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Simulation of Seismic Wave Propagation in Acoustic Medium using Staggered Grid Finite Difference Method

Emmanuel Anthony[1,*] , J.O Ozuomba[1] , Ikechuchu E. Nwosu[1] and I. Agbodike[1]
Department of Physics, Imo State University Owerri, Nigeria.

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ABSTRACT
Seismic wave propagation through acoustic medium allows us to understand response through a fluid saturated medium. This interaction has been described by the acoustic wave equation. In this work, the acoustic wave equation was written in coupled form and was discretized using the staggered grid finite difference (SGFD) method, which provides improved accuracy and efficiency of the numerical modeling and are naturally centered at the same point in space and time. To truncate artificial reflections from our computational boundary, we have applied the perfectly matched layer (PML) absorbing boundary condition. Our results show seismic wave propagating through a homogeneous medium and the effect of PML was clearly observed.

1. Introduction

In seismic exploration, numerical modeling plays an important role in studying the propagation of wavefields through the medium (or media). These wavefields are produced by a seismic source, which can be explosive and so on. These sources produce successive controlled energy, that propagates through the surface to the interior of the Earth. The earth is a non-homogeneous system made up of different layers and comprising of different strata. The mechanical and chemical properties of the earth change due to the change in the temperature and pressure with depth, and this results in the formation of strata based on density difference. The least dense outermost layer, the crust experiences most of the dynamics. The dynamics experienced by the earth result in several phenomena and these phenomena are random and non-linear in nature. It is easier to study a simple homogeneous system analytically, but to study a complex non-homogeneous system especially for exploration and their consequent processing, and inverse algorithm, numerical modeling is being done or required. Different types of models and schemes have been developed over the decades to carry out numerical modelling of the earth like inite difference method (FDM) (Zhu and McMechan, 1991; Dai et al., 1995; Wenzlau and Muller, 2009; Itz´a et al., 2016; O'Brien, 2010; Anthony and Vedanti, 2020), pseudo-spectral methods (Carcione, 1996b,a; Ozdenvar and McMechan, 1997), finite element method (Roberts and Garboczi, 2002) and spectral element (Morency, 2008). The FDM has been widely accepted in Geophysics especially exploration seismilogy as a good technique for numerical modeling. This method is easy to implement since the derivatives are represented as finite differences. It converts both the linear and non-linear differential equations into a system of equations, which can be further solved by the system of algebraic equations and can be easily computed by the modern gadgets. Unlike in the collocated finite difference scheme, were variables are defined at the same point on the grid, In the Staggered Grid Finite difference method (SGFD), variables are defined at different points on the grid. The staggered grid finite difference method which is placed halves the grid spacing, improves the accuracy and stability of the approximation. In this research work, we applied the SGFD to solved the coupled acoustic wave equation and truncate the artificial reflections from our computation boundary using perfectly match layer absorbing boundary conditions.

2. Theory

The first-order velocity-stress acoustic wave equation can be described as

$$\frac{\partial P}{\partial t} = \rho v^2 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\right)$$
$$\frac{\partial v_x}{\partial t} = \frac{1}{\rho}\left(\frac{\partial P}{\partial x}\right)$$
$$\frac{\partial v_z}{\partial t} = \frac{1}{\rho}\left(\frac{\partial P}{\partial z}\right) \qquad (1)$$

where P is the acoustic pressure, v is the wave propagation speed, ρ is the density and v_x and v_z are the particle velocities.

Substituting the above equations, we get the second-order acoustic wave equation which can be written as

$$\frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} \qquad (2)$$

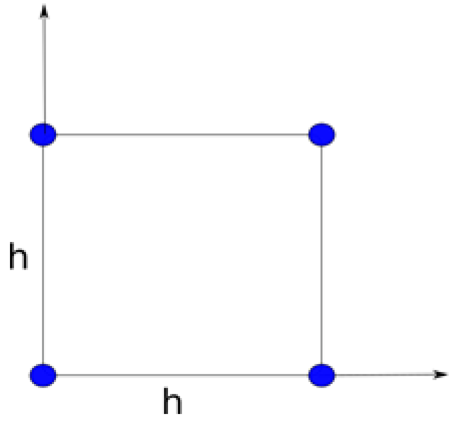
3. Staggered Grid Finite difference Method

The finite-difference (FD) method is a numerical methods or techniques most often ap- plied in geophysical research problems for wave propagation modelling (Virieux, 1984, 1986; Bohlen, 2002). The main or key concept of the FD method is that differential operators (spatial and temporal) are approximated on a discrete mesh. The approximate solution of the differential equation(s) is usually obtained at the mesh grid points.

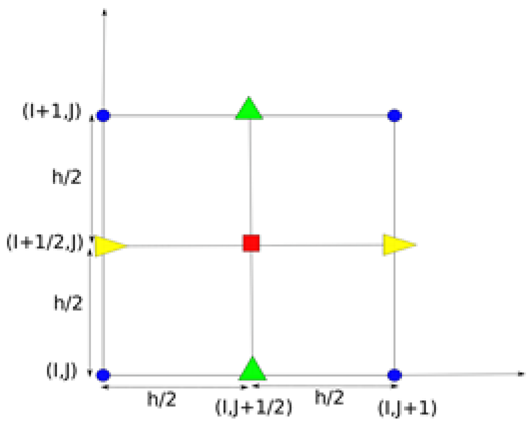
Tele:
E-mail address: kukuri2008@gmail.com

There are different types of finite difference schemes classified based on the stability, accuracy, and nature of the problem. Two of such schemes are:
1. Collocated Grid Finite difference method
2. Staggered Grid Finite difference method

In the collocated finite difference scheme, variables are defined at the same point on the grid, whereas in the staggered grid, variables are defined at different points on the grid (half point), as shown in Figure 1. Staggered grids are used to improve the accuracy and efficiency of the numerical modeling. One of the features that makes the staggered grid finite difference scheme outstanding compared to the collocated scheme is its ability to center differential operators naturally in space and time (Sheen et al., 2006).



(a)Collocated grid



(b) Staggered grid

Figure 1. Collocated and staggered finite difference.

To discretize the spatial derivatives of the governing equations, the staggered grids are used for variables and approximated by discrete operators.

If a spatial grid is defined by

$$v_{x=}v_x(x = i\Delta x, y = j\Delta y, z = k\Delta z, t) \quad (3)$$

The staggered grid finite-difference operators in the x-direction is written as

$$\frac{\partial v_x}{\partial t} = \frac{1}{\Delta x}\sum_{m=1}^{M} C_m \left(v_x\left(i+m-\frac{1}{2}, j\right) - v_x\left(i-m+\frac{1}{2}, j\right)\right) \quad (4)$$

where M is the length of the FD operator, c_m is the finite difference coefficients and the maximum number depends on the integer m

The staggered grid finite difference method is used in this work. Equation (1) is discretized using the staggered grid finite difference method and is given by

$$P^{n+1}_{i,j} = P^{n}_{i,j} + \frac{v^2\rho\Delta t}{\Delta x}\left(v^{n-\frac{1}{2}}_{i+\frac{1}{2},j} - v^{n-\frac{1}{2}}_{i-\frac{1}{2},j} + v^{n-\frac{1}{2}}_{i,j+\frac{1}{2}} - v^{n-\frac{1}{2}}_{i,j-\frac{1}{2}}\right.$$

$$vx^{n+\frac{1}{2}}_{i,j} = v^{n-\frac{1}{2}}_{i,j} + \frac{\Delta t}{\rho\Delta x}\left(P^{n}_{i+1,j} - P^{n}_{i,j}\right)$$

$$vy^{n+\frac{1}{2}}_{i,j} = v^{n-\frac{1}{2}}_{i,j} + \frac{\Delta t}{\rho\Delta x}\left(P^{n}_{i+1,j} - P^{n}_{i,j}\right) \quad (5)$$

The discretized form of the acoustic wave equation with the addition of the perfectly matched layer (PML) absorbing boundary condition in equation (7) is given by

$$Px^{n+1}_{i,j} = Px^{n}_{i,j}\left(\frac{1-0.5\sigma(x)\Delta t}{1+0.5\sigma(x)\Delta t}\right) + \frac{c^2\rho\Delta t}{\Delta x(1+0.5\sigma(x)\Delta t)}\left(vx^{n-\frac{1}{2}}_{i+\frac{1}{2},j} - vx^{n-\frac{1}{2}}_{i-\frac{1}{2},j}\right)$$

$$Py^{n+1}_{i,j} = Py^{n}_{i,j}\left(\frac{1-0.5\sigma(y)\Delta t}{1+0.5\sigma(y)\Delta t}\right) + \frac{c^2\rho\Delta t}{\Delta x(1+0.5\sigma(y)\Delta t)}\left(vy^{n-\frac{1}{2}}_{i+\frac{1}{2},j} - vy^{n-\frac{1}{2}}_{i-\frac{1}{2},j}\right)$$

$$P^{n+1}_{i,j} = Px^{n+1}_{i,j} + Py^{n+1}_{i,j} \quad (6)$$

$$vx^{n+\frac{1}{2}}_{i,j} = vx^{n-\frac{1}{2}}_{i,j}\left(\frac{1-0.5\sigma(x)\Delta t}{1+0.5\sigma(x)\Delta t}\right) + \frac{c^2\rho\Delta t}{\Delta x(1+0.5\sigma(x)\Delta t)}\left(p^{n}_{i+1,j} - p^{n}_{i,j}\right)$$

$$vy^{n+\frac{1}{2}}_{i,j} = vy^{n-\frac{1}{2}}_{i,j}\left(\frac{1-0.5\sigma(y)\Delta t}{1+0.5\sigma(y)\Delta t}\right) + \frac{c^2\rho\Delta t}{\Delta x(1+0.5\sigma(y)\Delta t)}\left(p^{n}_{i+1,j} - p^{n}_{i,j}\right)$$

4. Absorbing boundary condition (ABC)

To simulate an unbounded medium, ABC is often used to truncate reflections from the computational domain as seen in Figure 2. The perfectly matched layer(PML) proposed by Berenger (1994) is used in this research work to truncate the artificial reflections. This is given by

$$a_i = Log\left(\frac{1}{R}\right)\left(\frac{3v_p}{2}\right)\left(\frac{i^2}{L^3}\right)$$

where R is the reflection coefficient and L is the thickness of the PML

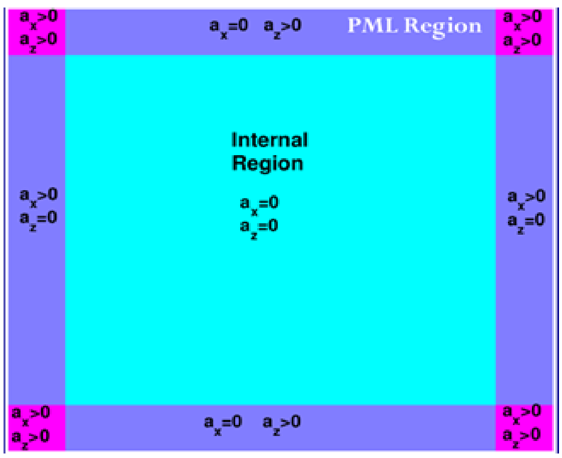


Figure 2. Computational domain and PML

The acoustic wave equation given in equation (1) is reformulated to include the perfectly matched layer (PML) boundary condition and is given by

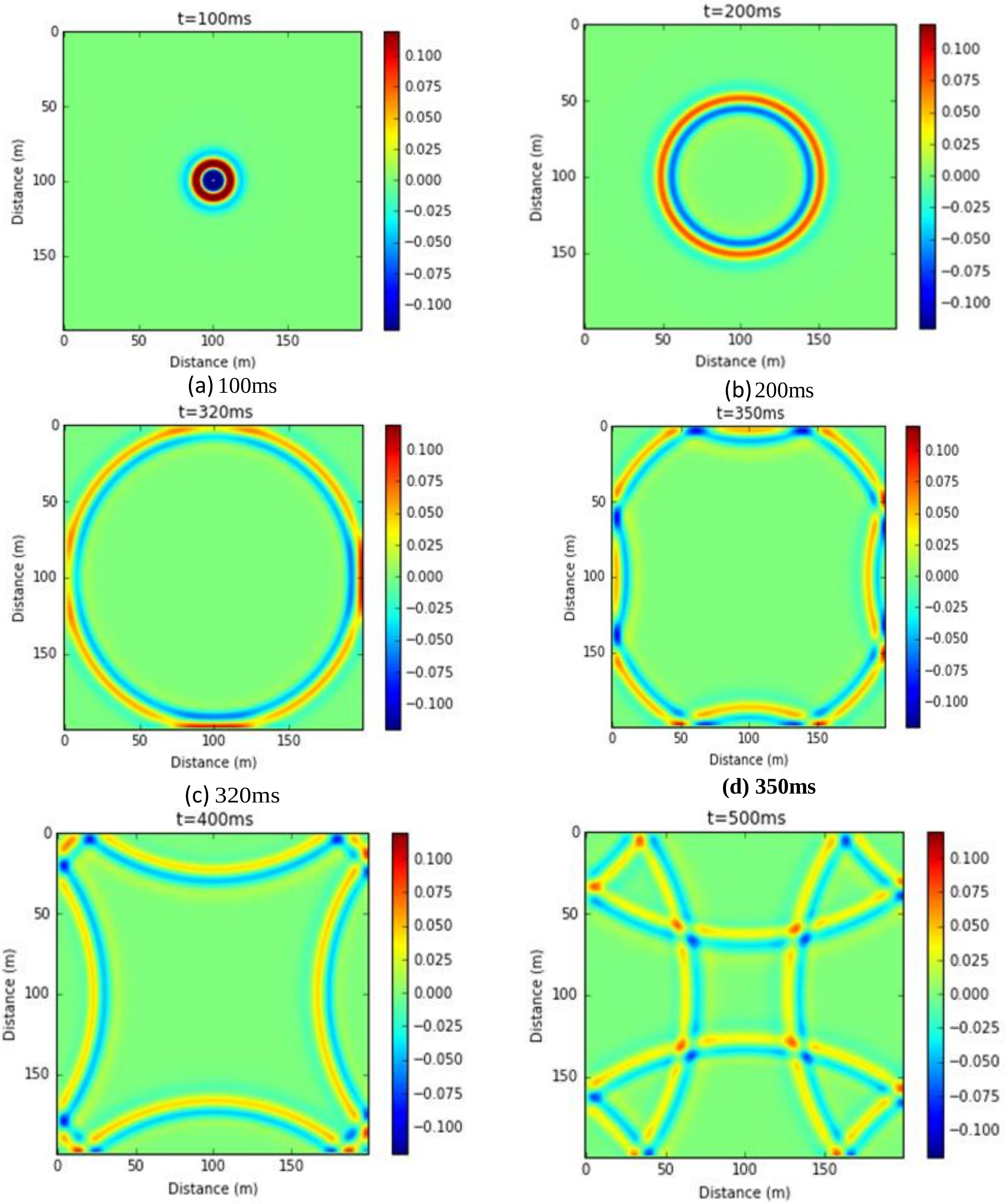
$$\frac{\partial p_x}{\partial t} + \sigma_x(x)p_x = \rho v^2 \left(\frac{\partial v_x}{\partial x}\right)$$
$$\frac{\partial p_y}{\partial t} + \sigma_y(y)p_y = \rho v^2 \left(\frac{\partial v_y}{\partial y}\right)$$
$$p = p_x + P_y$$
$$\frac{\partial v_x}{\partial t} + \sigma_x(x)p_x = \frac{1}{\rho}\left(\frac{\partial p_x}{\partial x}\right)$$
$$\frac{\partial v_y}{\partial t} + \sigma_y(y)p_y = \frac{1}{\rho}\left(\frac{\partial p_y}{\partial y}\right) \quad (7)$$

5. Numerical Examples

We considered a uniform homogeneous model of dimension 2000m x 2000m with grid spacing or size of 10m on both side, time step of 0.2 ms, velocity of 2000 m^2s and density of 2000 kg/m^3. Ricker wavelet is used as an explosive source with a dominant frequency of 10Hz. The source is located in the middle of the grid that is (*x*, *z*) = (1000*m*, 1000*m*). Figure 3 shows the snapshot of the seismic wave propagation in the acoustic medium for the homogeneous model considered in this work without the perfectly matched layer (PML) absorbing boundary condition. We can see that at time 100ms seen in Figure 3a, the wave continue to move until it got to the computational boundary at time 320ms in Figure 3c. From time 350ms and more, we see strong reflections from our computational boundary. This is expected has no boundary condition has been applied to truncate this effect. Figure 4 show the results from the seismic simulation of acoustic medium with the incorporation of PML absorbing boundary condition. This shows the effect of PML on the computational boundary as the seismic wave propagate. As we can see the wave begins to propagation at time 100ms (Figure 4a) just like in the previous case. When the wave got to the computational boundary in Figure 4c, we can see that the wave is being absorbed. This is unlike the previous case when no absorbing boundary condition was added and we can see reflections from our computational boundary. The situation is different with the incorporation of PML, as the artificial reflections are being absorbed. Figure 5 shows the shot gather for the acoustic wave medium with and without the incorporation of PML. We can also clearly see the waves getting absorbed in the computational boundary with the addition of PML.

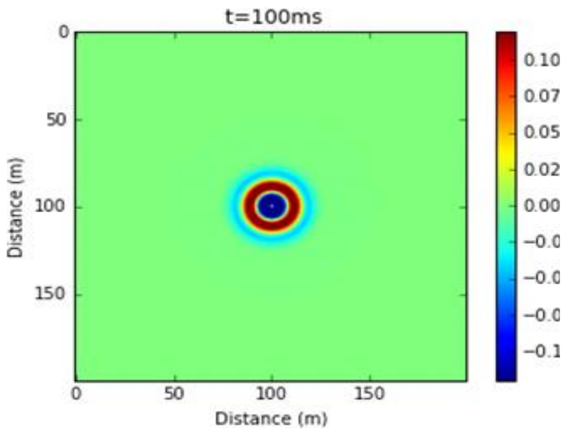
6. Conclusions

The coupled form of the acoustic wave equation was considered in this work. This equa- tion was written was discretized using the staggered grid finite difference (SGFD) method, which provide improved accuracy and efficiency of the numerical modeling and are natu- rally centered at the same point in space and time. The perfectly matched layer (PML) absorbing boundary condition was applied to truncate artificial reflections from our com- putational boundary. The results obtained show seismic wave propagating through a homogeneous medium and the effect of PML was clearly observed.

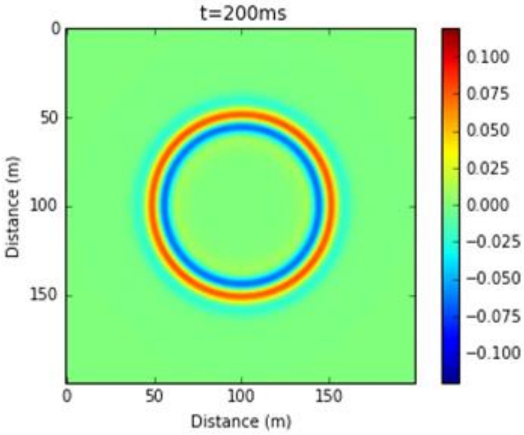


(a) 100ms (b) 200ms (c) 320ms **(d) 350ms** (e) 400ms (f) 500ms

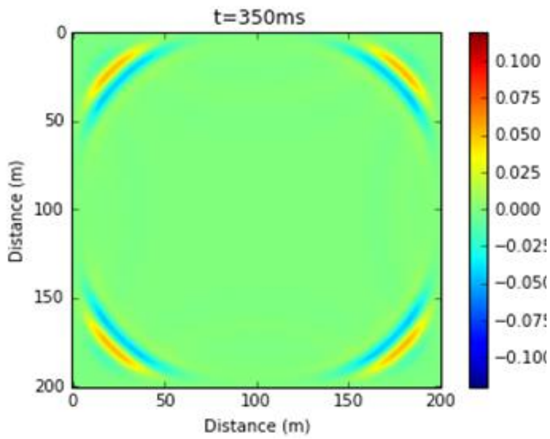
Figure 3. Snapshots of the acoustic wave simulation without PML for a homogeneous model (a) 100ms, (b)200ms, (c)320ms (d) 350ms (e) 400 and (f) 500ms.



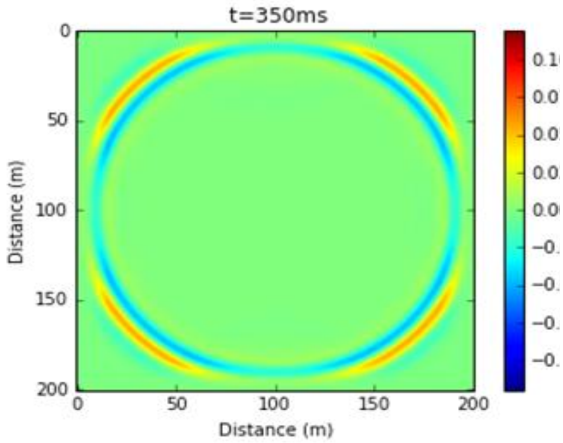
(a)100ms



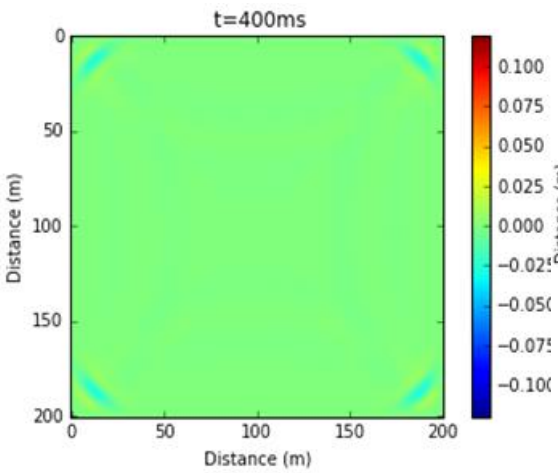
(b) 200ms



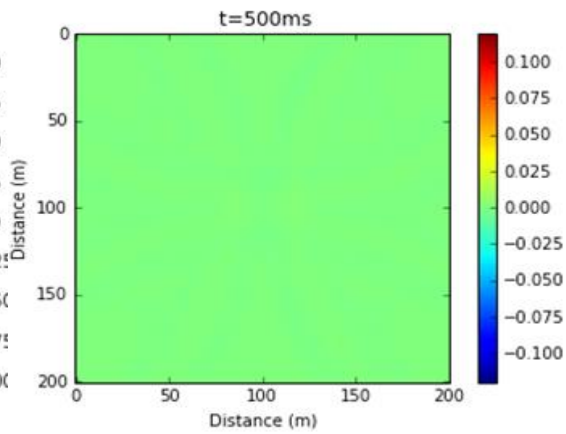
(c) 320ms



(d) 350ms

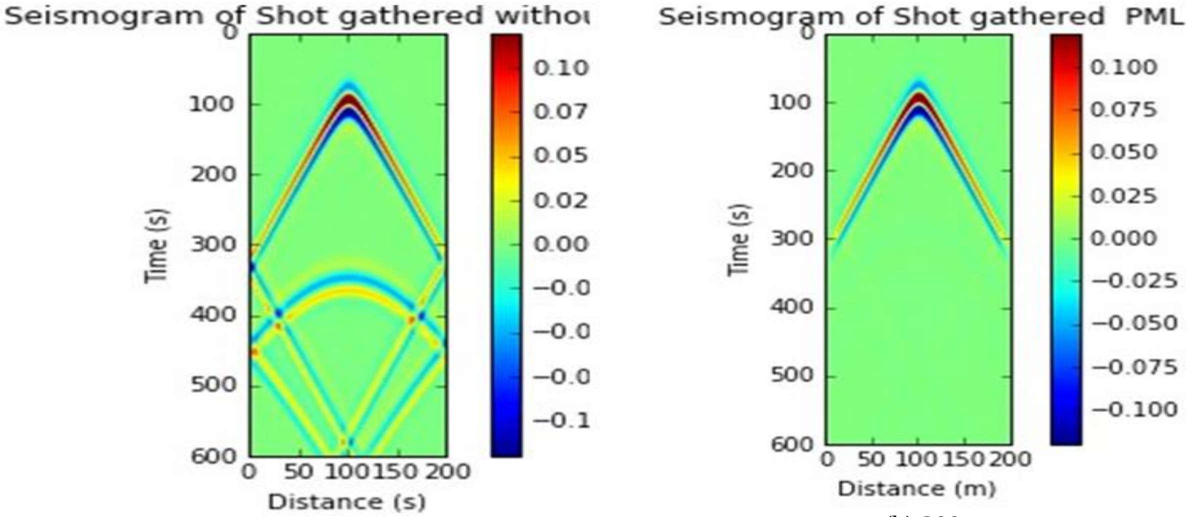


(e) 400ms



(f) 500ms

Figure 4. Snapshots of the acoustic wave simulation PML for a homogeneous model (a) 100ms, (b)200ms, (c)320ms (d) 350ms (e) 400 and (f) 500ms.



(a)100ms (b) 200ms

Figure 5. Shot gather for the acoustic wave equation (a) Without PML and (b) With PML.

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