Transient Response Performance Improvement of a Plastic Extrusion Process
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ABSTRACT
This paper has presented a control strategy for improving transient response performance for a plastic extrusion process. A dynamic equation of a single screw plastic extruder was obtained. The main control variable is the screw speed in relation with the pressure and the temperature. A high degree of speed accuracy is required to ensure the optimal performance of the extrusion process. In order to adequately control the screw speed in relation to the pressure, a model reference adaptive control (MRAC) which combines a proportional integral and derivative (PID) control strategy was developed and integrated with the obtained dynamic equation of a single screw plastic extrusion process considered in this context. The gain parameter of the MRAC was selected for different values and simulations were performed in Matlab/Simulink environment by altering the adaptive gain. The implemented system in Matlab/Simulink shows that the developed controller improved the transient response performance of the process. This resulted to a fast response, near zero-overshoot, and improved transient performance.

Introduction
The first synthetic polymer was first invented in 1869 by John Wesley. Ever since its invention, it has remained revolutionary, and thereby making plastic industry to develop more sophisticated one. The growth of plastic industry has been immensely enhanced by recent advancement in technology. This has contributed to the world economy most especially in various sectors like: Automotive, Construction, Electronics, Healthcare, Textiles e.t.c.

The application of plastics in the present age is quite critical. Today, electronics and communication devices such as computers, cell phones, televisions and radio are made sophisticated with the help of plastics. The fact that plastics have lightweight and are good for insulation purposes, has made them useful for saving fossil fuels used in heating and in transportation. Inexpensive plastics have raised the standard of living and made material abundance more readily available. Many of the things own by people so that they are taken for granted might be out of reach for all but the richest without plastics. Replacing natural materials with plastic has made many of our possessions cheaper, lighter, safer, and stronger.

Many techniques are involved in the production of quality plastic. The manufacturing process consists of equipment such as an extruder for material handling and quality control and includes other engineering activities such as product and system design, modeling and simulation. However, since plastics are made from polymers by combining selected monomers and requires high precision and smoothness, there is demand for high quality and efficient production. In order to improve the production rate of plastics, manufacturing industries should adopt a control technique that will result to improve product quality.

Dynamic modeling of manufacturing processes is one critical aspect that makes it possible to achieve active control of product quality. One technique of plastic production that has high quality products that will meet demands at the lowest cost is the plastic extrusion process. It solves the problems related to high quality which has a direct effect on the expected profit returns of a plastic manufacturing industry. Unfortunately, in many industrial processes involving plastic extrusion, inadequate control of screw speed, the pressure, the temperature has resulted in a poor performance of the overall system. This work intends to design an adaptive control technique that will improve product quality in a plastic production process by achieving an overall improvement in the screw speed and the pressure of the of single screw plastic extruder.

Single screw extruder
The single screw extruder is the most common equipment in the polymer industry. It can be part of an injection molding unit and is found in numerous other extrusion processes, including blow molding, film blowing and wire coating. Its function is to produce a homogeneous melt from the supplied plastics pellets and to press the melt through the shaping die. The tasks of a plasticating extruder are to transport the solid pellets or powder from the hopper to the screw channel, compact the pellets and move them down the channel, melt the pellets, mix the polymer into a homogeneous melt, and pump the melt through a die.

The overall structure of the plastic extrusion represented by with a block diagram is shown in Fig. 1. The polymer is fed into the hopper in solid pellet forms and it passes through the temperature zones where it is heated and melted. The melted polymer material is pushed forward by a powerful screw and it passes through the molding mechanism from the...
die. The quality of extrudates depends on uniform temperature distribution, physical property of raw material, and so forth.

![Block diagram of a plastic extrusion system](image)

**Figure 1. Block diagram of a plastic extrusion system.**

**Related Works**

Zhijun et al [9] performed a research on product quality control for single screw extrusion process. In their work, they stated that the extrusion product quality can be categorized into precision and accuracy. They maintained that the extrusion product precision control can be achieved by good process variable control, while the accuracy is a joint effect of the machine variable and process variable. The overall objective of the research was to employ a reliable method for control of quality accuracy. A generalized predictive control (GPC) algorithm was adopted to control the quality accuracy. The control scheme was implemented experimentally via a self-developed controller on an industrial pilot-scale single screw extrusion process. Some representative results were presented to highlight the advantages and limitations of the control strategy. The work of the GPC was to track a referenced diameter. Mbaocha et al [4] designed a plastic extrusion system controller. Their work was based on modeling and control of an overall plastic extrusion system. A digital controller was designed using Matlab software so as to improve the screw speed and the pressure of single screw plastic extruder. They stated that the designed controller ensures an optimum control of plastic extrusion process with 5.11% overshoot, 6.66 seconds settling time and a rise time of 1.96 seconds to a unit step input. They recommended that a more robust and intelligent control technique be employed which will yield better performance. Mohd [2] implemented a tuned proportional, integral and derivative (PID) controller using two model heuristic techniques that consists of Differential Evaluation (DE) and Genetic Algorithm (GA). Optimal parameters of the PID were applied for high order system that is, system with delay and non-minimum phase system. The objective functions of the techniques employed were set as Mean Square Error (MSE) and Integral Absolute Error (IAE) and were used for performance evaluation. A study on the reliability between DE and GA in consistently maintaining minimum MSE was carried out. The performance of the tuned PID control system using GA and DE are compared with Zeigler Nichol method. Oggunnaike et al [3] have managed to control some product qualities well, but they employ inference model to predict product quality. This prediction cannot accurately reflect the true product quality. Raju et al [10] have reviewed the research of the determination of process parameters and die design for plastic extrusion. Various approaches that have been researched on such as the Taguchi’s parameter design technique, artificial neural networks (ANN), Fuzzy logic, genetic algorithms (GA), non-linear modeling, and response surface methodology were discussed. Narasimha and Rejikumar [6] established a systematic approach to find the root causes for the occurrence of defects and wastes in plastic extrusion process. The cause-and effect diagram was implemented to identify the root causes of these defects. The extrusion process parameters such as vacuum pressure, temperature, take-off speed, screw speed of the extrusion process and raw material properties were identified as the major root causes of the defects from the cause-and-effect diagram. The quality loss for the current performance variation was calculated using Taguchi’s principle of loss function and requirement for improvement was verified. In this paper design of experiment (DOE) was applied to optimize the process parameters for the extrusion of high-density polyethylene (HDPE) pipe 50mm diameter and plain pipe 25mm diameter. Four independent process parameters viz. vacuum pressure, take-off speed, screw speed and temperature were investigated using Taguchi method. Minitab 15 software was used to analyze the result of the experiment. Based on the result of the analysis, optimum process parameters were selected. Deng et al [1] presented an approach in a soft computing paradigm for the process parameter optimization of multiple-input multiple-output (MIMO) plastic injection molding process. The proposed approach integrates Taguchi’s parameter design method, back propagation neural networks, genetic algorithms and engineering optimization concepts to optimize the process parameters. The results obtained indicated that the proposed approach can effectively help engineers to determine optimal process parameter settings and achieve competitive advantages of product quality and costs. Cirak and Kozan [8] presented knowledge based and neural network approaches to wire coating for polymer extrusion. The dependency of extrusion process parameters viz. barrel heating zones’ temperatures and screw speed on coating thickness of wire coating extrusion processes was investigated using ANN. A back propagation neural network model was used to predict the coating thickness. Mu et al [7] proposed an optimization approach for the processing design in the extrusion process of plastic profile with metal insert based on finite element simulation, back propagation neural network and genetic algorithm. The polymer melts flow in the extrusion process was predicted using finite element simulation. The simulated results were extracted for the establishment of neural network. The search for globally optimal design variable for the extrusion was done using GA with its objective function evaluated using the established neural network model. The uniformity of outlet flow distribution was taken as the optimization objective with a constraint condition on the maximum shear stress. The objective of flow balance was achieved by the optimal design of two processing parameters including the volume flow rate and the metal insert moving velocity.

**Methodology**

In this section, a mathematical modeling of the dynamic equations of an extrusion process is presented. The section also presents the controller design and the system configuration using Matlab/Simulink block.

**Single Screw Plastic Extrusion Model Dynamics**

A mathematical model of a typical single screw plastic extrusion process for polymer production presented in Mbaocha et al [4] is adopted in this context. Assuming a
typical dynamic equation of plastic polymer for single screw extruder in terms of the screw speed and the pressure can be derived as follows:

\[ Q = \alpha N - \beta \frac{d^2P}{dt^2} \]  

(1)

\[ Q = \text{Volumetric flow rate of polymer (m}^3\text{s)/}, \ N = \text{Extruder Screw Speed (rpm)}, \ P = \text{Pressure of polymer along the Extruder (Pa)}, \ \mu = \text{Polymer melt viscosity (Pa s)}, \ \alpha = \text{Drag flow coefficient (m}^3\text{s)/}, \ \beta = \text{Pressure flow coefficient (m}^3\text{s)/}. \]  

The flow rate of liquid plastic as a function of the pressure increase is given by:

\[ Q = \frac{dP}{dt} + \alpha P \]  

(2)

Equating (1) to (2) yields:

\[ \frac{dP}{dt} + \alpha P = \alpha N - \beta \frac{d^2P}{dt^2} \]  

(3)

Taking Laplace Transform of Equation (4) and yields:

\[ P(s)(\beta s^2 + \mu s + \alpha \mu) = \alpha \mu N(s) \]  

(5)

Hence, a typical transfer function of an extrusion plant, \( G_p(s) \):

\[ G_p(s) = \frac{P(s)}{N(s)} = \frac{\alpha \mu}{\beta s^2 + \mu s + \alpha \mu} \]  

(6)

In (“Experimental Validation of Models for Plastic Extrusion Process,” 1994), it was stated that \( \alpha \) and \( \beta \) are geometric factors that are constant for a given screw/barrel combination. For typical Single Screw Plastic Extrusion System, the Polymer melt viscosity \( \mu = 325 \text{Pa s} \), Drag flow coefficient \( \alpha = 0.13601 \text{m}^3 \text{s/} \), Pressure flow coefficient \( \beta = 498.75 \text{m}^3 \text{s/} \). Substituting these values into Eq. (6) yields:

\[ G_p(s) = \frac{0.089}{s^2 + 0.6525s + 0.089} \]  

(7)

The block diagram of a typical closed loop control system for single screw plastic extrusion process without a controller is shown in Fig.2.

**Figure 2. Control System Diagram for an Extruder.**

**Controller Design**

**Control Objective**

The control objective is to design an adaptive control system that will ensure a fast response tracking to a unit step input with a settling time of less than 7 seconds. 

**Design Specification**

(a) Percentage overshoot less than 5.50% to a unit step input. 
(b) Settling time less than 7.0 seconds to a unit step input. 
(c) Rise time of less than 5.0 seconds to a unit step input.

In this section a model reference adaptive controller (MRAC) is presented. The Massachusetts Institute of Technology (MIT) is used in this paper for developing the rule for the adaptive parameter in the controller. In order to design the MRAC, the approach used in [5] has been adopted.

In order to use the rule, the equations for the error and cost function are presented as follows. Let the difference between the actual output of the process and output of the referenced model be defined as the error, \( e \):

\[ e = P - P_{model} \]  

(8)

\( P \) and \( P_{model} \) are the output response of the plant and the out response of the referenced mode.

The equation for the cost function \( \theta \) is defined as:

\[ J(\theta) = \frac{1}{2} e^2 \]  

(9)

The cost function can be minimized such that the change in the parameter \( \theta \) can be maintained in the direction of the negative gradient of \( J \), and that is:

\[ \frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \frac{\partial e}{\partial \theta} \]  

(10)

The partial derivative expression \( \frac{\partial e}{\partial \theta} \) is called the sensitivity derivative. It shows the change in error with respect to parameter \( \theta \). Equation (10) describes the change in the parameter \( \theta \) with respect to time so as to be able to reduce the cost function to zero. The expression \( \gamma \) represents a positive quantity which indicates the gain of the adaptation mechanism of the controller. Now it is assumed that the plastic extrusion process is linear with transfer function \( KG_p(s) \), where \( K \) is a parameter whose value is unknown and \( G_p(s) \) is a second order system with a known transfer function (Eq.7). The objective is to design a controller such that the plant extrusion process could track a reference model whose transfer function, \( G_{pm}(s) = K_s G_p(s) \), where \( K_s \) is a parameter whose value is known.

Definitions:

\[ E(s) = KG_p(s)U(s) - K_s G_p(s)U_c(s) \]  

(11)

Stating a control law:

\[ U(s) = \theta \times U_c(s) \]  

(12)

Substituting Eq. (12) into Eq. (11) and taking partial differential yields:

\[ \frac{\partial E(s)}{\partial \theta} = KG_p(s)U_c(s) = \frac{K}{K_o} P_{model}(s) \]  

(13)

Combining Eq. (10) and Eq. (13) yields

\[ \frac{d\theta}{dt} = -\gamma \frac{K}{K_o} P_{model} = -\gamma \frac{e}{P_{model}} \]  

(14)

Equation (14) provides the law for adjusting the parameter \( \theta \) and the Simulink model of the control loop is shown in Fig. 4.
It is desired to generate the reference model. In order to do this, a second order transfer function is selected because the process under consideration is a second order system. Hence:

\[ G_p(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]  

(15)

where \( \omega_n \), \( \zeta \) are the natural frequency response and the damping ratio of the system. These are obtained as follows:

\[ M_p = e^{-\frac{\pi}{\sqrt{1-\zeta^2}}} \]  

(16)

where \( M_p \) is the peak overshoot in percentage which is equal to 5%. Substituting this value into Eq. (16) gives the damp ratio \( \zeta \) as 0.68. Equation (17) below defines the relationship between settling time \( T_s \), the damp ratio \( \zeta \), and the natural frequency response, \( \omega_n \).

\[ T_s = \frac{4}{\zeta \omega_n} \]  

(17)

Substituting the value \( T_s = 6.0 \) seconds and \( \zeta = 0.68 \) into (17) gives the value for \( \omega_n \) as 0.98. Substituting these values into Eq. (15) yields

\[ G_p(s) = \frac{0.96}{s^2 + 1.333s + 0.96} \]  

(18)

The Matlab/Simulink block diagram in Fig. 4 is the structure of the designed adaptive control system used for simulation in this context. The parameters of the designed PID controller that is added to the control loop of the MRAC are presented below in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional gain</td>
<td>( k_p )</td>
<td>15</td>
</tr>
<tr>
<td>Integral gain</td>
<td>( k_i )</td>
<td>0.01</td>
</tr>
<tr>
<td>Differential gain</td>
<td>( k_d )</td>
<td>9</td>
</tr>
</tbody>
</table>

**Simulation and Result**

Simulation is performed in this context for both the loop without controller and loop with controller using Matlab software. The Simulink blocks for both cases considered are shown in Fig. 3 and 4.

**Results**

The results obtained from the simulations performed in Matlab/Simulink environment are presented in Fig. 5, 6, 7, 8.
Integral and Derivative (PID) algorithm

Hence, the uncompensated plastic extrusion process. The adaptive controller designed in this context is a model reference adaptive control (MRAC) added to the loop, the results obtained from the simulation performed in Matlab/Simulink shows an optimal robust tracking controller is integrated as part of the components of the extrusion process, the response of the plastic extruder to a unit input will be faster rather than being sluggish. This can be achieved by varying the adaptation gain of the MRAC.

**Table 2. Time response performance comparison to unit step input.**

<table>
<thead>
<tr>
<th>Control loop</th>
<th>Settling Time</th>
<th>Rise Time</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without MRAC PID</td>
<td>9.4 s</td>
<td>9.0 s</td>
<td>Sluggish</td>
</tr>
<tr>
<td>With MRAC PID, $\gamma = -1.0$</td>
<td>6.0 s</td>
<td>2.0 s</td>
<td>Fast</td>
</tr>
<tr>
<td>With MRAC PID, $\gamma = -2.0$</td>
<td>5.5 s</td>
<td>2.0 s</td>
<td>Fast</td>
</tr>
<tr>
<td>With MRAC PID, $\gamma = -2.5$</td>
<td>5.0 s</td>
<td>2.0 s</td>
<td>Fast</td>
</tr>
</tbody>
</table>

**Discussion**

An optimal and robust tracking Model Reference Adaptive Control (MRAC) system which combines Proportional Integral and Derivative (PID) algorithm technique was developed and integrated with a plastic extrusion process model. Simulation was first performed without controller in the control loop as shown in Fig.3. The result obtained shows that the response of the uncompensated loop to unit step input is sluggish as shown in Fig. 5. Hence, there is a need to improve its response characteristics.

In Fig.4, with MRAC PID integrated in the loop, the closed loop responses obtained by adjusting the adaptation gain (gamma) of the MRAC show an optimal robust fast tracking system, with improved system performance characteristics: percentage overshoot, settling time and rise time (see Table 2).

Matlab/Simulink was used to perform simulation and selection of the controller model parameters. The whole process was based on altering the adaptation gain (gamma) of the MRAC.

It is obvious from the simulation Fig. of 6, 7, and 8 that for different values of adaptation gain $\gamma$, the actual plant (extruder process) and the reference model show continuous improvement as the value of $\gamma$ becomes large. Hence, for large value of $\gamma$ system responses are fast.

Finally, as shown in Fig. 5, the system has slow response when it has not been compensated. For this purpose, an adaptive control algorithm was adopted to take care of the sluggish response nature. With the model adaptive reference control (MRAC) added to the loop, the results obtained from the simulation performed in Matlab/Simulink shows that a fast response, near zero-overshoot, and precise tracking performance strategies is realized.

**Conclusion**

This paper has presented an adaptive control technique for improving product quality in a plastic extrusion process. The adaptive controller designed in this context is a model reference adaptive control (MRAC) which combines a proportion integral and derivative (PID) control algorithm to control a plastic extrusion process.

Initially, when the PID was not in the loop, the response of the system to a unit step input grows exponentially. Hence, it was necessary to add a PID control loop to the MRAC so as to improve the transient response characteristics of the process. The reason for the introduction of the PID algorithm was meant to reduce settling time, rise time and overshoots, and transient response.

It can be seen that the developed adaptive control has improved the response of a plastic extrusion process considerably and hence can be adopted in plastic manufacturing industry to improve product quality. When the controller is integrated as part of the components of the extrusion process, the response of the plastic extruder to a unit input will be faster rather than being sluggish. This can be achieved by varying the adaptation gain of the MRAC.

**Reference**

[1] Deng W.J., Wen-Chin C. and Pei-Hao Tai (2014), Process parameter optimization for MIMO plastic injection molding via soft computing. E-mail address: simond@chu.edu.tw (W.–J. Den)


