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Analyzing a Supplier's Decision under the Combination of Long-Term and Penalty Contracts

V.R. Marasinghe

Faculty of Engineering Technology, The Open University of Sri Lanka, Nawala, Nugegoda, Sri Lanka.

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ABSTRACT

Game-theoretic approach is common in supply chain management to analyze the effect of controllable factors on the decision parameters. This study examines a combination of a long-term contract and a penalty contract for multiple periods between a single supplier and a retailer. Demand is stochastic but uniformly distributed over the selling seasons of periods. Supplier's capacity decision is required to be taken at the beginning of period before the retailer placing the order. The supply chain performance is observed by analyzing committed order quantity and the sole total profits of supplier and retailer.

Introduction

Applications of game-theoretic analysis to supply chain efficiency has more tendency to be focused on the construction of legal agreement models between retailers and suppliers for sequential periods of supply chain. In this research, the focus is being placed on a combination of a long-term contract and a penalty contract between an individual supplier and a retailer. The demand is stochastic and assumed to be independently and normally distributed over the period. The supplier's capacity decision is required to be taken at the beginning of period when actual demand is unknown which means that before receiving the orders from the retailer. The contract is developed at the beginning of the planning horizon based on the retailer's demand forecast. In general, the retailer has more details about customer demand and may have a tendency to inflate demand forecast for her own advantage, so that supplier would be overstocked, Cachon and Lariviere (2001).

In the proposing model, the supplier needs to build capacity at the beginning of each period to supply the ordering quantity, so the choice on capacity amount should be taken well ahead of time before getting the orders from the retailer. According to Özer et al. (2007), a wholesale contract which prompts the retailer's wait-and-decide strategy makes the supply chain deficient. Dong and Zhu (2007) and Cachon (2004) show that, by making use of wholesale price contracts, it is possible to enhance the efficiency of a supply chain, considering the ordering opportunities available to the retailer even after the decision for production has been made by the supplier. The supplier must build at least minimum committed capacities at the beginning of periods right from the beginning of the planning horizon.

With the costs associated with building the capacity, the decision on quantity of capacity building is a difficult choice for supplier since she should avoid the risk of over-capacity. The supplier has to order built products from external sources at a higher price to provide the demand of the period if her inventory is insufficient to restrain the retailer from out-sourcing. Long- term contracts are beneficial in term of profit potential for the supply chain. Besides, the penalty contracts can ensure that the supplier select a build capacity as her full profit potential is accomplished (Frascatore and Mahmoodi, 2006). With a long-term contract, the manufacturer and the retailer do not need to engage in a negotiation every time the retailer needs more units of inventory.

Generally, the supplier is stimulus to keep her capacity relatively low to abstain from getting excess capacity, which can lead to a capacity depreciation. To ensure that the final demand is satisfied, the retailer may expect higher supplier's capacity. Therefore, the retailer prefers to stimulate demand information to increase the decision on supplier's production capacity, (Cachon and Lariviere (2001)). That incrementing potential in retailer's demand forecast may result in overstock for the supplier. To minimize that tendency, in this research, the retailer is charged a unit penalty for excess order quantities exceeding committed quantity at the end of each period. Frascatore and Mahmoodi (2007) show that introduction of penalty for long-term contracts increases the supply-chain profit potential for any well-ordered, continuous demand-distribution. Many researches have elaborated the issue of risk aversion to be important in supply chain operation, (Özer et al. (2007) and Agrawal and Seshadri (2000)).

Overall, this mathematical model helps the supplier to determine the capacity level that should be built at the beginning of each period to maximize his profit.

Objective of work

Determine the capacity levels of the supplier that should be built at the beginning of each period in a planning horizon

Tele:
E-mail address: vishala.marasingha@gmail.com

so as to maximize his profit under the combination of a long-term contract and a penalty contract.

Scope and Limitations

This study examines a long-term contract and penalty will be charged to the retailer if she orders more than the committed quantity. The supplier has to order built products from external sources at a higher price to provide the ordering quantity of the retailer actual demand) if she cannot fulfil the order quantity of the period by her own capacity. The demand for each period is independently and normally distributed. Capacity depreciation of the supplier will not be considered. The retailer places the order in advance before the selling season. The excess capacity at the end of a period of the supplier cannot be used for the following period.

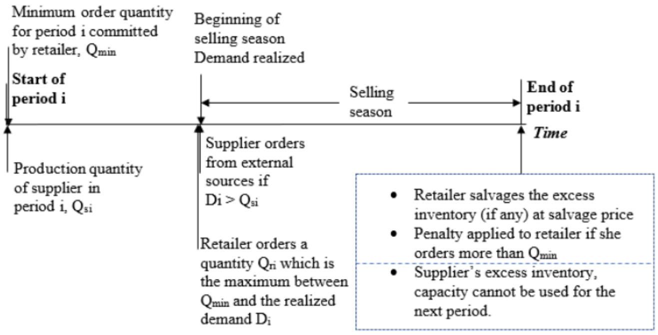

Figure 1. The sequence of events in a typical period i

Methodology

The research considers multiple periods during the planning horizon of the long-term contract. The model consists of a single supplier and a retailer. Let's denote the probability density function of demand Di (= x) to be f(x) and the cumulative density function as F(x), assuming that x~ N (μ, $б^2$) over the period. The following notations have been used in this research.

Q_{min} : Minimum committed order quantity of the retailer in a period

Q_i: Order quantity of retailer in period i (i = 1, 2, …, N)

Q_{si}: Built capacity of supplier at the beginning of period i, ($Q_{si} \geq Q_{min}$) (i = 1, 2, …, N)

D_i: Demand at retailer in period i (i = 1,2, …, N)

g_r : Unit order cost of retailer for order more than Qmin

C_c: Unit cost of capacity building of supplier

C_p: Unit production cost of supplier

w : Unit whole-sale price for committed order quantity (w < gr)

R : Retail-price (r > gr)

C_s: Unit- penalty cost of supplier for order from external source to fulfil retailer's order (cp + cs < gr)

v : Unit salvage price

- Retailer's profit function for a period i

Let Qi be the ordering quantity of the retailer for the period i. The retailer's ordering quantity should always exceed or be equal to the minimum committed quantity Q_{min}.

Qi = Max (Q_{min}, D_i)

There exist 2 scenarios;

$$Qi = \begin{cases} Q_{min}, & if\ Di \leq Q_{min} \\ Di, & if\ Di > Q_{min} \end{cases}$$

Equation 1. Ordering quantity of the retailer Qi

- Expected sales of the retailer for the period i -: E [Di]

In this study Qi ≥ Di for the both scenarios mentioned above. So, the retailer can always obtain a sufficient quantity that can fulfil the demand of each and every period i.

So, the expected sales would be;

E [Di] = $Di.\ f(Di)dDi$

Equation 2. Expected sales of the retailer for the period i

- Expected excess inventory of the retailer at the end of period i -: E [Qi - Di]

A remaining quantity would be existing for some particular periods at the end of the selling seasons, if and only if Qi > Di. Retailer sells that excess inventory at a unit salvage price at the end of the period.

$$\begin{aligned} E\,[Qi - Di] &= \int_0^{\infty} (\mathrm{Max}\,(Q_{min}, Di) - Di).\ f(Di)dDi \\ &= \int_0^{Qmin} (Q_{min} - Di).\ f(Di)dDi + \\ &\quad \int_{Qmin}^{\infty} (Di - Di).\ f(Di)dDi \\ &= \int_0^{Qmin} (Q_{min} - Di).\ f(Di)dDi \\ &= Q_{min}\int_0^{Qmin} f(Di)dDi - \int_0^{Qmin} Di.\ f(Di)dDi \\ &= Q_{min}.\ F(Q_{min}) - Di.\ F(Di)\,|_0^{Qmin} + \int_0^{Qmin} F(Di)dDi \\ &= \int_0^{Qmin} F(Di)dDi \end{aligned}$$

Equation 3. Expected excess inventory

- Expected amount ordered excess the minimum order quantity: E [Qi - Qmin]

In situations such that Di > Qmin, retailer orders the quantity of the demand Di for those particular periods. Retailer has to pay a higher unit cost gr (> whole-sale price) for each unit quantity that exceeds the minimum committed order quantity Qmin

$$\begin{aligned} E\,[Q_i - Q_{min}] &= \int_0^{\infty} [\mathrm{Max}\,(Q_{min}, Di) - Q_{min}].\ f(Di)dDi \\ &= \int_0^{Qmin} (Q_{min} - Q_{min}).\ f(Di)dDi + \\ &\quad \int_{Qmin}^{\infty} (Di - Q_{min}).\ f(Di)dDi \\ &= \int_{Qmin}^{\infty} (Di - Q_{min}).\ f(Di)dDi \\ &= \int_{Qmin}^{\infty} Di.\ f(Di)dDi - Qmin \int_{Qmin}^{\infty} f(Di)dDi \\ &= \int_0^{\infty} Di.\ f(Di)dDi - \int_{Qmin}^{\infty} Di.\ f(Di)dDi \\ &\quad - Q_{min}\,[1 - F(Q_{min})] \\ &= E[Di] - \{Q_{min}.\ F(Q_{min}) - F(Di)dDi - \\ &\quad Q_{min}\,[1 - F(Q_{min})] \\ &= E[Di] + \int_0^{Qmin} F(Di)dDi - Q_{min} \end{aligned}$$

Equation 4. Expected order quantity exceeding Q_{min}

- Expected profit of the retailer in period i

$$\pi r_i = r.E[Di] + v.E[Qi - Di] - w.Q_{min} - g_r.E[Qi - Q_{min}]$$

Equation 5. Expected retailer's profit for period i

- Total profit of the retailer

$$\pi_r = \sum_{i=1}^{N} \pi ri$$

Equation 6. Expected retailer's profit for period i

Decision variable of the retailer is: Q_{min}

- Supplier's profit function for a period i

Assume that production quantity of supplier in period i -: Min (Qsi, Qi)

- Expected production quantity of supplier in period i

$$\begin{aligned} E\,[\mathrm{Min}\,(Qsi, Qi)] &= \int_0^{\infty} [\mathrm{Min}\,(Qsi, \mathrm{Max}\,(Q_{min}, Di))].\ f(Di)dDi \\ &= \int_0^{Qmin} \mathrm{Min}\,(Qsi, Qi).\ f(Di)dDi + \\ &\quad \int_{Qmin}^{Qsi} \mathrm{Min}\,(Qsi, Qi).\ f(Di)dDi + \\ &\quad \int_{Qsi}^{\infty} (\mathrm{Min}\,(Qsi, Di)\ f(Di)dDi \\ &= \int_0^{Qmin} Qmin.\ f(Di)dDi + \int_{Qmin}^{Qsi} Di.\ f(Di)dDi \\ &\quad + \int_{Qsi}^{\infty} Qsi.f(Di)dDi \\ &= Qmin.\ F(Qmin) + \int_{Qmin}^{Qsi} Di.\ f(Di)dDi + \\ &\quad Qsi\,[1 - F(Qsi)] \end{aligned}$$

Equation 7. Expected supplier's production quantity for period i

- The amount the supplier should order from external sources to fulfil order of retailer in period i -: Qi – Min (Qsi, Qi)
- The expected order amount from external sources:

E [Qi – Min (Qsi, Qi)] = E[Qi] – E [Min (Qsi, Qi)]

Equation 8. Expected order amount from external sources

In which,

$$E[Q_i] = E\,[Max\,(Q_{min}, D_i)]$$
$$= \int_0^{\infty} Max\,(Q_{min}, D_i)\, f(Di) dDi$$
$$= \int_0^{Qmin} Q_{min}.\, f(Di).\, dDi + \int_{Qmin}^{\infty} Di.\, f(Di)dDi$$
$$= Q_{min}.\, F(Q_{min}) + Di.\, f(Di)dDi - Dif(Di)dDi$$
$$= Q_{min}.\, F(Q_{min}) + E[D_i] - [\, Q_{min}.\, F(Q_{min}) - F(Di)dDi]$$
$$= E[D_i] + \int F(Di)dDi$$

- Expected profit of supplier in period i

πsi = w. Qmin + gr. E [Qi - Qmin] – cc. Qsi - cp. E [Min (Qsi, Qi)] – cs. E [Qi – Min (Qsi, Qi)]

Equation 9. Qmin such that Пr is maximized

Total expected profit of supplier

$$\Pi_s = \sum_{i=1}^{N} \pi si$$

Equation 10. Q_{min} such that Π_r is maximized

Decision variables of supplier: Q_{si}'s

Qsi, such that πsi is maximized independently can be determined hence there is no relationship among periods in the profit function of the supplier

- Q_{si} such that Пsi is maximized

$$\pi_{si} = w.\, Q_{min} + gr.\{E[D_i] + \int_0^{Qmin} F(Di)dDi - Q_{min}\} - cc.\, Q_{si} -$$
$$cp.\, \{Q_{min}.F(Q_{min}) + \int_{Qmin}^{Qsi} Di.\, f(Di)dDi + Qsi\,[1- F(Qsi)]\}$$
$$- cs.\{E[D_i] + \int_0^{Qmin} F(Di)dDi - Q_{min}.F(Q_{min})$$
$$- \int_{Qmin}^{Qsi} Di.\, f(Di)dDi - Qsi\,[1- F(Qsi)]\}$$

$$\frac{d\Pi si}{dQsi} = - cc - cp.\, [Q_{si}.\, f(Q_{si}) + 1 - F_i\,(Q_{si}) - Q_{si}.\, f(Q_{si})]$$
$$- cs\,[- Qsi.\, f(Qsi) - 1 + F_i\,(Q_{si}) + Q_{si}.\, f(Q_{si})]$$

$$\frac{d\Pi si}{dQsi} = 0$$

$$F_i\,(Q_{si}) = 1 - \frac{cc}{cs - cp}$$

$$Q_{si} = F_i{-1}[\,1 - \frac{cc}{cs - cp}\,]$$

Equation 11. Q_{si} such that Π_{si} is maximized

- Q_{min} such that Пr is maximized

$$\Pi r = \sum_{i=1}^{N} (r.\, E[D_i] + v.\, E\,[Q_i - D_i] - w.Q_{min} - gr.E[Q_i - Q_{min}])$$
$$\Pi r = \sum_{i=1}^{N} (r.\, E[D_i] + v.\, \int_0^{Qmin} Fi(Di)dDi - w.\, Qmin$$
$$- gr.\{E[D_i] + \int_0^{Qmin} Fi(Di)dDi - Q_{min}\})$$
$$\frac{d\Pi r}{dQmin} = \sum_{i=1}^{N} (v.\, [Fi\,(Q_{min})] - w - gr.[Fi(Q_{min}) - 1])$$
$$\frac{d\Pi r}{dQmin} = \sum_{i=1}^{N} \{(v - gr).\, [Fi(Q_{min})] - (w - gr)\}$$

Equation 12. Q_{min} such that Π_r is maximized

Since Fi's are not the same for one period to another,

$(v - gr) \sum_{i=1}^{N} Fi\,(Q_{min}) - N\,(w - gr) = 0$

$\sum_{i=1}^{N} Fi\,(Q_{min}) = N\,(w{-}gr)\; v\,{-}gr\; N\; i{=}1$

Noted that:

$$\frac{d\Pi^2 r}{dQmin^2} = \sum_{i=1}^{N} fi\,(Q_{min}) > 0\; N\; i{=}1$$

$d\Pi r\; dQ_{min}$ is an increasing function w.r.t Q_{min}.

Also,

I. when Q_{min}=0, $d\Pi r\; dQ_{min}$ = -N(w-gr) > 0

II. when Q_{min} ∞, $d\Pi r\; dQ_{min}$ =N(v-gr) – N (w- gr) = N (v–w) < 0

So, the following equation has a unique solution of Q_{min}

$$y(Qmin) = (v - gr) \sum_{i=1}^{N} Fi(Qmin) - N(w - gr)$$

This equation is solved by MATLAB using bisection method

Numerical Experiments and Sensitivity Analysis

- Numerical Experiment

The numerical experiments are conducted in order to determine the effect of input parameters on minimum order quantity, supplier's capacity decision, total profit of retailer and supplier of the proposed contract

Assume that the demand is normally distributed over the period Di ~ N (μ, б 2)

4.1 Numerical Experiment

The given values are initially assigned to the parameters as follows

r = 200; Retail-price (r > gr)

w = 145; Unit whole-sale price for committed order quantity (w < gr)

gr = 150; Unit order cost of retailer for order more than Qmin

cc = 35 ; Unit cost of capacity building of supplier

cp = 25 ; Unit production cost of supplier

cs = 80 ; Unit- penalty cost of supplier for order from external source to fulfil retailer's order (cp + cs < gr)

v = 10 ; Unit salvage price

The solutions were obtained by using MATLAB to determine the effect of input parameters on the contract model. Total of twenty periods are considered (N=20) in the analysis.

Q_{min} = 40; Minimum committed order quantity of the retailer in a period

Π_s = 123110; Total profit of the supplier

Π_r = 76296 ; Total profit of the retailer

Q_{si}; Built capacity of supplier at the beginning of period ($Q_{si} \geq Q_{min}$), (i = 1, 2, …, N)

i	μ	б	Qsi
1	90	5	88
2	90	10	87
3	90	15	85
4	90	20	83
5	90	25	81
6	80	3	79
7	80	5	78
8	80	10	77
9	80	15	75
10	80	20	73

i	μ	б	Qsi
11	70	5	68
12	70	10	67
13	70	15	65
14	70	20	63
15	60	5	58
16	60	10	57
17	60	15	55
18	50	5	48
19	50	10	47
20	50	15	45

Figure 2. Supplier's initial build capacities

2. Sensitivity Analysis

Sensitivity-analysis is performed to analyze the effect of input parameters of the contract on the decision variables and total profits of retailer and supplier.

- Profit function w.r.t retail price

Figure 3. Profit functions vs. retail price

A linear increment in retailer's total profit can be observed when there is no change in the demand distribution of periods. There's no effect on supplier's profit and other variables when retail price changes.

• Qmin w.r.t. whole-sale price

Figure 4. Qmin vs whole-sale price

It can be seen that Q_{min} decreases when whole-sale price increases. This trend looks reasonable since the retailer's profit decreases simultaneously.

• Profit functions w.r.t. whole-sale price

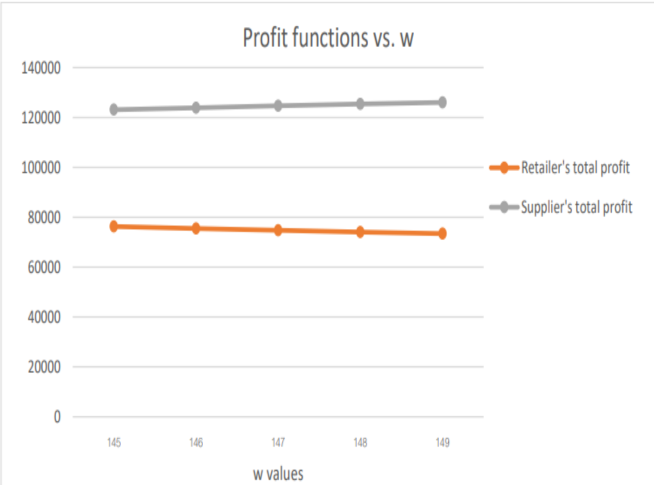

Figure 5. Profit functions vs. whole-sale price

Retailer's total profit shows a decreasing trend while supplier's total profit increases when whole-sale price increases. This trend is understandable because the increase in whole-sale price is favorable for the supplier but not for the retailer. It is noted that the change in whole-sale price will not affect the supplier's capacity decision

• Profit functions w.r.t. unit production cost

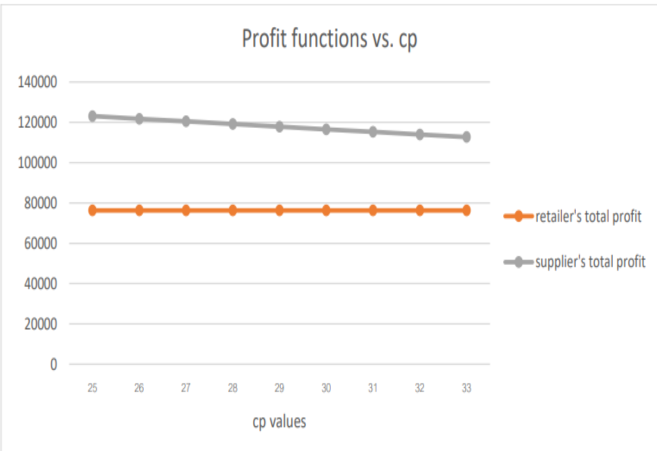

Figure 6. Profit functions vs. unit production cost

The graphs demonstrate that supplier's total profit decreases linearly while retailer's total profit remain constant when c_p increases. The supplier's capacity decision (Q_{si}) also decreases with the increase of c_p. There's no change in committed minimum order quantity: Q_{min}

• Profit functions w.r.t. unit capacity building cost

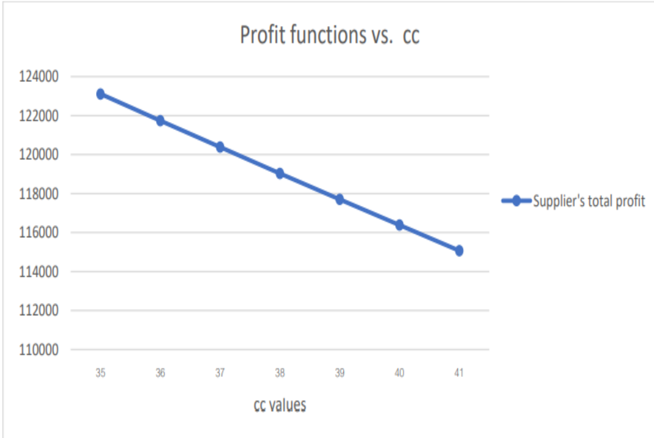

Figure 7. Profit functions vs. capacity building cost

Supplier's total profit shows a rapid linear decreasing trend when c_c increases. Supplier's capacity decision (Q_{si}) also decreases with the increase of c_c. There's no change in Q_{min} and retailer's total profit

• Q_{min} w.r.t. unit salvage cost

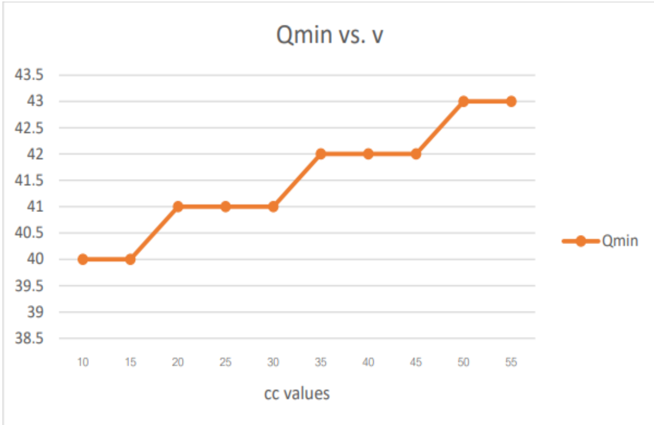

Figure 8. Qmin vs. salvage cost

From the results, it can be observed that Q_{min} increases with the when unit salvage value increases. This trend is reasonable since it is a profitable fact to the retailer, and exclusive of supplier's profit function

• Profit functions w.r.t. unit salvage value

Figure 9. Profit functions vs. salvage value

Retailer's total profit increases in the trend shown in the graph with the increase of unit salvage value. Supplier's total profit remains constant

•Q_{min} w.r.t. excess order unit cost

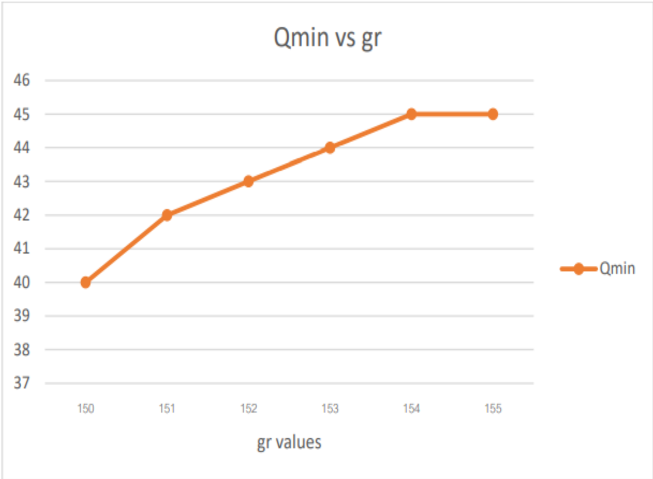

Figure 10. Qmin vs. excess order unit cost

Q_{min} increases with the increase of excess order unit price g_r. The trend is reasonable since ordering from external sources in order to fulfil the demand is unprofitable for the supplier.

•Profit functions w.r.t. order unit cost exceeding Q_{min}

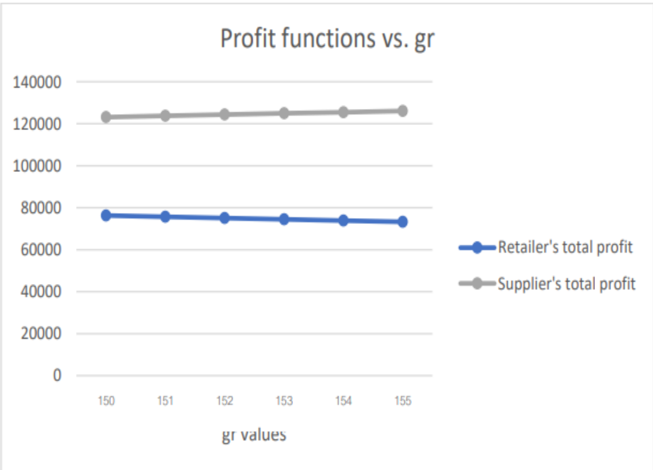

Figure 11. Profit functions vs. retailer's excess order unit cost

Supplier's total profit increases while retailer's total profit decreases. Retailer's total profit shows a higher decreasing rate, comparing with supplier's total profit increasing rate.

• Profit functions w.r.t. unit cost of orders from external sources

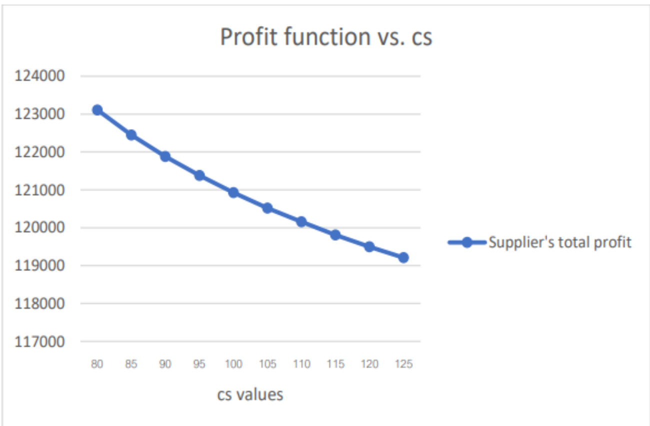

Figure 12. Profit functions vs. unit cost for orders from external sources

Q_{si} increases while supplier's total profit decreases when cs increases. It's reasonable that the effect of this cost on the decrease of the total profit of supplier. There's no change in Q_{min} and retailer's total profit.

4. Conclusion

The research investigates a combination of long-term and penalty contracts to determine supplier's capacity decision that should be built at the beginning of the period when demand is unknown. Demand of the model over the periods of the planning horizon is stochastic and assumed to be normally distributed. Mathematical analysis is conducted as a tool for examine the effect of contract parameters on total profits of both supply chain member individually for determine the optimal build capacity at the beginning of the period. The retail price(r), retailer's excess order price(gr) and whole-sale price(w) are exogenous, and the set of cost parameters (cc, cp, cs) depend on the supplier. The profit can be explored at the beginning of planning horizon based on expected demand distribution considering all the periods. Qmin is decided by the retailer based on an analysis to determine the minimum committed quantity where his total profit get maximized. Introducing such a quantity for the supply chain control and restrict the retailer from purchasing higher orders as desired which makes a capacity risk on the supplier and sub-stand the performance of the supply chain. Qsi should always be higher than Q_{min}.

Numerical experiment and sensitivity analysis of this research inspect the influence of each contract parameters on profit functions and decision variables. It is so important to study the trend and rate that is described in the chapter 4. The following table shows the effect that happens when increase the value of the input parameters in the model.

Table 1. Effect of Contract Parameters

Decision Variable	Effect on Contract Parameters			
	Qmin	Qsi	πr	πs
r	-	-	↑	-
w	↓	-	↓	↑
cp	-	↓	-	↓
cc	-	↓	-	↓
v	↑	-	↑	-
gr	↑	-	↓	↑
cs	-	↑	-	↓

It can be noted that increase of all the cost cp, cc, cs have decreased the total profit of the supplier. It is noted that cc makes slightly higher influence on decreasing supplier's capacity decision Qsi than cp. Increment in w is beneficial to the supplier. Its negative effect on retailer gets minimized since his decision on Qmin decreases. So, the profit loss of retailer becomes less than profit gain by the supplier due to increment of whole-sale price. w and gr can be adjusted through coordination when one of the parties is not satisfied with her profit.

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