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# Laplace Transform (Definition of Differential Influencer Method) and Application

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# ABSTRACT

After we covered the subject of Laplace transformations through two previous manuscripts. In this manuscript, we will discuss the definition of (Differential Influencer Method), which is a type of method for solving differential equations. As we explained through the example of the difference between (normal method) and (Laplace transform method) as mathematically supported solution methods.

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## Keywords

Differential Influencer, Normal Method, Laplace Transform Method.

## Introduction

In the first and second part of our series of articles on Laplace Transformations, we were working with researcher Alaa Aris Abdel-Raouf. Now in this part, which is the third part, our work will be with researcher Afifa Youssef Jaafar, where we will complete the last concepts related to Laplace transformations and conclude the topic, after this work we will intensify our efforts on other topics in the field of mathematics and non-mathematics.

## **Definition of Differential Influencer Method**

The first-order differential operator is defined as the one that takes the form  $d = \frac{d}{dx}$  and if it affects a function, what is the

product of its first derivative, dx?

Similarly, differential operators of the second and third order can be defined....., the nth order can be defined as:

$$D^{2} = \frac{d^{2}}{dx^{2}}; D^{3} = \frac{d^{3}}{dx^{3}}, \dots \dots D^{n} = \frac{d^{n}}{dx^{n}}$$
$$L(D) = a_{n}D^{n} + a_{n-1}D^{n-1} + D_{n-2}D^{n-2} + \dots + a_{0}$$

where:  $[a_i]_{i=0}^n$ 

aiin=0 are the coefficients of L(D)

Theory

If LD is a differential operator, then the following theorem is correct  $1 - L(D)e^{ax} = e^{ax}l(a)$ The proof :

$$\begin{split} L(D)e^{ax} &= (a_nD^n + a_{n-1}D^{n-1} + \dots + a_0)e^{ax} \\ &= a_na^ne^{ax} + a_{n-1}a^{n-1}e^{ax} + \dots a_1ae^{ax} + a_0e^{ax} \\ e^{ax}(a_na^n + a_{n-1}a^{n-1} + \dots + a_0) &= e^{ax}L(a) \\ &2 - L(D)e^{ax}f(x) &= e^{ax}L(D+a)f(x) \\ &\text{the proof} \\ D(e^{ax}f(x) &= e^{ax}(Df(x) + af(x) = e^{ax}(D+a)f(x)) \end{split}$$

 $e^{ax}(Df(x) + af(x) = e^{ax}L(D + a)f(x)$ 

$$\begin{split} l(d)e^{ax}(a_n(D+a)^n+a_{n-1}(D+1)^{n-1}+\dots+a_0)f(x) \\ &= e^{ax}(a_n(D+a)^n+a_{n-1}(D+a)^{n-1}+\dots+a_0)f(x) \\ &= e^{ax}(a_nD^n+a_{n-1}D^{n-1}+\dots+a_1D+a_0)e^{ax}f(x) \\ &= e^{ax}L(D+a)f(x) \end{split}$$

 $\begin{array}{l} 3-L(D^2)cos\ 3t=L(-k^2)cos\ kt\\ the\ proof:-\\ D^2(cos(kt))=D(-ksin\ kt)=-k^2cos(kt)\\ \mbox{Application}\\ Find\ the\ general\ solution\ to\ the\ differential\ equation:\\ y''-7y'+12y=8e^{3x}sin(3x)\\ The\ solution:\\ The\ characteristic\ equation\ is\ given:\\ \lambda^2-7\lambda+12=0\Rightarrow\lambda_1=3,\lambda_2=4\\ If\ the\ possible\ solution\ is\\ y_c=c_1e^{3x}+c_2e^{4x} \end{array}$ 

And your solution is  $y_p = \frac{1}{D^2 - 7D + 12}$ 

$$=\frac{8e^{3x}}{(D+3)^2-7(D+3)+12}sin(2x)=\frac{8e^{3x}}{D^2-D}sin(2x)$$
$$=\frac{2}{5}e^{3x}(2cos(2x)-4sin(2x))$$

So, the general solution to the given equation is:

$$y_p = c_1 e^{3x} + c_2 e^{4x} + \frac{2}{5} e^{3x} (2\cos(2x) - 4\sin(2x))$$
  
Theory

This theorem studies the method of finding the solution yp when Q(x) consists of two functions, one of which is the function x and the other is a function, let it be h(x) that is:

 $\begin{aligned} \mathbf{Q}(\mathbf{x}) &= \mathbf{x} \cdot \mathbf{h}(\mathbf{x}) \\ \text{Her private will be on the picture} \\ \mathbf{y}_p &= \frac{1}{f(D)} \mathbf{x} \mathbf{h}(\mathbf{x}) = \mathbf{x} \frac{1}{f(D)} \mathbf{h}(\mathbf{x}) - \frac{f(D)}{[f(D)]^2} \mathbf{h}(\mathbf{x}) \\ \text{the proof :-} \end{aligned}$ 

By influencing the function with the operator 1f(D) a number of times, we get Dxh(x) = xDh(x) + h(x)

 $\begin{aligned} D^{2}xh(x) &= xD^{2}(x) + 2Dh(x) \\ D^{n}xh(x) &= xD^{2}h(x) + \lambda D^{n-1}h(x) \\ \text{If the influencer image is:} \\ n \end{aligned}$ 

$$\mathbf{F}(\mathbf{D}) = \sum_{\lambda=0} \mathbf{a}_{\lambda} \mathbf{D}^{\lambda}$$

then:  $F(D)xh(x) = \sum_{\lambda=0}^{n} a_{\lambda}D^{\lambda}xh(x)$ 

$$= \sum_{\lambda=0}^{n} a_{\lambda} (x D^{\lambda} h(x) + \lambda D^{\lambda-1} h(x))$$
  
$$\Rightarrow F(D) x h(x) = x F(D) \cdot h(x) + F'(D) \cdot h(x)$$

Suppose that

 $h(x) = \frac{1}{F(D)}H(x)$ 

=

Substituting this relationship into the previous relationship:  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ 

$$\mathbf{F}(\mathbf{D})\mathbf{x}\left[\frac{\mathbf{I}}{\mathbf{F}(\mathbf{D})}\mathbf{H}(\mathbf{x})\right] = \mathbf{x}\mathbf{H}(\mathbf{x}) + \mathbf{F}'(\mathbf{D}) \dots \left[\frac{\mathbf{I}}{\mathbf{F}(\mathbf{D})}\right]\mathbf{H}(\mathbf{x})$$

 $\begin{aligned} xh(x) &= F(D)x\frac{1}{F(D)}h(x) - \frac{f'(D)}{f(D)}[H(x)] \\ \text{By acting on both sides with the operator} \frac{1}{f(D)} \text{ we get} \end{aligned}$ 

$$\left[\frac{1}{F(D)}xh(x)\right] = x\frac{1}{F(D)}H(x) - \frac{F'(D)}{[F(D)]^2}[H(x)]$$

Whereas your solution is:

$$y_{p} = \frac{1}{F(D)}xh(x) = x\frac{1}{F(D)}h(x) - \frac{F'(D)}{[F-(D)]^{2}} = h(x)$$

It is the required relationship

Compare the following equations with the two methods (normal method - Laplace transform method) Application

Solve the following equation **dT** 

$$\frac{\mathrm{d}T}{\mathrm{d}t} + \mathrm{KT} = 100\mathrm{K}, \mathrm{T}(0)$$

where k is a constant The solution : First: - Solving the equation in the normal way: Let's say p(t)=k $I(t, T) = e^{\int Kdt} = e^{Kt}$ 

We get I(t,T) we multiply the differential equation by the coefficient

## $d(Te^{Kt}) = 100Ke^{Kt|}gTe^{Kt} + Ke^{Kt}T = 100e^{Kt}$

We complete both sides and we get to

 $Te^{Kt} = \int 100 Ke^{Kt} dt = 100e^{Kt} + c$ 

 $T = 100 + Ce^{-Kt} \Rightarrow 50 = 100 + c \Rightarrow c = -50$ Secondly:

Solve the equation using the Laplace transform:

By taking the Laplace transform of both sides of the equation and setting  $\mathcal{L}[T] = t(s)$  We get  $\mathcal{L}\left[\frac{dT}{dT}\right] + K\mathcal{L}[T] = 100K\mathcal{L}[1]$ 

$$t(s) = \frac{50}{s+K} + \frac{100K}{S(S+K)}\rho[st(s) - 50] + Kt(s) = 100K\left(\frac{1}{s}\right)$$
  
We take the inverse Laplace transform

$$\mathbf{T} = \mathcal{L}^{-1}[\mathbf{st}(\mathbf{s})] = \mathcal{L}^{-1}\left[\frac{\mathbf{s}}{\mathbf{s}+\mathbf{K}}\right] + \left[\frac{100\mathbf{K}}{\mathbf{s}(\mathbf{s}+\mathbf{K})}\right]$$

Partial fractions are used =  $\mathcal{L}^{-1}\left[\frac{50}{S+K} + \frac{100}{S} + \frac{-100}{S+K}\right]$ 

$$=\mathcal{L}^{-1}\left[\frac{-50}{s+k}+\frac{100}{s}\right]=-50\mathcal{L}^{-1}\left[\frac{1}{s+k}\right]+100\mathcal{L}^{-1}\left[\frac{1}{s}\right]$$

 $\therefore -50e^{-Kt} + 100$ 

# Application:

Find the solution to the following equation

 $\frac{dQ}{dt} + 0.04Q = 3.2e^{-0.04t}$ The solution :-First: - Solve the equation in the normal way: -We assume that:  $Q(t) = 0.04, g(T, Q) = e^{\int 0.04dt} = e^{0.04t}$ We multiply the equation I(t Q)

we multiply the equation 
$$I(t,Q)$$
  
 $e^{0.04t} \frac{dQ}{dt} + 0.04e^{0.04t}Q = 3.2, \frac{d}{dt}(Qe^{0.04t}) = 3.2$ 

Integrate both sides  $Qe^{0.04t} = 3.2t + c$ 

The solution is:

Second: - The solution using the Laplace transform: The solution:

$$\frac{\mathrm{d}Q}{\mathrm{d}T} + 0.04\mathrm{Q} = 3, 2\mathrm{e}^{-0.04\mathrm{t}}, \mathrm{Q}(0) = 0$$

By taking the Atlas transform for both sides of the equation and setting L(Q)=q(s)

$$\begin{split} \mathcal{L} \left[ \frac{dQ}{dT} \right] &+ 0.04 \mathcal{L}[Q] = 3.2 \mathcal{L}[e^{-0.04t}] \\ &= [sq(s) - 0] + [0.04q(s)] = 3.2 \frac{1}{s + 0.04} \cdot q(s) = 3.2 \frac{1}{(s + 0.04)^2} \end{split}$$

We take the inverse Laplace transform

Q = 3.2
$$\mathcal{L}^{-1}\left[\frac{1}{(s+0.04)^2}\right]$$
 = 3.2te<sup>-0.04t</sup>

The solution is  $3.2te^{-0.04t3}$ Application (3-9):-Solve the following differential equation:

$$\frac{d^2Q}{dt^2} + 8\frac{dQ}{dt} + 25Q = 0$$
  
Solution

# **First: Normal method**

The characteristic equation is:  $\mathcal{L}(\mathbf{Q}) = \mathbf{q}(\mathbf{s})$ 

We use the general law to analyze this equation of the second degree in order to get the roots of the equation

 $\mathcal{L} \bigg[ \frac{dQ}{dT} \bigg] + 0.04 \mathcal{L}[Q] = 3.2 \mathcal{L}[e^{-0.04t}]$ 

We substitute a,b,c into the law

So the roots of the equation are two imaginary and conjugated

 $Q = e^{-4t}(c_1 \cos 4t + c_2 \sin 3t)$ 

The special solution  $iQ_p = A_0 \sin 3t + B_0 \cos 3t$ s

We substitute Qp and derivative into the differential equation and arrange the terms

# $(16A_0 - 24B_0)\sin 3t + (24A_0 + 16B_0)\cos 3t = 50\sin 3t + 0\cos 3t$

The boundary coefficients are A0= $\frac{50}{5^2}$ , B0= $\frac{-75}{5^2}$ We add (1) and (2) and then we substitute A0, B0 So the general solution is

$$Q = e^{-4t}(C_1\cos 3t + C_2\sin 3t) + \frac{50}{5^2}\sin 3t - \frac{75}{5^2}\cos 3t$$

Second: The solution using the Laplace transform

$$\frac{d^2Q}{dt^2} + 8\frac{dQ}{dt} + 25Q = 50\sin 3t$$
 ,  $Q'^{(0)} = 0$  ,  $Q(0) = 0$ 

$$s^{2}Lq(s) - sLq(0) - Lq'^{(0)} + 8sLq(s) - Lq(0) + 25Lq(s) = 50L[\sin 3t]$$

$$= Lq(s)[s^2 + 8s + 25] = \frac{150}{s^2 + 9}$$

$$q(s) = \frac{150}{(s^2 + 9)(s^2 + 8s + 25)}$$

$$=\frac{75}{26}\frac{1}{s^2+9}-\frac{75}{5^2}\frac{s}{s^2+9}+\frac{75}{26}\frac{1}{(s+4)^2+9}+\frac{75}{5^2}\frac{s+4}{(s+4)^2+9}$$

$$\therefore q' = \frac{25}{25} \sin 3t - \frac{75}{5^2} \cos 3t + \frac{25^{-4t}}{2^6} \sin 3t + \frac{25^{-4t}}{2^6} \sin 3t + \frac{75^{-4t}}{5^2} \cos 3t$$

$$\frac{25}{7^2} (2\sin 3t - 3\cos 3t) + \frac{25}{7^2} e^{-4t} (3\cos 3t + \sin 3t)$$

$$\frac{25}{5^2}(2\sin 3t - 3\cos 3t) + \frac{25}{5^2}e^{-4t}(3\cos 3t + \sin 3t)$$

## Application

Find the solution to the initial value problem:  $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 3\frac{dy}{dy} = 0 \quad y(0) = 4, y'^{(0)} = -4$ The solution

First, the normal method: We put the equation in the form  $(\triangle^3 + 2 \triangle^2 - 3 \triangle)y = 0$ We assume that y=e $\lambda$ x auxiliary equation is 3+22-3 $\lambda$ =0 $\Rightarrow$  $\lambda$ =0,1,3 56488 Mohammad Abdul Hameed Jassim Al Kufi / Elixir Applied Mathematics 170 (2022) 56484 - 56493  $\therefore \lambda(\lambda-1)(\lambda+3)=0$ And the solution is:  $y = C_1 + C_2 e^x + C_3 e^{-3x}(1)$ To find the solution that satisfies the initial condition, we find  $y' = c_2 e^x - 3 c_3 e^{-3x}$  $\mathbf{v}'' = \mathbf{c}_2 \mathbf{e}^{\mathbf{x}} + 9\mathbf{c}_3 \mathbf{e}^{-3\mathbf{x}}$ Substitute the initial conditions from equation: :.4=c1+c2+c3 4⇐y0=4  $8=c2-3c35 \Leftarrow y0=8$ 0=c2+9c36⇐y"0=-4 Solving equations 6,5,4, we find that c1=0, c2=5, c3=-1 And the solution is in the form  $\mathbf{v} = \mathbf{5}\mathbf{e}^{\mathbf{x}} - \mathbf{e}^{-3\mathbf{x}}$  $\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} + 2\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 3\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ y''' + 2y'' - 3y' = 0 $y(0) = 4, y'^{(0)} = 8, y''^{(0)} = -4$ The solution : By taking Laplace L(y''') + 2L(y'') - 3L(y') $s^{3}y(s) - s^{2}y(0) - y^{\prime\prime(0)} + sy^{\prime(0)} + 2\big[s^{2} - sy(0) - y^{\prime}(0)\big] - 3[sy - y(0)] = 0$ Using the terms  $p''y - 4p' - 8p + 4 + 2p^2y - 8p - 16 - 3py + \lambda = 0$  $p''y(s) + 2p^2y - 3py = 4p^2 + 16p$  $y(p^3 + 2p^2 - 3p) = \frac{4p^2 + 16p}{p(p^2 + 2p - 3)}$  $y = \frac{4p^2 + 16p}{p(p-1)(p+3)}$ Using partial fractions:  $\frac{A}{P} + \frac{B}{P-1} + \frac{C}{P+3}$  $\frac{4P^2 + 16P}{P(P-1)(P+3)} = \frac{A(P-1)(P+3) + BP(P+3) + CP(P-1)}{P(P-1)(P+3)}$  $4P^{2} + 16P = AP^{2} - 2AP - 3A + BP^{2} + 3B + CP^{2} - C4 = A + B + C$ 16 = 2A + 3B - C0 = -3A (3)  $A = 0 \ 20 = 4B$ , B = 5We substitute the value of A, B to find C 4 = 0 + 5 + C C = -1

 $\frac{0}{P}+\frac{5}{P-1}-\frac{1}{P+3}$ 

Finding inverse Laplace

 $5e^x - e^{-3x1}$ 

#### Conclusion

This part relates to the problems that have been solved by Laplace and the equations, and we note the clear difference in the ease of solving one, but in differential equations we solve first and then apply the initial conditions to find the optional constants and we find that it takes place in two steps instead of one step as in Laplace.

This does not mean that we do not use the normal method, meaning that it is not useful, but rather with the aim of reaching the solution in an easier and faster way, in addition to the stressful mental operations, and that we fully know that mathematics is a cumulative science that depends on what has been reached in advance, and that if we reached the normal method, we would not have reached the methods. Others such as Laplace and others.

We talk about solving applied problems (physics, chemistry, and engineering) using differential equations and the Laplacian transformation.

#### **Applications-First the physics**

A 120-ib weight is suspended in a 6416f tail. The weight starts the motion without initial velocity with a displacement of 6in above the equilibrium position and at the same time the effect of an external force  $f(t)=8\sin 4t$  on the weight, find the resulting motion of the weight assuming ignoring the air resistance.

The solution :

We have a=0,k=4,m=64,ft=8sin 4t

So, the equation becomes

x+16x=2sin 4t

Thus, this problem is an example of a coercive diminishing motion, and the solution to the accompanying homogeneous equation is:

xh=c1cos 4t + c2sin 4t

Find the solution for the method of unrelated equations

$$x_p = -\frac{1}{4}t\cos 4t$$
$$X = c_1\cos 4t + c_2\sin 4t - \frac{1}{4}t\cos 4t$$
Note that x, you at two

Note that  $x \rightarrow \infty$  at  $t = \infty$ 

this phenomenon is called net resonance. It is the result of the coercive function f(t) having the same circular frequency as that of the concomitant undiminished free system

As for the Laplace transform:

$$x + 16x = 2\sin 4t$$
  $x(0) = -\frac{1}{2}, X(0) = 0$ 

This unknown equation for the unknown function x(t) in the independent variable t, we put xs=Lx(t) by taking the Laplace transform of the given differential equation

$$[s^{2}x(s) - sx(0) - x(0)] + 16x(s) = 2\left(\frac{4}{s^{2} + 16}\right)$$

$$\left[s^{2}x(s) - s\left(\frac{1}{2}\right) - 0\right] + 16x(s) = \frac{8}{s^{2} + 16}$$

$$(s^{2} + 16)x(s) = \frac{8}{s^{2} + 16} - \frac{s}{2}$$
$$x(s) = \frac{8}{(s^{2} + 16)^{2}} - \frac{1}{2} \left(\frac{s}{s^{2+16}}\right)$$

and with a=0

$$\mathbf{x}(t) = \mathcal{L}^{-1}[\mathbf{x}(s)] = \mathcal{L}^{-1}\left[\frac{8}{(s^2 + 16)^2} - \frac{1}{2}\left(\frac{s}{s^2 + 16}\right)\right]$$

$$= \frac{1}{16} \mathcal{L}^{-1} \left[ \frac{128}{(s^2 + 16)^2} \right] - \frac{1}{2} \mathcal{L}^{-1} \left[ \frac{s}{s^2 + 16} \right]$$
$$= \frac{1}{16} (\sin 4t - 4t \cos 4t) - \frac{1}{2} \cos 4t$$

#### **Application:**

A body of mass 64 pounds falls from rest under the influence of gravity and is also affected by the force of air resistance R = 8v in pounds, where V is the velocity in feet per second V in terms of time t, use g = 32 feet per second

where g is acceleration due to gravity

# 56490 Mohammad Abdul Hameed Jassim Al Kufi / Elixir Applied Mathematics 170 (2022) 56484 - 56493 Applying Newton's second law, we get:

 $\frac{64}{32}x'^{(t)} = 64 - 8x(t)$ 

x'(t) + 4x(t) = 32This equation is linear and its solution is when x=0,t=0 And therefore:  $x = ce^{-4t} + 8$ 

 $\mathbf{x} = \mathbf{c}\mathbf{e}^{-1} + \mathbf{a}$ 

 $0 = ce^{-4(0)} + 8$ 

 $\mathbf{s}\mathbf{x}(\mathbf{s}) + \mathbf{4}\mathbf{x}(\mathbf{s}) = \mathbf{32}$ 

 $x(s) = \frac{32}{s(s+4)} = \frac{8}{s} - \frac{8}{s+4}$ 

Using the inverse Laplace transform, we get:

$$x(s) = 8 - 8e^{-4t}$$

Second, chemistry.

## Application:

A reservoir initially contains 100 gallons of brine containing one pound of salt, another brine containing one pound of salt per gallon flows into the reservoir at a rate of 3 gallons/min when t = 0, and at the same moment a well-mixed mixture comes out of the reservoir at the same rate

Find The amount of salt in the warehouse at any moment t

The solution :

We have b=1 ,a=1, v=100, e=f=3 So the equation becomes

# $\frac{\mathrm{dx}}{\mathrm{dt}} + 0.03\mathrm{x} = 3$

The solution to this linear differential equation is  $\mathbf{x} = \mathbf{c}\mathbf{e}^{-0.03t} + \mathbf{100}$ 

t = 0, X = 0

 $1 = ce^0 + 100$ , c = -99The equation can be written in the form

 $X = 100 - 99e^{-0.03t}$ As for Laplace

Taking inverse Laplace:

$$sx(s) - s(0) + 0.03x(s) = 3\frac{1}{s}$$
  

$$s(0) = 1$$
  

$$x(s)(s + 0.03) = \frac{3}{s} + 1$$
  

$$x(s) = \frac{3}{s(s + 0.03)} + \frac{1}{(s + 0.03)}$$
  

$$\frac{100}{s} - \frac{100}{s + 0.03} + \frac{1}{s + 0.03}$$

$$\mathcal{L}^{-1}\left(\frac{3}{s(s+0.03)}\right) + \mathcal{L}^{-1}\left(\frac{1}{(s+0.03)}\right)$$

 $100 - 100_{e^{-0.03t}} + e^{-0.03t}$ 

 $100 - 99e^{-0.03t}$ 

## **Application:**

A 50\_gal tank contains 10 gallons of fresh water. A brine containing one pound of salt per gallon flows into the tank at a rate of 4 gallons/min when t = 0 and at the same moment the mixture well mixed with the tank comes out at a rate of 2 gallons/min. The solution:

The equation in this problem becomes:

 $\frac{dx}{dt} + \frac{2}{10+2t}x = 4$ 

It is a linear equation and its solution is  $\frac{10 + 21}{10 + 21}$ 

 $\mathbf{x} = \frac{\mathbf{40t} + \mathbf{4t}^2 + \mathbf{c}}{\mathbf{10} + \mathbf{2t}}$ When x=0, t=0 then c=0 So

 $\mathbf{x} = \frac{40t + 4t^2\mathbf{1}}{10 + 2t}$ Third engineering:

## **Application:**

In the figure shown it is: I1(0)=I2(0)=I3(0)=0 Set I1,I2,I3 at the moment tsec I after the circuit is closed The solution:

Applying Kirchhoff's law to the left circle, we get:



 $10I_1(t) + 10I_3(t) + 10I'_1(t) = 100$ It is the right circle

 $\begin{aligned} 10I_2(t) &+ 10I'_2 - 10I_3(t) = 0 \\ & \text{Using that} \\ & \text{We get:} \\ I_{3(t)} &= I_1(t) - I_2(t) \end{aligned}$ 

 $2I_1(t) - I_2(t) + {I'}_1(t) = 10$ 

 $2I_2(t) - I_1(t) + I'_2(t) = 0$ 

$$2I_1(s) - I_2(s) + sI_1(s) = \frac{10}{s}$$

 $2I_2(s) - I_1(s) + sI_2(s) = 0$ 

56492 Mohammad Abdul Hameed Jassim Al Kufi / Elixir Applied Mathematics 170 (2022) 56484 - 56493  $(2+s)I_1(s) - I_2(s) = \frac{10}{s}$  $(2+s)I_1(s) - I_1(s) = 0$  $[-1 + (2 + s)^2]I_2(s) = \frac{10}{s}$  $I_2(s) = \frac{10}{s[(2+s)^2 - 1]}$  $I_2(t) = 10e^{-2t}\mathcal{L}^{-1}\left[\frac{1}{(s-2)(s^2-1)}\right]$  $10e^{-2t}\mathcal{L}^{-1}\left[\frac{1}{3}\frac{1}{(s-2)}-\frac{1}{2}\frac{1}{(s-1)}+\frac{1}{6}\frac{1}{(s+1)}\right]$  $= 10e^{-2t}\left(\!\frac{e^{2t}}{3}\!-\!\frac{e^t}{2}\!+\!\frac{e^{-t}}{6}\!\right)$ Then: =  $\frac{5}{3}(2 - 3e^{-t} + e^{-3t})$  $\frac{10}{3}(2-3e^{-t}+e^{-3t})+\frac{5}{3}(3e^{-t}-3e^{-3t})$  $= \frac{5}{3}(4 - 3e^{-t} - e^{-3t})$  $I_3(t) = i_1(t) - I_2(t)$  $= \frac{5}{3}(4 - 3e^{-t} - e^{-3t}) - \frac{5}{3}(2 - 3e^{-t} + e^{-3t})$  $\frac{10}{3}(1-e^{-3t})^1$ **Application:** 

The RL circuit has a voltage of 5 volts and a resistance of 50. The circuit at any moment t The solution:

We have

 $\frac{dX}{dt} + 50x = 5$ 

It is a partial equation, and its solution is when x=0, t=0

$$x = ce^{-50t} + \frac{1}{10}$$
  

$$0 = ce^{0} + \frac{1}{10}$$
  

$$c = -\frac{1}{10}$$
  

$$x = \frac{1}{10} - \frac{1}{10}e^{-50t}$$
  
In Laplace, at X(S) =0

$$\mathbf{s}\mathbf{x}(\mathbf{s}) - \mathbf{x}(\mathbf{0}) + \mathbf{50}\mathbf{x}(\mathbf{s}) = \mathbf{5}\left(\frac{1}{\mathbf{s}}\right)$$

 $\mathbf{x}(\mathbf{s}) = \frac{5}{\mathbf{s}(\mathbf{s} + 5\mathbf{0})}$ Using Partial Fractions and Inverse Laplace

$$\mathbf{x}(\mathbf{t}) = \mathcal{L}^{-1} \left[ \frac{5}{\mathbf{s}(\mathbf{s} + \mathbf{50})} \right]$$

$$\mathcal{L}^{-1}\left[\frac{710}{s} - \frac{710}{s+50}\right] = \frac{1}{10} - \frac{1}{10}e^{-50t}$$

#### Results

Through the research, the researchers were able to identify the solution of differential equations by algebraic methods and to solve differential equations by Laplace method separately and then compare the solution with the two methods. The researchers concluded that the comparison can be summarized in the following points:

1) The Laplace method contains fewer solving steps than the algebraic method, which helps to reduce the error rate.

2) Ease of dealing with the properties that help in solving problems, as there is a direct formula for finding the solution, unlike the algebraic method, where each problem has a limited method different from the other in the solution.

3) For the algebraic method, there are basic forms of the solution, as each problem can be distinguished by its method of solving it just by looking. I must use the appropriate solution method as a solution to the Laplace method, which has no limited method for solving.

4) The possibility of using the Laplace transform in solving problems of physical, chemical, engineering and chemical techniques. (4-4) Recommendations:

The recommendations are summarized as follows:

1) The researchers recommend to expand the study of the Laplace transform in a broader way to know the advantages of the Laplace transform in solving differential equations and other uses.

2) To conduct a separate research in solving higher order differential equations, especially the influencer method, to get to know this method more broadly.

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