Knot theory is a branch of topology which is important in studying of the 3 dimensional geometry. In this study, we have proved a theorem which is related to Knot theory. Also, using some known results, one theorem has been well developed and adapted to Knot Theory.
1. Using the following Murasugi’s generalized signature lower bound for transforming metric on knots. i.e. using \( u(K_1, K_2) \geq \frac{1}{2} |\sigma(K_1) - \sigma(K_2)| \) \[1\]

Now, \( u(K, K') \geq \frac{1}{2} |\sigma(K) - \sigma(K')| \)

Since, \(|\sigma(K) - \sigma(K')| \geq 0\), we have \( u(K, K') \geq 0 \).

2. \( u(K, K') = 0 \iff 0 \geq \frac{1}{2} |\sigma(K) - \sigma(K')| \geq 0 \)
   \( \iff |\sigma(K) - \sigma(K')| = 0 \)
   \( \iff \sigma(K) = \sigma(K') \)
   \( \iff K = K' \)

3. \( u(K, K') \) is the minimum number of unknotting operations which are needed to transform \( K \) and \( K' \) where the minimum is taken over all diagrams.
   
   \[ u(K, K') = \min_{D \in D, D' \in D'} \left\{ u(D, D') \right\} \]

   \[ = \min_{D \in D, D' \in D'} \left\{ u(D', D) \right\} \]

   = minimum number of unknotting operations which are needed to transform \( K' \) and \( K \) where the minimum is taken over all diagrams.

   \( u(K, K') \)

4. For any three knots \( K_0, K, K' \in X \).
   
   \[ u(K, K_0) \geq \frac{1}{2} |\sigma(K) - \sigma(K_0)| \] and \( u(K_0, K') \geq \frac{1}{2} |\sigma(K_0) - \sigma(K')| \).

   Then, \( u(K, K_0) + u(K_0, K') \geq \frac{1}{2} |\sigma(K) - \sigma(K_0)| + \frac{1}{2} |\sigma(K_0) - \sigma(K')| \)
   \( = \frac{1}{2} \left[ |\sigma(K) - \sigma(K_0)| + |\sigma(K_0) - \sigma(K')| \right] \)
   \( \geq |\sigma(K) - \sigma(K_0) + \sigma(K_0) - \sigma(K')| \)
   \( = |\sigma(K) - \sigma(K')| \)
   \( = u(K, K') \)

i.e. \( u(K, K') \leq u(K, K_0) + u(K_0, K') \) for any \( K_0, K, K' \in X \).

Therefore, the four properties of a metric are satisfied.
So, \( u \) is a metric on \( X \) and \( (u, X) \) is the metric space.

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References