Introduction
Graph labeling has many applications in Computer Science and Engineering. Prime labeling is a special case of graph labeling which is very useful for research in graph theory. In our previous researches prime labeling of known graphs, Roach graph and Scorpion graph have been done. For the present work, we have defined a new graph with 3 parameters called Snow graph, which is a shape of snowflake and it is a union of wheel and star graphs.

The concept of prime labeling was introduced by Roger Entringer around 1980’s in the setting of trees where he stated the conjecture; every tree is prime. The theory was developed to the current setting by A. Tout et al. in 1982 [3]. Prime labeling can be applied for various types of graphs. In this paper, we have given prime labeling of Snow Graphs. Since the Snow Graph represents a structure of snowflake, it can be used to model as a computer network system structure or as a part of it. In our work, we have introduced an Algorithm for prime labeling of Snow graphs. Some most important definitions in this research are given below.

Definition 1 (Cycle graph; \(C_n\))
A connected graph that is regular of degree 2 is a Cycle graph and it is denoted by \(C_n\) for \(n\) vertices.[7]

Definition 2 (Wheel graph; \(W_n\))
The graph obtained from \(C_{n-1}\) by joining each vertex to a newly introduce vertex at its center is the Wheel on \(n\) vertices, denoted by \(W_n\) (\(C_{n-1}\) is denoted by cycle graph on \(n-1\) vertices.) [7]

Definition 3 (Star graph; \(K_{1,m}\))
A complete bipartite graph of the form \(K_{1,m}\) is a Star graph with \(m+1\) vertices where a single vertex belongs to one set and all the remaining vertices belong to the other set and each edge joins single vertex to all the other vertices.

Definition 4 (Snow graph; \(S_{n,1,m}\))
A graph obtained from the wheel graph \(W_n\) by adjoining one vertex of each of \(n\) star graph \(K_{1,m}\) to the outer cycle \(C_{n-1}\) of \(W_n\) is called a Snow graph.

Number of vertices of the Snow graph = \(mn + n - m\)
Number of edges of the Snow graph = \((m + 2)(n - 1)\)

Figure 1. The Snow graph of the form \(S_{6,1,7}\).
Definition 5 (Prime Labeling)

Let \( G = (V,E) \) be a simple graph where \( V \) and \( E \) denotes the set of vertices and edges respectively. A **Prime labeling** of \( G \) is a labeling of vertices of \( G \) with distinct integers from the set \( \{1,2,\ldots,n\} \) in such a way that the labels of any two adjacent vertices are relatively prime. Such a graph is called a **Prime graph**.

Definition 6 (Greatest Common Divisor)

Let \( a \) and \( b \) be integer that are not both zero. Then, the greatest common divisor \((\gcd)\) of \( a \) and \( b \) is the largest positive integer that divides both \( a \) and \( b \), and is denoted by \( \gcd(a,b) \).

Two integers \( a \) and \( b \) are relatively prime if \( \gcd(a,b) = 1 \).[5]

Materials and Methods

**Theorem:** The Snow graph of the form \( S_{6,1,m} = W_6 \cup K_{1,m} \cup K_{1,m} \cup K_{1,m} \cup K_{1,m} \) with \( 5m+6 \) vertices has a prime labeling.

**Algorithm**

**Data:** \( S_{6,1,m} \), \( 5(m+1) + 1 = P; \) \( P \) is a prime.

**Result:** \( S_{6,1,m} \) has a prime labeling.

**Step 1**

Count the number of vertices in the graph \( S_{6,1,m} \).

Total number of vertices of Snow graph \( S_{6,1,m} = mn + n - m \).

For \( S_{6,1,m} \); the number of vertices = \( 5(m+1) + 1 = P \); \( P \) is a prime number.

**Step 2**

Take the largest 10 prime numbers in the interval \( [1,P-1] \).

Denote those prime numbers as \( P_1, P_2, P_3, \ldots, P_{10} \) and \( P = P_{11} \).

Here, \( P_1 < P_2 < P_3 < \cdots < P_9 < P_{10} < P_{11} = P \) and define a set \( L \) such that \( L = \{P_1, P_2, P_3, \ldots, P_9, P_{10}, P_{11}\} \).

**Step 3**

- Label the middle vertex of the wheel using \( \min\{P_1, P_2, P_3, \ldots, P_9, P_{10}, P_{11}\} \).
- Next, label the other vertices of the wheel graph such that \( P_2, P_3, P_4, P_5, P_6, P_7, P_8, \ldots, P_{11}, P_{11} = P \).
- Label the head of the star graph \( P_7, P_8, P_9, P_{10}, P_{11} = P \).

**Step 4**

Divide other integers in \([1,P]\) into five groups.

Define those five group as \( F_1, F_2, F_3, F_4, F_5 \) such that

\[
F_i = \left\{ x \in [1,P] \setminus \left( L \cup \sum_{j=1}^{i-1} F_j \right) \mid x < 2P_{i+6} \land \gcd(x, P_{i+6}) = 1 \land |F_i| = m - 1 \land i = 1,5 \right\}
\]

Note: \(|F_1| = |F_2| = |F_3| = |F_4| = |F_5| = m - 1\).

**Step 5**

Label other vertices in the \( S_{6,1,m} \) using corresponding \( F_i \) to \( P_{i+6}, i = 1,5 \) such that

\[
\begin{align*}
F_1 & \rightarrow P_7 \\
F_2 & \rightarrow P_8 \\
F_3 & \rightarrow P_9 \\
F_4 & \rightarrow P_{10} \\
F_5 & \rightarrow P_{11}
\end{align*}
\]

Now, consider each step carefully. It is shown that the graphs \( S_{6,1,m} \) have prime labeling.
Results and Discussion

We divide the labeling into two parts as follows:

i. A part of wheel graph and a part of star graph (\(K_{1,m}\)) of head vertices.

ii. Other vertices in \(K_{1,m}\) graph.

Let \(A = \{W_0, W_1, W_2, W_3, W_4, W_5\} \cup \{H_1, H_2, H_3, H_4, H_5\} : |A| = 11\) and

\(B = \{H_i \in \mathcal{V}(K_{1,m}) | j \in \{1, 5\}; \ i \in \{1, m - 1\}; |B| = 5(m - 1)\)

In step 3, labeling can be done using \(P_1, P_2, P_3, \ldots, P_{10}, P_{11} = P\) which are prime.

So, it is clear that those adjacent pairs are relatively prime.

Therefore, \(\gcd(W_i, W_j) = 1; i \neq j ; i, j = 0, 1, 2, 3, 5\)

Furthermore,

\(\gcd(H_i, H_j) = 1; i = 1, 2, 3, 4, 5\)

Now, to label set B,

Defined set \(F_i \) which is corresponding to prime number \(H_i = P_{i+6}; i = 1, 2, 3, 4, 5\) and

\(|F_i| = m - 1; i = 1, 2, 3, 4, 5\)

Clearly, \(\forall x \in F_i\) \(\gcd(x, P_{i+6}) = 1\), since \(P_{i+6}\) is prime and \(x < 2P_{i+6}\).

Therefore, a labeling is a prime labeling.

Conclusions

We have obtained a theorem for prime labeling of a new graph, which is known as Snow graph. Prime labeling can be obtained by the constructed algorithm for Snow graphs. As a future work, planning to write a computer program for this algorithm and generalize this theorem for different \(n\) integer values such that \((m + 1)(n - 1) + 1\) is a prime.

References