric

We thus see that the event horizon is the point at which *x* and *t* exchange their respective signatures (positive and negative). In region *II* we have to move forward in *r* i.e. to $r = 0$, which means one inevitably hits the singularity. This is reminiscent of particles always moving forward in time (after setting our conventions of time direction) in region *I*. An outside observer will never see a signal reach the event horizon since signals close to the event horizon get infinitely red-shifted i.e. signals slow down until they become stationary close to the event horizon. Infinite redshift means that light loses all its energy as it climbs out of the gravitational potential of the mass at $r = 0$, which is why black hole is an appropriate name for this object. At $r = 2M$, the light-cone flips since metric signature changes from *(−,+,+,+)* to *(+,−,+,+)*.We now consider a co-moving observer that is propagating toward a black hole. Such an observer crosses the event horizon in finite proper time τ and does not experience anything special when crossing the event horizon. This observer also reaches the space-time singularity in finite time. Consider a radial null geodesic i.e. $\boldsymbol{ds^2 = 0}$ and $\frac{d\theta}{dt} = \boldsymbol{0} = \frac{d\varphi}{dt}$. This gives [5]

$$\frac{dr}{dt} = \pm\left(\mathbf{1} - \frac{2M}{r}\right) \tag{3}$$

Equation (3) tells us that the light cone indeed 'closes up' when approaching the event horizon. We now change to a coordinate system which does not contain the coordinate singularity we have in the Schwarzschild solution. We first go to *tortoise coordinates*. We first solve Eq. 4.3, which gives

$$t = \pm\left[r + 2M\ln\left(\frac{r}{2M} - \mathbf{1}\right)\right] \tag{4}$$

We then introduce $r* = r + 2M\ln\left(\frac{r}{2M} - 1\right) \xrightarrow{r\to 2M} -\infty$. Our metric then becomes

$$ds^2 = \left(1 - \frac{2M}{r}\right)(-dt^2 + (dr^*)^2) + r^2 d\Omega^2 \tag{5}$$

This metric is conformally equivalent to the Minkowski metric, which means that light cones are given by $r* = \pm t$. However, this metric is not quite what we are looking for since $g_{rr}, g_{rr} \xrightarrow{r\to 2M} 0$.

We then introduce $v = t + r^*$ and $u = t - r^*$ such that v=constant and u=constant describe outgoing and infalling radial null curves, respectively. We can then express our metric as follows

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dv^2 + 2dvdr + r^2 d\Omega^2 \tag{6}$$

This is the so-called *Eddington–Finkelstein metric*. For a null curve, we have
$ds2 = 0$. For $d\_ = 0$ (i.e. radial null curves), this gives $ds2 = -\_1 - 2M\,r \quad dv2 + 2drdv = 0$. The solutions are

$$\frac{dv}{dr} = \begin{cases} 0, \\ \frac{2}{1 - 2M/r} \end{cases} \tag{7}$$

Hence for $r < 2M$ all future-directed paths are in the direction of decreasing r, which means that a signal at $r < 2M$ will inevitably hit the space-time singularity at $r = 0$.

Cosmic censorship conjecture CCC: an assumption, first proposed by Penrose in 1969 It assumed that whenever singularity occur, it will be covered within event horizons, in other words under reasonable physical conditions the final outcome of gravitational collapse will always be a black hole in spacetime, with the curvature singularity of collapse always being hidden inside it[5].

Black Hole Mechanics and Thermodynamics

In the 1970's, Bekenstein realized that, assuming the second law of thermodynamics holds, a black hole must carry entropy. This follows from the fact that if we throw an object with some entropy into a black hole, the entropy of the total system may not decrease; hence the entropy of a black hole must grow when we throw an object into it. Such thought experiments can be used to derive the laws of black hole mechanics, which turn out to be profoundly connected to the laws of thermodynamics.

In particular, certain parameters from black hole physics will be seen to correspond to thermodynamic quantities. We state the laws of black hole mechanics and the corresponding laws of thermodynamics

Law	Thermodynamics	Black hole mechanics
0	T constant in equilibrium	κ =constant
1	$\delta E = T\delta S$	$\delta M = \frac{k}{8\pi}\delta A + \Omega\delta J + \phi\delta Q$
2	$\delta S \geq 0$	$\delta A \geq 0$
3	Cannot let $T \to 0$ in a finite number of steps	Cannot let $k \to 0$ in a finite number of steps

First Law of Black Hole Mechanics

The *first law of black hole mechanics* tells us how the mass of a black hole changes with its horizon area, charge, and angular momentum. It reads

$$\delta M = \frac{k}{4\pi}\delta A + \Omega\delta J + \phi\delta Q \tag{8}$$

We will show that this is the correct expression for stationary, axisymmetric, and asymptotically flat space-times. We need our space-time to have these properties so that mass and angular momentum are well-defined. To find an expression for the mass of a black hole, we first consider the example of electromagnetism. The electric charge inside a volume V is given by

$$Q(V) = \int_V \rho dV = \int_V \nabla.\vec{E}dV = \oint_{\partial V} \vec{E}.\hat{n}dS \tag{9}$$

Where $\hat{n}$ is the normal vector to ∂V. The covariant generalization of this expression is

$$Q(V) = \int_V dV\sqrt{\gamma}\,n_\mu j^\mu = \int_V dV\sqrt{\gamma}\,n_\mu \nabla_v F^{\mu v} = \oint_{\partial V} dS\sqrt{\tilde{\gamma}}\,n_\mu \sigma_v F^{\mu v} \tag{10}$$

Where γ and $\tilde{\gamma}$ are induced three- and two-dimensional metrics with corresponding unit normal given by n_μ and σ_v, receptively.

We now consider general relativity. The *Kumar integral* associated to a Killing vector ξ is given by

$$Q_\xi(V) = -\frac{1}{4\pi}\oint_{\partial V} dS\sqrt{\tilde{\gamma}}n_\mu \sigma_v (\nabla^\mu \xi^v)_{\mu v} \tag{11}$$

Comparing this expression with that of electromagnetism, we note that $\nabla^\mu \xi^v = \nabla^v \xi^\mu$, similar to $F^{\mu v} = -F^{v\mu}$. We now use $\nabla_v \nabla_\mu \xi^v = R_{\mu v}\xi^v$, which holds for any Killing vector ξ^v, to rewrite

$$Q_\xi(V) = -\frac{1}{4\pi}\int_V dV\sqrt{\gamma}\,n_\mu R^v \xi^\mu \tag{12}$$

The zeroth law states that surface gravity is constant over an event horizon. For our discussion of the first law we considered Kumar quantities. For some surface $S \subset M$ with volume element dS_μ with boundary ∂S with volume element $dS_{\mu v}$ and ξ a Killing vector, we have general Kumar quantity

$$Q_k(V) = -\frac{1}{4\pi}\int_{\partial V = S} dS\sqrt{\tilde{\gamma}}\,n_\mu \sigma_v \nabla^\mu \xi^v \tag{13}$$

E.g. for $k\sim\partial_t$ or $\widetilde{k}\sim\partial_\phi$, we find the Komar mass or angular momentum, respectively

$$M=-\frac{1}{4\pi}\int_S dS\sqrt{\widetilde{\gamma}}\, n_\mu \sigma_v \nabla^\mu k^v,$$

$$J=-\frac{1}{8\pi}\int_S dS\sqrt{\widetilde{\gamma}}\, n_\mu \sigma_v \nabla^\mu \widetilde{k}^v. \quad (14)$$

The total mass inside some *S* is then

$$M=\phi Q+2\Omega_H J+\frac{k}{4\pi}A. \quad (15)$$

Combining these gives the first law

$$dM=\frac{k(M,J,Q)}{8\pi}dA+\Omega(M,J,Q)dJ+\phi(M,J,Q)dQ, \quad (16)$$

Where Ω is the angular velocity at the horizon, and κ and ϕ are the acceleration and electric potential at the horizon with respect to a stationary asymptotic observer.

Second Law

A natural question concerning black holes is whether there are restrictions on the possible charges, areas, angular momenta, and masses of black holes for a given set of boundary conditions. This question is relevant for matter that collapses to a black hole, or generally for dynamical problems. The answer is that there are indeed restrictions, for example, the BPS-bound gives $J^2 \leq M^2$, and the same for *Q*. Further, we have Hawking's area theorem, which states that

$$\frac{dA}{dt}\geq 0 \quad (17)$$

This expression holds assuming that our space is asymptotically flat, we have cosmic censorship i.e. no naked singularities, and the weak energy condition holds i.e. $T_{\mu v}v^\mu v^v \geq 0$, where v^μ is an arbitrary time-like vector.

Proof (relies on causality): A *causal curve* is a curve that is nowhere space-like.

We define the *causal past* of some surface *S* as $J^-(S) \coloneqq \{x\in \mathcal{M} | \exists s \in S, x\leq s\}$, where $x \leq s$ means that the time coordinate of *x* is less than or equal to *x*.

The *causal future* is then defined as $J^+(S) \coloneqq \{x\in \mathcal{M} | \exists s \in S, x< s\}$. We now consider *chronological curves* which are defined to be everywhere time-like.

The *chronological past* and *future* are then given by $I^- := \{x\in | \exists s\in S, x<s\}$ and $I^+(S) \coloneqq \{x\in | \exists s\in S, x>s\}$. The boundary of the causal past is $\partial J^-(s)=J^-(s)/I^-(S), \partial J^-(S)$ is generated by a set of null geodesics, which are referred to as the *null geodesic generators* of $\partial J^-(S)$. There is a lemma due to Penrose which states that in any subset *S* ⊂ *M*, a null geodesic generator of $\partial J^-(S)$ cannot have future endpoints on $\partial J^-(S)$. In other words, the area of a trapped surface cannot decrease.Since the event horizon of a black hole is such a trapped surface, Hawking's area theorem follows trivially.

Third Law of Black Hole Mechanics

The third law of black hole mechanics states that it is impossible to let the surface gravity κ go to zero in a finite number of operations. We give this statement without proof, but it can be seen from the fact that we need to let *M* →∞or *(a*2 + *Q*2 + *P*2*)* → *M*2. We thus need to add an infinite amount of (BPS-saturated) matter. We thus see that extremal (BPS) black holes cannot be formed continuously since they have κ = 0. This is related to supersymmetry, since BPS black holes can be made invariant under supersymmetry while the non-BPS black holes cannot.

Entropy of Black Holes

In thermodynamics, we may consider e.g. the grand canonical ensemble, where the system is characterized by quantities *μ, V, T,* namely chemical potential, volume, and temperature, respectively. The first law of thermodynamics is then

$$dE=TdS-pdV+\mu dN, \quad (18)$$

Where *p* and *N* are pressure and particle number, respectively. The first law of black hole mechanics is

$$dM=\frac{k}{8\pi}dA+\Omega dJ+\phi dQ. \quad (19)$$

By comparing the two expressions, we see that $-pdV+\mu dN$is analogous to $\Omega dJ+\phi dQ$ so that $\frac{k}{8\pi}dA$ is the entropy term. Moreover, Hawking's area theorem tells us that $\frac{dA}{dt}\geq 0$ which further solidifies the analogy between A and S. The precise relation will involve ħ, signaling the importance of quantum effects. The full expression is Birkenstein-Hawking area law

$$s=\frac{c^3 A}{4G_N \hbar} \quad (20)$$

Hence *S* diverges for _ → 0. We can rewrite this as

$$S=\frac{A}{4\pi L_P^2}\sim\frac{1}{4\pi}\frac{A}{(10^{-33})^2} \quad (21)$$

L_P Is the Planck length $\sim 10^{-33}$ cm. e.g. for a solar mass black hole with $r_s\sim 3$ km, *S* ≈ 10^{77}.In statistical physics, entropy is interpreted as information. The von Neumann entropy is given by $S=-\sum_n p_n \log p_n$, where p_n are probabilities satisfying $\sum_n p_n = 1$. Entropy is related to information (quantum bits). One naturally wonders what the quantum bits of black holes are i.e. what are the carriers of black hole information. People have speculated that the quantum bits are somehow 'distributed' over the horizon, since *S* ∝ *A*. This area law has inspired the principle of holography, which is the idea that gravitational degrees of freedom are dual to degrees of freedom on a holographic 'screen' which has one dimension lower than the gravitational system [5].

Consequences for Coalescing Black Holes

We consider the limits of mass-energy conversion of a black hole collision (such as observed by LIGO). Consider two black holes with masses M_1 and M_2, respectively.

They coalesce to form a third black hole, which has mass M_3. During this process, part of the total energy is converted to gravitational waves; this fraction of the total energy is given by $M_1 + M_2 - M_3$. We can then define an *efficiency coefficient* $\eta := \frac{M_1+M_2-M_3}{M_1+M_2} = 1 - \frac{M_3}{M_1+M_2}$. Black hole surface area is $A = 16\pi M^2$, so Hawking's area theorem tells us that i.e.

$$\eta \leq 1 - \frac{\sqrt{M_1^2+M_2^2}}{M_1+M_2} = 1 - \frac{1}{\sqrt{2}} \quad (22)$$

One then easily sees that a single black hole cannot split up into two separate black holes. The exceptions are BPS black holes, which can freely split up without decreasing total entropy.

Rotating (Kerr) Black Hole

So far, we considered spherically symmetric black holes. We will now consider a black hole with some non-zero angular velocity along azimuthal angle φ. The metric will then depend on θ in a non-trivial way, but it will not depend on φ i.e. it will be axially symmetric.

Remark: A slowly rotating body at large r has the following metric

$$ds^2 = \underbrace{ds_0^2}_{non-rotating\ metric} + \frac{4J}{r} sin^2\theta dt d\phi \quad (23)$$

Then J is the angular momentum of the rotating body. We now check that this is reproduced by the full solution, like we did in the case for the metric of a spherical body at large r. The full solution is

$$ds^2 = -\left(1 - \frac{2Mr}{r^2+a^2cos^2\theta}\right)(du + asin\theta d\phi)^2 + 2(du + sin^2\theta d\phi)(dr + asin^2\theta d\phi) + (r^2a^2cos^2\theta)\underbrace{(d\theta^2 + sin^2\theta d\phi^2)}_{d\Omega_{(2)}^2} \quad (24)$$

Where

$$u = t - 2M\log(\frac{r}{2M} - 1)\cdot$$

This solution was found by Kerr in 1963. The fact that it took almost 50 years since Einstein published his gravitational equations for someone to discover this solution is probably due to the fact that it has a rather complicated mixing between t and φ. If we redefine our coordinates as $t \rightarrow t - 2M\int \frac{rdr}{r^{2-2Mr+a^2}}$, $\phi \rightarrow -\phi - a\int \frac{rdr}{r^2-2Mr+a^2}$ the metric becomes

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right)dt^2 - \frac{4Marsin^2\theta}{\rho^2}d\phi dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2sin^2\theta}{\rho^2}\right)sin^2\theta d\phi^2 \quad (25)$$

With $\rho^2 = r^2 + a^2cos^2\theta$, $\Delta = r^2 - 2Mr + a^2$, $a = \frac{J}{M}$.

We remark on two limits of parameters a and M

i- $a \rightarrow 0$. Then $\rho \rightarrow r$, $\Delta = r^2 - 2Mr$, this gives the Schwarzschild metric.

ii- $M \rightarrow 0$. This gives flat Minkowski in so-called *oblate spheroidal coordinates*.

The Kerr metric does not depend on t or ϕ and is invariant under the combination

$t \rightarrow -t$ and $\phi \rightarrow -\phi$. The horizons are located at the points where $g_{rr} \rightarrow \infty$.

Where $\Delta \rightarrow \infty$. We thus find two horizons, located at[5]

$$r_{\pm} = M \pm \sqrt{M^2 - a^2} \quad (26)$$

Hawking Radiation

Near extremal Kerr black holes are subject to the Thorne limit $a < a^*_{lim} = 0.998$ in the case of thin disc accretion or some generalized version of this in other disc geometries. However any limit that differs from the thermodynamics limit $a^* < 1$ can in principle be evaded in other astrophysical configurations, and in particular if the near extremal black holes are primordial and subject to evaporation by Hawking radiation only. We derive the lower mass limit above which Hawking radiation is slow enough so that a primordial black hole with a spin initially above some generalized Thorne limit can still be above this limit today. Thus, we point out that the observation of Kerr black holes with extremely high spin should be a hint of either exotic astrophysical mechanisms or primordial origin.

Hawking showed that BHs are not as black as was first supposed. Throughout, we use a natural system of units where $G = c = K_B = \hbar = 1$. Hawking used a semiclassical treatment, that is to say the general relativity Kerr (or Schwarzschild) metric for space-time

$$ds^2 = \left(1 - \frac{2Mr}{\Sigma^2}\right)dt^2 + \frac{4aMr\sin\theta^2}{\Sigma^2}dt d\phi - \frac{\Sigma^2}{\Delta}dr^2 - \Sigma^2 d\theta^2 - \left(r^2 + a^2 + \frac{2a^2Mr\sin\theta^2}{\Sigma^2}\right)\sin\theta^2 d\phi^2 \quad (27)$$

where M is the BH mass, $a \equiv J/M$ is the BH spin parameter (J is the BH angular momentum), $\Sigma \equiv r^2 + a^2\cos(\theta)^2$ and $\Delta \equiv r^2 - 2Mr + a^2$, and a quantum mechanics treatment of Standard Model (SM) particles through a wave function ψ satisfying the Dirac equation for fermions (spin 1/2)

$$(i\partial - \mu)\psi = 0 \quad (28)$$

$\partial \equiv \gamma_v \partial^v$ is the standard Feynman notation, and the Proca equation for bosons (spin 0, 1 or 2)

$$(\Box + \mu^2)\psi = 0 \quad (29)$$

Where μ is the particle rest mass. Setting $\mu = 0$ in these equations of motion also allows to compute the propagation of the massless fields (in the following neutrinos, photons, gravitons). In these equations, we neglect the couplings between the fields, since they do not affect the probability of emission of (primary) SM particles via Hawking radiation, but we consider them to obtain the abundance of the final (secondary) particles at infinity, which come from the hadronization or decay of the primary particles. Solving these equations shows that there is a net emission of particles of type i called the Hawking radiation (HR). The number of particles emitted per unit time and energy is

$$\frac{d^2N_i}{dtdE} = \frac{1}{2\pi}\sum_{dof.}\frac{\Gamma_i^{lm}(E,M,a^*)}{e^{\acute{E}/T}\pm 1} \quad (30)$$

Where T is the Kerr BH Hawking temperature

$$T \equiv \frac{1}{2m}\left(\frac{r_+ - M}{r_+^2 + a^2}\right) \quad (31)$$

And $r\pm \equiv M\left(1 \pm \sqrt{1-(a^*)^2}\right)$ are the Kerr horizons radii; $a^* \equiv a/M$ is the Kerr dimensionless spin parameter, it is 0 for a Schwarzschild -non rotating- BH and 1 for a Kerr extremal BH; $\acute{E} \equiv E - m\Omega$ is the energy of the particle that takes into account the horizon rotation with angular velocity $\Omega \equiv a^*/(2r_+)$ on top of the total energy $E \equiv E_{kin} + \mu$; m is the particle angular momentum projection $m \in [-l, +]$. The sum of Eq. (4) is on the degrees of freedom (dof.) of the particle considered, that is to say the color and helicity multiplicity as well as the angular momentum l and its projection m. The quantity $\Gamma_i^{lm}(E, M, a^*)$ is called the graybody factor and has been extensively studied in the literature.

It encodes the probability that a particle of type i with angular momentum l and projection m generated at the horizon of a BH escapes its gravitational well and reaches space infinity [5].

Stellar Black Holes

It is believed that stellar black holes (BHs) can be formed in two different ways: Either a massive star collapses directly into a BH without a supernova (SN) explosion, or an explosion occurs in a proto-neutron star, but the energy is too low to completely unbind the stellar envelope, and a large fraction of it falls back onto the short-lived neutron star (NS), leading to the delayed formation of a BH. Theoretical models set progenitor masses for BH formation by implosion, namely, by complete or almost complete collapse, but observational evidences have been elusive. Here are reviewed the observational insights on BHs formed by implosion without large natal kicks from[6]:

i- The kinematics in three dimensions of space of five Galactic BH X-ray binaries (BH-XRBs)

ii- The diversity of optical and infrared observations of massive stars that collapse in the dark, with no luminous SN explosions, possibly leading to the formation of BHs.

iii- The sources of gravitational waves produced by mergers of stellar BHs so far detected with LIGO.

Multiple indications of BH formation without ejection of a significant amount of matter and with no natal kicks obtained from these different areas of observational astrophysics, and the recent observational confirmation of the expected dependence of BH formation on metallicity and redshift, are qualitatively consistent with the high merger rates of binary black holes (BBHs) inferred from the first detections with LIGO.

The formation of stellar BHs is of topical interest for several areas of astrophysics. Stellar BHs are remnants of massive stars, possible seeds for the formation of supermassive BHs, and sources of the most energetic phenomena in the universe, such as the gravitational waves produced by fusion of BHs.

BHs and NSs are the fossils of stars with masses above ~8$M_{\odot}$. It is known that some fraction of NSs have large runaway motions, probably due to strong natal kicks (NKs) imparted to the compact object. NKs have also been invoked in models of the core collapse of massive stars that lead to the formation of BHs. Such models predict in addition, that under specific conditions, BHs can also be formed by implosion with no energetic kicks **i,ii,** depending on mass, binarity, metallicity, angular momentum, and magnetic fields, among others properties of the progenitor star. NKs are of interest in Gravitational Wave Astrophysics since from population synthesis models of isolated binary evolution it is inferred that the merger rate of BBHs increases by a factor of ~20 when BH NKs are decreased from a kick distribution typical of NSs to zero.

It is believed that the runaway velocity of a BH-XRB can be due to the following mechanisms.

i- The sudden baryonic mass-loss in the SN explosion of the primary star of a binary (Blaauw kick). In this case the ejected matter will continue to move with the orbital velocity of the progenitor, and to conserve momentum the resulting compact binary will move in the opposite direction. A sudden mass loss would unbind the binary only when more than half the binary's total mass is instantaneously lost, which is not expected.

ii- NKs can also be imparted to the compact object, by anisotropic emission of neutrinos and GWs during core-collapse. If formed in a dense stellar cluster, other possible causes for the runaway velocity of a compact BH-XRB could be either one of several possible dynamical interactions in the stellar cluster, or the explosion of a massive star that before its collapse formed a multiple bound system with the runaway compact binary.

Kinematics of Galactic black hole X-ray binaries

The kinematics of BH-XRBs can provide clues on the formation of BHs. If a compact object is accompanied by a mass-donor star in an X-ray binary, it is possible to determine the distance, radial velocity, and proper motion of the system's barycenter, from which can be derived the velocity in three dimensions of space, and in some cases the path to the site of birth may be tracked. Among the estimated 3×10^8 stellar BHs in the Galaxy, about 20 BH-XRBs have been dynamically confirmed, and until present for only five of those BH-XRBs it was possible to determine their velocity in the three dimensions of space. High-precision astrometric observations with Very Long Baseline Interferometry (VLBI) at radio wavelengths provides model-independent distances from geometric parallaxes, from which can be gathered insights on the X-ray binary systems, and on the formation mechanism of BHs.

Table 2

BH-XRB	M_{BH} $M_{\odot}$	M_{donor} $M_{\odot}$	Sp type	V_{pec} kms^{-1}	e (galactic)	e (orbital)	P (days)
GRO J1655-40	5.3±0.7	2.4±0.7	F6-7IV	112±18	0.34±0.05	0.0	2.62
XTE J1118+480	7.6±0.7	0.5±**0.3**	K7-M1V	183±31	0.54±0.05	0.0	0.17
V 04 Cyg	9.0±0.6	0.75±**0.25**	K0 IV	39.9±5.5	0.16±0.02	0.0	6.47
Cygnus x-1	14.8±1.0	19.2±1.9	O9.7Iab	<9±2		0.18±0.03	5.6
GRS 1915+105	10.1±0.6	0.5±0.3	K-M III	22±24	0.28±0.05	0.0	34

In Table 2 are listed the known parameters of these five black hole binary systems and the estimated peculiar velocities relative to their local environment and/or birth place.

Black Holes Formed by Implosion

The end of massive stellar evolution depends on metallicity, binarity, angular momentum, nucleosynthesis and neutrino transport for the explosions, among other possible factors. Despite multiple uncertainties, most models predict that massive stars may collapse forming BHs directly, when no proto-neutron star is formed, a transient proto-neutron star is formed but unable to launch a SN shock, and a black hole is formed by fallback of mass after an initial SN shock has been launched. Stellar BHs may be formed with or without explosive ejection of a significant amount of baryonic matter, and with or without natal kicks.

Cygnus X-1 is an X-ray binary at a distance of 1.86 ± 0.1 kpc composed of a BH of 14.8 ± 1.0 $M_{\odot}$ and a 09.7lab donor star of 19.2 ± 1.9 $M_{\odot}$ with an orbital period of 5.6 days and eccentricity of 0.018 ± 0.

Classification of black holes

By the assumptions of general relativity, there are no constraints on the value of the mass of a black hole, which can thus be arbitrarily small as well as arbitrarily large. But from astronomical observations, scientists have collected strong evidence of at least two classes of astrophysical black holes:

i- Stellar-mass black holes.

ii- Supermassive black holes.

There is also some evidence of intermediate-mass black holes, with a mass filling the gap between the stellar- mass and the supermassive ones. Black holes should form from the complete gravitational collapse of a system, when there is no mechanics capable of balancing the gravitational force and the system shrinks until the formation of the event horizon. The collapse of the core of heavy stars is expected to produce black holes with a mass $M \geq 3M_{\odot}$ because for cores of lower mass the quantum pressure of neutrons should stop the collapse and the final product should be a neutron star.

However, there are cosmological scenarios in which it is possible to produce primordial black holes with any mass, even much lower than $3M_{\odot}$. Nevertheless, for the moment there is no evidence for the existence of such objects[6].

Stellar-Mass Black Holes

From stellar evolution simulations, we expect that in our Galaxy there is a population of about 10^8 -0^9 black holes formed at the end of the evolution of heavy stars, and the same number can be expected in similar galaxies. The initial mass of a stellar-mass black hole should depend on the properties of the progenitor star: on its mass, its evolution, and the supernova explosion mechanism. A crucial quantity is the metallicity of the star, namely the fraction of mass of the star made of elements heavier than helium.The maximum mass of black hole remnants critically depends on the metallicity. The final mass of the remnant is indeed determined by the mass loss rate by stellar winds, which increases with the metallicity because heavier elements have a larger cross section than lighter ones, and therefore they evaporate faster. For a low- metallicity star, there may be a mass gap in the remnant, roughly between 50 and $150M_{\odot}$, namely the mass of the black hole remnant can be $M \leq 50M_{\odot}$ or $M \geq 150M_{\odot}$. As the metallicity increases, black holes with M $M \geq 150M_{\odot}$ disappear, mainly because of the increased mass loss rate. Note, however, that some models do not find remnants with a mass above the gap, because stars with $M \geq 150M_{\odot}$ may undergo a runaway thermonuclear explosion that completely destroys the system, without leaving any black hole remnant.The lower bound may come from the maximum mass for a neutron star: the exact value is currently unknown, because it depends on the equation of state of matter at super-nuclear densities, but it should be around 2-$3M_{\odot}$.For bodies with a mass lower than this limit, the quantum neutron pressure can stop the collapse and the final product is a neutron stars. For bodies exceeding this limit, the final product is a black hole. Note, however, that there may be a mass gap between the maximum neutron star mass and the minimum black hole mass.

Stellar-mass black holes may thus have a mass in the range 3-$100M_{\odot}$. At the moment, all the known stellar-mass black holes in X-ray binaries have a mass $M \approx 3 - 20M_{\odot}$. Gravitational waves have shown the existence of heavier stellar-mass black holes. In particular, the event called GW150914 was associated to the coalescence of two black holes with a mass $M \approx 30M_{\odot}$ that merged to form a black hole with $M \approx 60M_{\odot}$.While we expect a huge number of stellar-mass black holes in the Galaxy, we only know about 20 objects with a dynamical measurement of the mass and about 50 objects without a dynamical measurement of their mass (it is thus possible that some of them are not black holes but neutron stars). This is because their detection is very challenging. The simplest case is when the black hole is in a binary system and has a companion star. The presence of a compact object can be discovered from the observation of X-ray radiation emitted from the inner part of the accretion disk. If we can study the orbital motion of the companion star, we may be able to measure the mass function

$$f(M) = \frac{K_c^3 P_{orb}}{2\pi G_N} = \frac{M \sin^3 i}{(1+q)^2} \quad (32)$$

Where $K_c = v_c \sin i$, v_c is the velocity of the companion star, i is the angle between the normal of the orbital plane and our line of sight, P_{orb} is the orbital period of the system, $q = M_c/M$, M_c is the mass of the companion, and M is the mass of the dark object. If we can somehow estimate i and M_c, we can infer *M*, and in this case we talk about dynamical measurement of the mass. The dark object is a black hole if $M > 3M_\odot$.

Supermassive Black Holes

Astronomical observations show that at the center of a large number of galaxies there is a large amount of mass in a relatively small volume. The standard interpretation is that these objects are supermassive black holes with $M \sim 10^5 - 10^{10} M_\odot$.The strongest constraints come from the center of our Galaxy and of NGC 4258 by studying the motion of individual stars or of gas in their nuclei. In the end, we can exclude the existence of a cluster of compact non-luminous bodies like neutron stars and therefore we can conclude that these objects are supermassive black holes. In the case of other galaxies, it is not possible to put such constraints with the available data, but it is thought that every middle-size (like our Galaxy) or large galaxy has a supermassive black hole at its center. For lighter galaxies, the situation is more uncertain. Most models predict supermassive black holes at the center of lighter galaxies as well, but there are also models predicting the existence of a population of faint low-mass galaxies with no supermassive black hole at their center.

Observations suggest that some small galaxies have a supermassive black hole and other small galaxies do not. In the case of stellar-mass black holes, it is easy to argue that they are the final product of the evolution of very heavy stars. In the case of supermassive black holes, at the moment we do not know their exact origin.We observe supermassive objects in galactic nuclei with a mass $M \sim 10^5 - 10^{10} M_\odot$. More puzzlingly, we observe objects with masses $M \sim 10^{10} M_\odot$ even in very distant galaxies, when the Universe was only 1 billion years old, and we do not know how such objects were created and were able to grow so fast in a relatively short time. The Eddington accretion rate can be exceeded in some accretion models, and this may indeed be a possible path to the rapid growth of supermassive black holes. The possibility of super-Eddington accretion is confirmed, for instance, by the observation of a neutron star in the galaxy M82 with a luminosity exceeding its Eddington[1] limit. It is also possible that supermassive black holes formed from the collapse of heavy primordial clouds rather than of stars, or that they formed from the merger of several black holes [6].

Intermediate-Mass Black Holes

Intermediate-mass black holes are, by definition, black holes with a mass between the stellar-mass and the supermassive ones, say $M \sim 10^2 - 10^4 M_\odot$. At the moment, there is no dynamical measurement of the mass of these objects, and their actual nature is still controversial. Some intermediate-mass black hole candidates are associated to ultra-luminous X-ray sources. These objects have an X-ray luminosity $L_X > 10^{39}$ erg/s, which exceeds the Eddington luminosity of a stellar-mass object, and they may thus have a mass in the range $10^2 - 10^4 M_\odot$. However, we cannot exclude they are actually stellar-mass black holes (or neutron stars) with non-isotropic emission and a moderate super-Eddington mass accretion rate [6].

Runaway Black Hole X-Ray Binaries

The physical mechanisms that may impart the runaway velocity of a BH are of topical interest to observationally constrain SN models and population-synthesis models of BH binary evolution. The runaway velocities of BH-XRBs can be caused by different physics mechanisms; a variety of mechanisms based on those originally proposed to explain the runaway massive stars, namely, baryonic mass loss in the sudden SN explosion of the primary star of a massive binary (Blaauw kick), dynamical interactions in high density stellar environments Galactic diffusion by random gravitational perturbations from encounters with spiral arms and giant molecular clouds for anomalous velocities up to 20-30 km s^{-1}, and BH NKs. There are two types of BH NKs, those imparted intrinsically to the BH by asymmetric gravitational waves and/or asymmetric neutrino emission during core-collapse, and those imparted to the transient NS that turn into a BH by mass fallback.

Several efforts were recently undertaken to estimate BH NKs from the observations of low-mass BH-XRBs. From the statistical analysis and model binary evolution of low mass BH-XRBs binaries with determined positions it has been proposed that in order to achieve their distances from the Galactic disk, BHs may receive high NKs at birth. This motivated the proposition based on theoretical calculations that by the gravitational pull from asymmetric mass ejecta, BHs can be accelerated to velocities comparable to those of NSs. However, because of the unknown origin and several other uncertainties of the samples of sources, it has been argued that from only the existing observations of the spatial locations of low-mass X-ray binaries, it is not possible to confidently infer the existence of high BH NKs. In the following are reviewed the observations of the three runaway low mass BH-XRBs for which the space velocities in three dimensions have been determined: GRO J1655-40, XTE 1118+480 and V404 Cyg.**GRO J1655-40** is an X-ray binary with a BH of 5.3 ± 0.7 $M_\odot$ and a F6-F7 IV donor star with a runaway velocity of 112 ± 18 km s^{-1} moving in a highly eccentric (e=0.34 ± 0.05) Galactic orbit. The overabundance of oxygen and alpha-elements in the atmosphere of the donor star has been interpreted as evidence for SN ejecta captured by the donor star. The runaway linear momentum of this X-ray binary is similar to those of solitary runaway neutron stars and millisecond pulsars with the most extreme runaway velocities. It had been proposed that the large runaway velocity of this BH-XRB may be accounted for by a successful SN in which a mass of ~4 $M_\odot$ was ejected, that a natal kick is not needed to explain its large space velocity, and that a fallback of 3$M_\odot$ after the SN explosion would be required in order to explain the kinematics of the system. In a more recent model, it is found that although a symmetric BH formation event cannot be formally excluded, the associated system parameters would be marginally consistent with the currently observed binary properties. It has been argued that BH formation mechanisms involving an asymmetric SN explosion with associated BH kick velocities of a few tens of km s^{-1}, may satisfy the constraints much more comfortably. However, it should be mentioned that the generally assumed distance and SN origin of the overabundances of α-elements observed in this Galactic BH-XRB binary have been challenged [6].

Stellar black holes in globular clusters

It is expected that in a typical globular cluster (GC) with present day masses of 105–106$M_\odot$, hundreds of stellar-mass BHs should be born during the first ∼10 Myr after formation. The specific frequency of X-ray binaries in GCs is ∼100 times larger than in the field and provides strong evidence that in GCs mass-transferring binaries are dynamically formed with high efficiency. If BHs are present in GCs, a subset should be detectable as accreting binaries. However, it is known that the majority of the most luminous X-ray binaries in GCs are accreting NSs since they are detected as sources of Type I X-ray bursts, which are thermonuclear explosions on the hard surface of accreting NSs. In the last decade BH candidates may have been identified in galactic and extragalactic GCs. It was assumed that these sources are candidate BH binaries because they have X-ray luminosities above the Eddington luminosities of NSs, and they vary significantly on short timescales, making it implausible that the luminosity could come from a superposition of several NS X-ray binaries. The most luminous ULXs so far identified are associated with a GC in the massive Virgo elliptical galaxy NGC 447260, which has a peak X-ray luminosity LX ∼ 4 × 1039 erg s^{-1}. Optical spectroscopy of the associated GC exhibits broad (1500 km s^{-1}) [OIII] emission but no Balmer lines. The few BHs possibly observed in extragalactic GCs would be those with either the most extreme accretion rates or very massive. They likely represent only the very tip of the iceberg in terms of BH X-ray binaries in GCs. Many BHs with lower accretion rates are almost certain to exist among X-ray sources in GCs, but they are greatly outnumbered by NS binaries and are difficult to distinguish from NSs using X-ray data alone. Because ULXs may also be powered by NSs, a new strategy for identifying quiescent BH X-ray binaries in Milky Way GCs makes use of both radio and X-ray data. Stellar-mass BHs accreting at low rates have compact jets which emit radio continuum via partially self-absorbed synchrotron emission. BHs are much more luminous in the radio than NSs with comparable X-ray luminosity; in fact LR/LX is ∼2 orders of magnitude higher for BHs than NSs. Before the recent upgrade to the VLA, the radio emission from a quiescent BH like V404 Cyg would not have been detectable at high significance at typical GC distances. The upgraded VLA can now readily detect the expected flux densities (tens of μJy) in reasonable exposure times. Using observations at radio frequencies two candidate stellar mass BHs in the core of the Galactic GC cluster M 22 have been found. These sources have flat radio spectra and 6 GHz flux densities of 55–60 μJy. As these sources are not detected in shallow archival Chandra imaging, they cannot yet be placed directly on the LX–LR relation; nevertheless, their overall properties are consistent with those expected from accreting BH binaries. Another BH candidate in a second Galactic GC, M62 (NGC 6266; D = 6.8 kpc) named M62-VLA1 was discovered. Unlike the former cases for the M22 sources, M62-VLA1 has clear X-ray and optical counterparts, and so it is the most compelling candidate BH X-ray binary in a Milky Way GC. It was believed that BHs formed by implosion in GCs fall to the center, where accreting BH X-ray binaries as XTE J1118+48043 and BH–BH binaries like GW150914 may then be formed through three-body interactions10 that lead to recoil velocities much larger than the escape velocities from typical GCs of few tens of km s^{-1}. However, it has been shown recently that core collapse driven by BHs (through the Spitzer "mass segregation instability") is easily reverted through three-body processes, and involves only a small number of the most massive BHs, while lower mass BHs remain well-mixed with ordinary stars far from the central cusp, suggesting that stellar BHs could still be present today in large numbers in many GCs[7].

Intermediate mass black holes in globular clusters

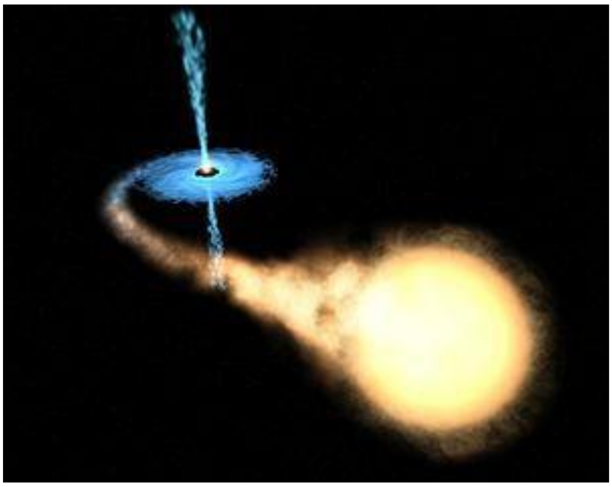
The formation process of supermassive BHs is still uncertain and one of the possible clues to understand their origin may reside in the evolutionary connection between stellar mass and supermassive BHs through the formation of intermediate mass BHs (IMBHs) of 102 to 104$M_\odot$. Although it is believed that such objects should be formed in dense stellar systems such as GCs, the observational evidences for their existence had been elusive. For instance, based on the lack of electromagnetic counterparts in X-rays64 and radio waves, in the GC 47 Tucana, which is at distance of ~4 kpc, upper limits of 470 and 2,060 $M_\odot$ had been placed on the mass of a putative IMBH. Recently, probing the dynamics of the GC 47 Tucana with pulsars it has been inferred the existence of a gas-starved IMBH with a mass of 2,200 (+1,500/-800) M□. The authors conclude that this BH is electromagnetically undetectable due to the absence of gas in the core within the radius of influence of the IMBH, and that IMBHs as the one in 47 Tuc may constitute a subpopulation of progenitor seeds that formed supermassive BHs in galaxy centers.

The Formation of Binary Stellar Black Holes

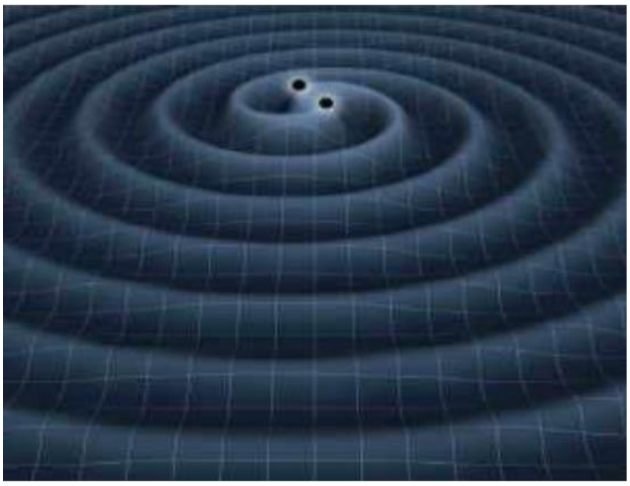
The first sources of gravitational waves detected by LIGO were mergers of stellar BHs101 and the question on how these BBHs may be formed is of topical interest. In previous sections have been presented observational results that are consistent with the idea that stellar black holes may be formed by the implosion of massive stars in the dark, without luminous natal SNe, with no ejection of a significant amount of baryonic matter, and with no energetic kicks, binary stars can be formed by one of the following three methods[8]:

Bbhs Formed by Isolated Evolution of Massive Stellar Binaries

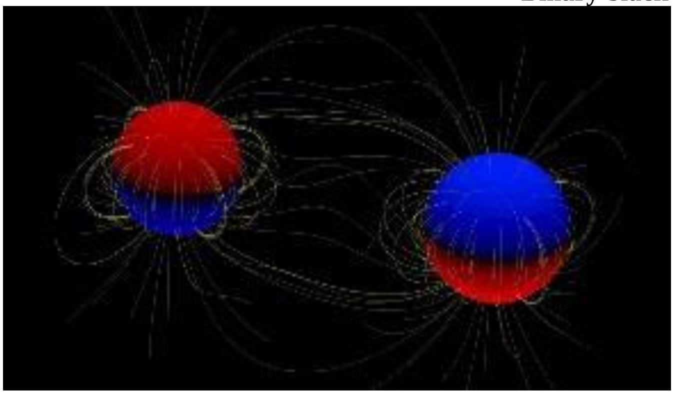
BH high mass X-ray binaries (BH-HMXBs) are an evolutionary stage of isolated massive stellar binaries, before BBH formation The BHs that merged producing the source of gravitational waves GW150914103 had masses of ~30$M_\odot$, much larger than the 5$M_\odot$ to 15$M_\odot$ stellar BHs found so far in the Milky Way, and it has been proposed67 that those BHs were formed by direct collapse. In Figure a is shown an example of a field binary evolution leading to a BH-BH merger similar to GW15091467. This model invokes mass transfer from the secondary to the first BH during common envelope, which still is a poorly understood evolutionary phase of BH-HMXRBs that leads to large uncertainties.



BH-HMXB



Binary black hole



Massive Stellar Binary

Bbhs Formed from Tight Binaries with Fully Mixed Chemistry

Another scenario is that of a massive over contact binary (MOB) that remains fully mixed as a result of their tidally induced spins. This chemically homogeneous evolutionary channel for BBH formation in tight massive binaries is insensitive to kicks smaller than the binary's orbital velocity. In this model BBHs originate from binaries in or near contact at the onset of hydrogen burning, which allows mixing of the burning products in the center throughout the stellar envelope, a process originally proposed for rotating single stars. At low metallicities MOBs will produce BBHs that merge within Hubble time with mass ratios larger than 0.55, as in GW150914.

Bbhs Formed by Dynamical Interaction in Dense Star Clusters

Alternative paths for the formation of the BBH progenitor of GW150914 by dynamical interactions in GCs. The paths assume that members of the BBH are formed with no energetic SNe or NKs that would disrupt the binary system or eject the BH components out from the cluster before BBH formation. The escape velocity from a typical GC is a few tens of km s^{-1} and BHs with kick velocities of hundreds of km s^{-1} as observed in some NSs would be ejected from typical GCs, unless BBHs are preferentially formed by dynamical interactions in nuclear clusters of $107M_{\odot}$ or more with sizes of only a few parsecs108. However, due to uncertainties in the conversion of luminosity to mass, the actual existence and frequency of such super-massive clusters is a matter of debate. For instance, from HST observations of nuclear clusters in the starburst galaxies M82 and NGC 5253 it has been estimated that the most massive nuclear clusters in these nearby galaxies have typical sizes of a few parsecs but masses between 7 x $104M_{\odot}$ and up to 1.3 x $106M_{\odot}$. Anyway, it is known that large numbers of NSs have been retained in GCs and must have been born with low or no overall kicks [9].

Conclusion

Stars run on hydrogen fusion and as anything that is not infinite in this universe, this fuel will come to end and the star will collapse inward due to gravity resulting in a black hole, but formation of black holes require stars with mass that only exceed the critical limit which is 3 M⊙ and any star that does not possess the critical mass will not eventually become a black hole rather it will become a neutron star.

Also black holes can be characterized by their mass, spin, and charge and can be classified into three categories regarding their stellar mass which can be either a stellar mass BH which has the size of 3-5M⊙ and a supermassive BH that possess the mass of 10-100 M⊙ and an intermediate BH with the mass range from 5-10 M⊙ which is not yet to be discovered.

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