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Introduction of Laplace Transforms and their Relationship to Differential Equations

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ABSTRACT

In this manuscript we will show what are the Laplace transforms. What is their relationship to differential equations? Where we will explain some general examples of this relationship through a brief explanation of some applications. This will be clarified through the articles of the manuscript as below.

Introduction

Mathematicians and scientists realized that more than physical laws are better expressed by equations containing not only the unknown variables, but also equations of their verbal change.

The interest in differential equations and their applications is still continuing so far, and it resulted from the efforts made to solve various theoretical issues related to differential equations following mathematical analysis, especially the study of infinite processes. Researchers continued to discover good applications of differential equations not only in the physical sciences but in various fields.

Differential Equations

It is an equation that contains functions and their derivatives. If the functions are true in one variable, then the derivatives that appear are ordinary derivatives, and they are called ordinary differential equations. There are some methods that contribute to solving the differential equation, including the Laplace method, and the Laplace transform in relation to the French mathematician and physicist Pierre Simon Laplace

Transformation in general is a tool for transforming functions and equations from their original form to another simpler form, at least known to us. The integral Laplace transform when affecting the function transforms it into another function completely different from the original function, where the independent variable of the function is converted to another variable and thus changes the scope and extent of the function.

The Laplace transforms depend entirely on the integration, so it is necessary to know the integral well ... that is, the Laplace transform is defined as an integral over the range from small to infinity, and the Laplace transform is particularly effective in solving elementary value problems containing linear differential equations with second equations.

The operator range D is the class of differentiable functions and the operator D is used to solve differential equations and the integral operator Y defined by the Laplace transform gives another way to solve differential equations

Artecal problem

Sometimes we find there are some problems in differential equations that are difficult to solve by known methods, so we resort to other ways to solve equations, including solving differential equations using the Laplace transform.

Importance of artecal

* Laplace transform helps to solve related functions on intervals whose solutions can be obtained using traditional methods.
* When changing the form of the original complex function that has another form that is easier and simpler to deal with by converting the differential equations into an algebraic equation that can be solved, by finding the inverse Laplace transform, we get the solution of the original differential equation.
* Laplace is used in solving some differential and integral problems, as well as in dealing with probability theory.
* The differential equation is used in useful analytical methods to solve engineering problems and it is required to be solved by Laplace.
* Differential equations help in understanding many complex phenomena in our daily life, most notably the electromagnetic phenomenon.

Artecal objectives

* Solve problems that are not homogeneous.

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* The Laplace transform enables us to know some basic facts about integrals, especially defective integrals
* The Laplace transform aims to know the convergent and divergent series.
* The Laplace transform helps in solving the given initial value problems with linear differential equations with constant coefficients.

Artecal Questions

* Does the Laplace transform help in finding the solution of differential equations in a faster and easier way?
* Does the Laplace transform help in physical, engineering and chemical applications?

Artecal Methodology

The researchers used in this research the descriptive method and the experimental method.

Search terms

* differential equations :
* It is an equality relationship between an independent variable let x and a dependent variable let y be one or more y′, y″ derivatives.
* Laplace transform:
* Two independent functions of equation (1), then:- y″,y′ If y″+a1y′+a2y=0 …..(1)

Rank of the equation:
is the highest differential coefficient in the equation.
Degree of Equation:
It is the degree of strength of the highest differential coefficient in the equation provided that all coefficients are free of military force.
The general solution of the differential equation of order: N
The general solution to an n-order differential equation is a solution that contains n optional constants and of course satisfies the differential equation.
Special Solution:
It is any solution that achieves a differential equation that does not include any optional constants, and we may obtain it sometimes by substituting for the optional constants in the general solution with limited values.

Differential Equations and Laplace Transforms

Differential Equations

Definition of Differential Equations

It is an equal relationship between two independent variables, say x and a dependent variable, and let (x)y one or more differential derivatives y′,y″, meaning they are in the general form

Among these equations, it is called an ordinary differential equation if it contains functions of one variable and the derivatives of this variable. As for partial, it contains mathematical functions of more than one independent variable with their partial derivatives (not the subject of our study).

* Rank of the equation:

It is the highest differential coefficient in the equation.

Equation degree:

It is the degree of the highest differential coefficient in the equation provided that all differential equations are free of the fractional power.

Differential Equations

As we studied in the previous courses or chapters some types of differential equations and we will mention some of them for remembrance.

How to separate variables:

Its general form or we put the equation in the following form:

$$\mathbf{F(x)dx + g(x)dy = 0}$$

Then we use direct integrals, so the solution is

Homogeneous differential equations:

It is said that differential equations

$$\mathbf{M(x,t)dx + N(N,y)dy = 0}$$

is homogeneous if M,N is a homogeneous function of the same degree that F(x,y) is a homogeneous function of degree n if

$$\mathbf{f(\lambda x \lambda y) = \lambda^2 f(x,y) \lambda \in R}$$

Ordinary differential equations devolve into homogeneous equations These ordinary differential equations are in the form:

$$\mathbf{\frac{dy}{dx} = (a_1x + b_1y + c_1)/(a_2x + b_2y + c_2)}$$

` constants a1,b1,c1,a2,b2,c2 where
Perfect differential equations:

The differential equations for the function f(x,y) are of the form:

$$\mathbf{df(x,y) = \frac{dy}{dx}dx + \frac{df}{dy}dy}$$

i n $\mathbf{M(x,y)dx + w(x,y)dy = 0}$

It is complete if its left side is completely differential and if the equation is complete then there is a function u(x,y) where $\mathbf{du = M(x, y)dx + w(x, y)dy}$

So u(x,y) where c is a constant.

Linear differential equations:

We will focus on this part and separate it because it is the subject of our research, and the differential equation is linear if the dependent variable and its derivatives are from the equation of the first degree.

The general form of a linear equation of the first order is

$$\frac{dy}{dx} + P(x, y) = Qx$$

It is called linear in y And the equation of the first order is linear in x in the form:

$$\frac{dx}{dy} + a(y)x = B(y)$$

Application:

Find the solution to the equation:

$$x\frac{dy}{dx} + 2y = x^3$$

The solution :-

We put the equation on the image: $\frac{dy}{dx} + p(x) = Q(x)$.... (1)

That is, $\frac{dy}{dx} + \frac{2}{x}y = x^2$.......(2)

Comparing with (2) (1), we find that:

$$Q(x) = x^2, P(x) = \frac{2}{x}$$

We find :

$$1 - \int p(x)dx = \int \frac{2}{x}dx = \ln x^2$$

$$I(x) = e^{\int p(x)d(x)} = e^{\ln x^2} = x^2$$

$$2 - \int u(x)Qxdx = \int x^2x^2dx = \int x^4dx = \frac{1}{5}x^5$$

And the solution to the given equation is $I(y) = \int IQdX + C$ That is: $Xy^2 = \frac{1}{5}X^5 + C_0 + 1$

6. Differential equations that turn out to be linear:

1. Bernoulli equation:

The equation will be in the form: $\frac{dy}{dx} + P(x) = Q(x)y^n$

Where 0,1≠n is called Bernoulli's equation n is a real number, and this equation can be transformed into a linear equation:

1/ By dividing by yn, we get: -

$$y^{-n}\frac{dy}{dX} + P(X)y^{-n+1} = Q(X)(1)$$

2\ Assume that y-n+1=z, then differentiating both sides with respect to x,we get

dz/dx=(-n+1)y-ndy/dx

3/ By multiplying both sides of equation (1) by (-n+1) and substituting for y in terms of Z, we find that

$$\frac{dz}{dX} + (-n + 1)p(X)Z = (-n + 1)Q(X)$$

4\We put (-n+1)Q(x)=qx,(-n+1)p(x)=p(x)

We put the equation in the form: $\frac{dz}{dx}$+p(x)z=q(x):

It is a linear differential equation in z

5/ The solution to the equation is: $I(x)z=\int I(x)q(x)dx+c$

6\ Then replace $Z=Y^{-n+1}$

$$\mathbf{I(x)y^{-n+1} = \int I(x)q(x)dx + c}$$

where $\mathbf{I(x) = e^{\int p(x)dx}}$

Application (2-2):

$$\mathbf{dy + zxyd\,x = xe^{-x^2}y^3dx}$$

The solution :

The equation can be put into the picture:

It is the Bernoulli equation.

$$\mathbf{y^{-3}\frac{dy}{dx} + 2xy^{-2} = xe^{-x^2}}$$

Putting y-2=Z, we get that

$$\mathbf{-2y^{-3}\frac{dy}{dx} = \frac{dz}{dx}}$$

We multiply equation (1) by -2 and by substituting y for y in terms of z, we get:

$$\mathbf{\frac{dz}{dx} - 4xz = 2xe^{-x^2}}$$

It is a linear equation in the form:

$$\mathbf{\frac{dz}{dx} + p(x)z = q(x)}$$

$$\mathbf{p(x) = -4x, q(x) = 2xe^{-x^2}}$$

$$\mathbf{\int p(x)dx = -2x^2}$$

$$\mathbf{SO \int\ I(x)q(x)dx = \int\ e^{-2x^2}\left(-2xe^{-x^2}\right)dx_{\dot{9}}}$$

$$\mathbf{-2\int\ xe^{-3x^2}dx = \frac{1}{3}e^{-3x^2}}$$

Solve the equation on the picture:

$$\mathbf{e^{-2x^2}z = \frac{1}{3}e^{-3x^2} + c}$$

And since z=y-2 is

$$\mathbf{e^{-2x^2}y^{-2} = \frac{1}{3}e^{-3x^2} + c\ gle^{x^2}y^{-2} = \frac{1}{3}ce^{3x^2}}$$

Application:

Find the general solution to the differential equations:

y-2dydx-1+1xy-1=ex

-y-2dydx=dzdx

$$\mathbf{\frac{dy}{dx} - \left[1 + \frac{1}{x}\right]y = -2e^xy^2}$$

The solution /

The equation given in Bernoulli form and multiplied by y2

We get

$$\mathbf{y^{-2}\frac{dy}{dx} - \left[1 + \frac{1}{x}\right]y^{-1} = -2e^x}$$

We put y-1=z, so y-2

$$\mathbf{-y^{-2}\frac{dy}{dx} = \frac{dz}{dx}}$$

And by multiplying the equation by -1 by substituting y for y instead of Z, the equation becomes:

It is a linear equation of the picture

where $\mathbf{G(x) = 2e^xp(x) = 1 + \frac{1}{x}}$

So $\mathbf{\int p(x)dx = \int\ \left[1 + \frac{1}{x}\right]dx = x + \ln x}$

$$\mathbf{I(x) = e^{x+\ln x} = xe^x}$$

By retail integration $\mathbf{u = 2xdv = e^{2x}dx^4jx}$

In the general solution to the equation: $\mathbf{x^3 \frac{x}{xy-1} = -\frac{1}{4}x^4 + c}$

$$\mathbf{xy - 1 = \frac{4x^3}{4c - x^4}xy \Rightarrow y = \frac{4x^3}{4c - x^4} + \frac{1}{x}}$$

ricati equation

Take the equation

$$\mathbf{\frac{dy}{dx} = p(x)y^2 + Q(x)y + R(x)}$$

where p,Q,R are functions of x only

We find that the Riccati equation is the most general of the Bernoulli equation and the linear equation.

And the general solution to the Rickett equation is using the substitution y=y+$\mathbf{\frac{1}{Z}}$

$$\mathbf{\frac{dy}{dx} = \frac{dy_1}{dx} - \frac{1}{z^2}\frac{dz}{dx}}$$

Substituting in the equation

$$\mathbf{\frac{dy_1}{dx} - \frac{1}{z^2}\frac{dz}{dx} = p(x)\left[y_1 + \frac{1}{z}\right]^2 + Q(x)\left[y_1 + \frac{1}{z}\right] + R(x)}$$

$$\mathbf{\frac{dy_1}{dx} - \frac{1}{z^2}\frac{dz}{dx} = p(x)y_1^2 + 2p(x)y_1\frac{1}{z} + p(x)\frac{1}{z^2} + Q(x)y_1 + Q(x)\frac{1}{z}}$$

Since y_1 is a specific solution to the equation, then:$\mathbf{\frac{1}{z^2}\frac{dz}{dx} = 2p(x)y_1\frac{1}{z} + p(x)\frac{1}{z^2} + Q(\ x)\frac{1}{z}}$

And by multiplying by z2, we get:

$$\mathbf{\frac{dz}{dx} + (2p(x)y_1z + Q(x)z = -p(x)}$$

It is a linear equation in z

Application:

Find the general solution to the equation

where y=$\mathbf{\frac{1}{x}}$ is a special solution for it

The solution:

Put the equation in the form $\mathbf{\frac{dy}{dx} = y^2 + \frac{1}{x}y - \frac{3}{x^2}}$

Which is the Rickett equation $\mathbf{\frac{dy}{dx} = p(x)y^2 + Q(x)y + R(x)}$

which turns into a linear equation:

$$\mathbf{\frac{dz}{dx} + p(x)y_1 + Q(x)z = -p(x)}$$

where Q(x)=$\mathbf{\frac{1}{x}}$y1=$\mathbf{\frac{1}{x}}$ $\quad \mathbf{p(x) = 1}$

That is, $\mathbf{\frac{dz}{dx} + \left[\frac{2}{x} + \frac{1}{x}\right]z = -1}$

where y=$\mathbf{\frac{1}{x}}$+$\mathbf{\frac{1}{Z}}$

It is a linear equation of the form $\mathbf{\frac{dz}{dx}}$+ $\mathbf{\frac{3}{x}}$z=-1

$$\mathbf{q(x) = -1 \qquad p(x) = \frac{3}{x}}$$

$$\mathbf{\int p(x)d(x) = \int \frac{3}{x}dx = \ln x^3}$$

So $\mathbf{I\Big((x) = e^{\ln x^3} = x^3}$

$$\mathbf{\int I(x)q(x)d(x) = \int -x^3dx = \frac{1}{4}x^4}$$

$$\mathbf{x^3z = -\frac{1}{4}x^4 + c}:$$

Thus, we get the given equation:

$$\mathbf{y = \frac{1}{x} + \frac{1}{z}}$$

where

$$\mathbf{Iz} = \int \mathbf{Iqdx} + \mathbf{c}$$

That is:

$$\mathbf{xe^{2x}z} = \mathbf{e^{2x}}\left[\mathbf{x} - \frac{\mathbf{1}}{\mathbf{2}}\right] + \mathbf{c}$$

where z≠y-1

In the general solution $\frac{\mathbf{x}}{\mathbf{y}}\mathbf{e^{x}} = \mathbf{e^{2x}}\left[\mathbf{x} - \frac{\mathbf{1}}{\mathbf{2}}\right] + \mathbf{c}$

Application:

Find the general solution to the differential equation:

$$\mathbf{2x^2}\frac{\mathbf{dy}}{\mathbf{dx}} = (\mathbf{x} - \mathbf{1})\left(\mathbf{y^2} - \mathbf{x^2}\right) + \mathbf{2xy}$$

where y = x is a special solution for it

The solution :

By investigating, we find that y=x is a solution to the equation in..we assume that

$$\mathbf{y} = \mathbf{x} + \frac{\mathbf{1}}{\mathbf{2}}$$

$$\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{z^2}}\frac{\mathbf{dz}}{\mathbf{dx}}$$

Since the equation is Rickett's equation in substitution in the equation we find that:

$$\mathbf{2x^2}\left[\mathbf{2} - \mathbf{z^2}\frac{\mathbf{dz}}{\mathbf{dx}}\right] = (\mathbf{x} - \mathbf{1})\left[\left(\mathbf{x} + \frac{\mathbf{1}}{\mathbf{z}}\right)^2 - \mathbf{x^2}\right] + \mathbf{2x}\left(\mathbf{x} + \frac{\mathbf{1}}{\mathbf{z}}\right)$$

$$\mathbf{2x^2} - \mathbf{2}\frac{\mathbf{x^2}}{\mathbf{z^2}}\frac{\mathbf{dz}}{\mathbf{dx}} = (\mathbf{X} - \mathbf{1})\left[\frac{\mathbf{2x}}{\mathbf{z}} + \frac{\mathbf{1}}{\mathbf{z^2}}\right] + \mathbf{2x^2} + \frac{\mathbf{2X}}{\mathbf{X}}$$

$$\mathbf{2x^2} - \mathbf{2}\ \frac{\mathbf{x^2}}{\mathbf{z^2}}\frac{\mathbf{dz}}{\mathbf{dx}} = \mathbf{2}\frac{\mathbf{x^2}}{\mathbf{z}} + \frac{\mathbf{x}}{\mathbf{z^2}} - \frac{\mathbf{2x}}{\mathbf{z}} - \frac{\mathbf{1}}{\mathbf{z^2}} + \frac{\mathbf{2x}}{\mathbf{z}}$$

$$\frac{\mathbf{dz}}{\mathbf{dx}} = -\mathbf{z} - \frac{\mathbf{1}}{\mathbf{2x}} + \frac{\mathbf{1}}{\mathbf{2x^2}}$$

$$\frac{\mathbf{dz}}{\mathbf{dx}} + \mathbf{z} = \frac{\mathbf{1}}{\mathbf{2x}} + \frac{\mathbf{1}}{\mathbf{2x^2}}$$

linear equation on the form:

$$\frac{\mathbf{dz}}{\mathbf{dx}} + \mathbf{p}(\mathbf{x})\mathbf{z} = \mathbf{q}(\mathbf{x})$$

$$\mathbf{p}(\mathbf{x}) = \mathbf{1}$$

$$\mathbf{q}(\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{2}}\left[\frac{\mathbf{1}}{\mathbf{x^2}} - \frac{\mathbf{1}}{\mathbf{x}}\right]$$

$$\int \mathbf{p}(\mathbf{x})\mathbf{dx} = \mathbf{x} \Rightarrow \mathbf{I}(\mathbf{x}) = \mathbf{e^x}$$

$$\int \mathbf{I}(\mathbf{x})\mathbf{q}(\mathbf{x})\mathbf{dx} = \frac{\mathbf{1}}{\mathbf{2}}\left[\frac{\mathbf{e^x}}{\mathbf{x^2}} - \frac{\mathbf{e^x}}{\mathbf{x}}\right]\mathbf{dx}$$

There is hash: $\int \frac{\mathbf{e^x}}{\mathbf{x}}\mathbf{dx}$

$$\mathbf{u} = \frac{\mathbf{1}}{\mathbf{x}}\ ,\ \mathbf{dv} = \mathbf{e^x}\,, \mathbf{du} = \frac{\mathbf{1}}{\mathbf{x^2}}\mathbf{dx}, \mathbf{v} = \mathbf{e^x}$$

$$\int \mathbf{I}(\mathbf{x})\mathbf{q}(\mathbf{x})\mathbf{d}(\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{2}}\left[\int \frac{\mathbf{e^x}}{\mathbf{x^2}}\mathbf{dx} - \frac{\mathbf{e^x}}{\mathbf{x}} - \int \frac{\mathbf{e^x}}{\mathbf{x^2}}\mathbf{dx}\right]$$

$$= -\frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{e^x}}{\mathbf{x}}$$

That is, solving the equation in the form:

$$= \int \mathbf{1(x)q(x)dx + c}$$

$$\mathbf{e^x z = -\frac{1}{2}\frac{e^x}{x} + c}$$

$$\mathbf{y = x + \frac{1}{z} \Rightarrow \frac{1}{z} = y - x \Rightarrow z = \frac{1}{y - x}}$$

du=1x2dx,v=ex , dv=ex , u=1x

So the general solution to the equation is:

$$\mathbf{\frac{e^x}{y - x} = \frac{1}{2}\frac{e^x}{x} + c}$$

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