Introduction
Most of the problems in engineering, medical science, economics, environments, etc. have various uncertainties. To exceed these uncertainties, some kind of theories were given like theory of fuzzy sets, intuitionistic fuzzy sets and so on. Fuzzy set was introduction by L.A.Zadeh[16], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R[5, 6]. In this paper, we introduce the some theorems in fuzzy subfield of a field with respect to T-norm. It is denoted as T-fuzzy subfield of a field.

1. Preliminaries
1.1 Definition
A T-norm is a binary operations T: [0, 1] × [0, 1] → [0, 1] satisfying the following requirements;
(i) T(0, x) = 0, T(1, x) = x (boundary condition)
(ii) T(x, y) = T(y, x) (commutativity)
(iii) T(T(x, y), z) = T(T(x, y), z)(associativity)
(iv) if x ≤ y and w ≤ z, then T(x, w) ≤ T(y, z)( monotonicity).

1.2 Definition
Let X be a non-empty set. A fuzzy subset A of X is a function A : X → [0, 1].

1.3 Definition
Let (F, +, ∙) be a field. A fuzzy subset A of F is said to be a T-fuzzy subfield of F if the following conditions are satisfied:
(i) μA(x+y) ≥ T(μA(x), μA(y)), for all x and y in F,
(ii) μA(−x) ≥ μA(x), for all x in F,
(iii) μA(xy) ≥ T(μA(x), μA(y)), for all x and y in F,
(iv) μA(x^−1) ≥ μA(x), for all x ≠ 0 in F,

1.4 Definition
Let (F, +, ∙) be any two fields. Let f : F → F be any function and A be a T-fuzzy subfield in F, V be a T-fuzzy subfield in f(F) = F, defined by μV(y) = sup x∈ F μA(x), for all x in F and y in F. Then A is called a preimage of V under f and is denoted by f^−1(V).

1.5 Definition
Let A and B be any two fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as AxB = { (x, y) : μAxB(x, y) } / for all x in G and y in H }, where μAxB(x, y) = min{ μA(x), μB(y) }, for all x in G and y in H.

1.6 Definition
Let A be a fuzzy subset in a set S, the strongest fuzzy relation on S, that is a fuzzy relation on A is V = { (x,y) : μV(x,y) } / for x and y in S given by μV(x, y) = min{ μA(x), μA(y) }, for all x and y in S.

1.7 Definition
A fuzzy subset A of a set X is said to be normalized if there exist x in X such that μA(x) = 1.
1.8 Definition

Let $A$ be a T-fuzzy subfield of a field $(F, +, \cdot)$ and $a$ in $F$. Then the pseudo T-fuzzy coset $(aA)^{\mu}$ is defined by $((aA)^{\mu})(x) = p(\mu_{A}(x))$, for every $x$ in $F$ and for some $p$ in $F$.

2. Properties of T-Fuzzy Subfields of a Field

2.1 Theorem

If $A$ is a T-fuzzy subfield of a field $(F, +, \cdot)$, then $\mu_{A}(x) = \mu_{A}(y)$, for all $x$ in $F$ and $\mu_{A}(x) = \mu_{A}(y)$, for all $x \neq 0$ in $F$ and $\mu_{A}(x) = \mu_{A}(y)$, for all $x \neq 0$ in $F$, where 0 and 1 are identity elements in $F$.

Proof

For $x$ in $F$ and 0, 1 are identity elements in $F$. Now, $\mu_{A}(x) = \mu_{A}(y) \geq \mu_{A}(x) \geq \mu_{A}(y)$. Therefore, $\mu_{A}(x) = \mu_{A}(y)$, for all $x$ in $F$. Now, $\mu_{A}(x) = \mu_{A}(y) \geq \mu_{A}(x) \geq \mu_{A}(y)$. Therefore, $\mu_{A}(x) = \mu_{A}(y)$, for all $x \neq 0$ in $F$. Now, $\mu_{A}(0) = \mu_{A}(x) \geq \mu_{A}(x)$, $\mu_{A}(x) = \mu_{A}(x)$. Therefore, $\mu_{A}(0) = \mu_{A}(x)$, for all $x$ in $F$. Now $\mu_{A}(x) = \mu_{A}(x) \geq \mu_{A}(x) = \mu_{A}(x)$. Therefore, $\mu_{A}(1) = \mu_{A}(x)$, for all $x \neq 0$ in $F$.

2.2 Theorem

If $A$ is a T-fuzzy subfield of a field $(F, +, \cdot)$, then (i) $\mu_{A}(x-y) = \mu_{A}(0)$ gives $\mu_{A}(x) = \mu_{A}(y)$, for all $x$ in $F$, and (ii) $\mu_{A}(x^{-1}) = \mu_{A}(1)$ gives $\mu_{A}(x) = \mu_{A}(y)$, for all $x$ and $y \neq 0$ in $F$, where 0 and 1 are identity elements in $F$.

Proof

Let $x$ and $y$ in $F$, 0, 1 are identity elements in $F$. (i) Now, $\mu_{A}(x) = \mu_{A}(y) \geq \mu_{A}(x) \geq \mu_{A}(y)$. Therefore, $\mu_{A}(x) = \mu_{A}(y)$, for all $x$ in $F$. Now, $\mu_{A}(x) = \mu_{A}(y) \geq \mu_{A}(x) \geq \mu_{A}(y)$. Therefore, $\mu_{A}(x) = \mu_{A}(y)$, for all $x \neq 0$ in $F$. Now, $\mu_{A}(0) = \mu_{A}(x) \geq \mu_{A}(x)$, $\mu_{A}(x) = \mu_{A}(x)$. Therefore, $\mu_{A}(0) = \mu_{A}(x)$, for all $x$ in $F$. Now $\mu_{A}(x) = \mu_{A}(x) \geq \mu_{A}(x) = \mu_{A}(x)$. Therefore, $\mu_{A}(1) = \mu_{A}(x)$, for all $x \neq 0$ in $F$.

2.3 Theorem

Let $A$ be a fuzzy subset of a field $(F, +, \cdot)$. If $\mu_{A}(x) = \mu_{A}(y) = \eta \neq 0$ and $\mu_{A}(x-y) = \mu_{A}(x), \mu_{A}(y)$, for all $x$ and $y$ in $F$ and $\mu_{A}(x^{-1}) \geq \mu_{A}(x), \mu_{A}(y)$, for all $x$ and $y \neq e$ in $F$, then $A$ is a T-fuzzy subfield of $F$, where $e$ and $e'$ are identity elements in $F$.

Proof

Let $x$ and $y$ in $F$ and $e, e'$ are identity elements of $F$. Now $\mu_{A}(x) = \mu_{A}(e-x) \geq \mu_{A}(e-x) \geq \mu_{A}(x)$. Therefore, $\mu_{A}(x) \geq \mu_{A}(x)$, for all $x$ in $F$. Now $\mu_{A}(x) = \mu_{A}(e-x) \geq \mu_{A}(e-x) \geq \mu_{A}(x)$. Therefore, $\mu_{A}(x) \geq \mu_{A}(x)$, for all $x \neq e$ in $F$. Now, $\mu_{A}(x-y) = \mu_{A}(x-y) \geq \mu_{A}(x-y) \geq \mu_{A}(x), \mu_{A}(y)$. Therefore, $\mu_{A}(x-y) \geq \mu_{A}(x), \mu_{A}(y)$. Therefore, $\mu_{A}(x-y) \geq \mu_{A}(x), \mu_{A}(y)$. Therefore, $\mu_{A}(x-y) \geq \mu_{A}(x), \mu_{A}(y)$, for all $x$ and $y \neq e$ in $F$. Hence $A$ is a T-fuzzy subfield of $F$.

2.4 Theorem

If $A$ is a T-fuzzy subfield of a field $(F, +, \cdot)$, then $H = \{ x \in \mu_{F}(x) = 1 \}$ is either empty or a subfield of $F$.

Proof

If no element satisfies this condition, then $H$ is empty. If $x$ and $y$ in $H$, then $\mu_{A}(x-y) \geq \mu_{A}(x-y) \geq \mu_{A}(x), \mu_{A}(y) = T(1, 1) = 1$. Therefore, $\mu_{A}(x-y) = 1$, for all $x$ and $y$ in $H$. And $T(1, 1) = \mu_{A}(x-y) \geq \mu_{A}(x), \mu_{A}(y)$. Therefore, $\mu_{A}(x-y) = T(\mu_{A}(x), \mu_{A}(y))$. Therefore, $\mu_{A}(x-y) = T(\mu_{A}(x), \mu_{A}(y))$. Therefore, $\mu_{A}(x-y) = T(\mu_{A}(x), \mu_{A}(y))$, for all $x$ and $y \neq e$ in $F$. Hence $H$ is a T-fuzzy subfield of $F$.

2.5 Theorem

If $A$ is a T-fuzzy subfield of a field $(F, +, \cdot)$, then $H = \{ x \in \mu_{F}(x) = \mu_{A}(e) = \mu_{A}(e) \}$ is either empty or a subfield of $F$, where $e$ and $e'$ are identity elements of $F$.

Proof

It is trivial.

2.6 Theorem

Let $A$ be a T-fuzzy subfield of a field $(F, +, \cdot)$. Then (i) if $\mu_{A}(x-y) = 1$, then $\mu_{A}(x) = \mu_{A}(y)$, for all $x$ and $y$ in $F$, and (ii) if $\mu_{A}(x^{-1}) = 1$, then $\mu_{A}(x) = \mu_{A}(y)$. Therefore, $\mu_{A}(x) = \mu_{A}(y)$, for all $x \neq e$ and $y \neq e$ in $F$, where $e$ and $e'$ are identity elements of $F$.

Proof

Let $x$ and $y$ in $F$. (i) Now, $\mu_{A}(x) = \mu_{A}(y) \geq \mu_{A}(x-y) \geq \mu_{A}(x), \mu_{A}(y) = T(1, 1) = 1$. Therefore, $\mu_{A}(x-y) = 1$, for all $x$ and $y$ in $F$. And, $\mu_{A}(x^{-1}) \geq \mu_{A}(x^{-1}) \geq \mu_{A}(x), \mu_{A}(y)$. Therefore, $\mu_{A}(x^{-1}) \geq \mu_{A}(x), \mu_{A}(y)$. Therefore, $\mu_{A}(x^{-1}) \geq \mu_{A}(x), \mu_{A}(y)$. Therefore, $\mu_{A}(x^{-1}) = \mu_{A}(x), \mu_{A}(y)$, for all $x \neq e$ and $y \neq e$ in $F$.

2.7 Theorem

If $A$ be a T-fuzzy subfield of a field $(F, +, \cdot)$, then if $\mu_{A}(x-y) = 0$, then either $\mu_{A}(x) = 0$ or $\mu_{A}(y) = 0$, for all $x$ and $y$ in $F$ if $\mu_{A}(y) = 0$, then either $\mu_{A}(x) = 0$ or $\mu_{A}(y) = 0$, for all $x$ and $y \neq e$ in $F$, where $e$ and $e'$ are identity elements of $F$.

Proof

Let $x$ and $y$ in $F$. By the definition $\mu_{A}(x-y) \geq T(\mu_{A}(x), \mu_{A}(y))$, which implies that $0 \geq T(\mu_{A}(x), \mu_{A}(y))$. Therefore, either $\mu_{A}(x) = 0$ or $\mu_{A}(y) = 0$, for all $x$ and $y$ in $F$. And, by the definition $\mu_{A}(y) = T(\mu_{A}(x), \mu_{A}(y))$, which implies that $0 \geq T(\mu_{A}(x), \mu_{A}(y))$. Therefore, either $\mu_{A}(x) = 0$ or $\mu_{A}(y) = 0$, for all $x$ and $y \neq e$ in $F$.

2.8 Theorem

Let $(F, +, \cdot)$ be a field. If $A$ is a T-fuzzy subfield of $F$, then $\mu_{A}(x-y) = T(\mu_{A}(x), \mu_{A}(y))$, for all $x$ and $y$ in $F$ and $\mu_{A}(y) = T(\mu_{A}(x), \mu_{A}(y))$, for all $x$ and $y \neq 0$ in $F$, where 0 and 1 are identity elements of $F$. 
The intersection of a family of T-fuzzy subfields of a field (F, +, \cdot) is a T-fuzzy subfield of F.

**Proof**

It is trivial.

**2.11 Theorem**

Let A be a T-fuzzy subfield of a field (F, +, \cdot). If \( \mu_A(x) < \mu_A(y) \), for some x and y in F, then \( \mu_A(x+y) = \mu_A(x) + \mu_A(y) \), for all x and y in F, and \( \mu_A(xy) = \mu_A(x) \cdot \mu_A(y) \), for all x and y \neq 0 in F.

**Proof**

Let A be a T-fuzzy subfield of a field F. Also we have \( \mu_A(x) < \mu_A(y) \), for some x and y in F. Hence, \( \mu_A(x+y) \geq \min(\mu_A(x), \mu_A(y)) \) and \( \mu_A(x-y) \geq \min(\mu_A(x), \mu_A(y)) \) for all x and y in F. Therefore, \( \mu_A(x+y) = \mu_A(x) + \mu_A(y) \), for all x and y in F.

**2.12 Theorem**

Let A be a T-fuzzy subfield of a field (F, +, \cdot). If \( \mu_A(x) > \mu_A(y) \), for some x and y in F, then \( \mu_A(x+y) = \mu_A(x) + \mu_A(y) \), for all x and y in F, and \( \mu_A(xy) = \mu_A(x) \cdot \mu_A(y) \), for all x and y \neq 0 in F.

**Proof**

It is trivial.

**2.13 Theorem**

If A and B are T-fuzzy subfields of the fields G and H respectively, then A\times B is a T-fuzzy subfield of GxH.

**Proof**

Let A and B be T-fuzzy subfields of the fields G and H respectively. Let \( x_1 \) and \( y_1 \) be in G and \( x_2 \) and \( y_2 \) be in H. Then (\( x_1, y_1 \)) and (\( x_2, y_2 \)) are in GxH. Now, \( \mu_{A\times B}(x_1, y_1 - (x_2, y_2)) = \mu_{A\times B}(x_1-x_2, y_1-y_2) = \min(\mu_A(x_1-x_2), \mu_H(y_1-y_2)) = \min(\mu_A(x_1), \mu_H(y_1)) = \mu_A(x_1) \cdot \mu_H(y_1) \), where \( \mu_A \) and \( \mu_H \) are the identity elements of A and H respectively. Therefore, \( \mu_{A\times B}(x_1, y_1-x_2, y_1-y_2) = \mu_{A\times B}(x_1-x_2, y_1-y_2) = \min(\mu_A(x_1-x_2), \mu_H(y_1-y_2)) = \min(\mu_A(x_1), \mu_H(y_1)) = \mu_A(x_1) \cdot \mu_H(y_1) \), for all x and y in G and H.

**2.14 Theorem**

Let A and B be fuzzy subsets of the fields G and H respectively. Suppose that 0, 1 and 0', 1' are the identity elements of G and H respectively. If A is a T-fuzzy subfield of GxH, then at least one of the following two statements must hold.

(i) \( \mu_A(0') \geq \mu_A(x) \), for all x in G and \( \mu_B(1') \geq \mu_B(y) \), for all y \neq 0 in G.

(ii) \( \mu_A(0) \geq \mu_A(y) \), for all y in H and \( \mu_B(1) \geq \mu_B(y) \), for all y \neq 0 in H.

**Proof**

Let A\times B be a T-fuzzy subfield of GxH. By contraposition, suppose that none of the statements (i) and (ii) hold. Then we can find a in G and b in H such that \( \mu_A(a) > \mu_B(0') \) and \( \mu_B(b) > \mu_A(1) \). Let \( \mu_B(a, b) = \min(\mu_A(a), \mu_B(b)) = \min(\mu_A(0), \mu_B(0')) \), where \( \mu_A \) and \( \mu_B \) are the identity elements of G and H respectively. Thus, A\times B is not a T-fuzzy subfield of GxH. Hence either \( \mu_A(0') \geq \mu_A(x) \), for all x in G and \( \mu_B(1') \geq \mu_B(y) \), for all y \neq 0 in G, or \( \mu_A(0) \geq \mu_A(y) \), for all y in H and \( \mu_B(1) \geq \mu_B(y) \), for all y \neq 0 in H.

**2.15 Theorem**

Let A and B be fuzzy subsets of the fields G and H respectively. If A is a T-fuzzy subfield of G, then \( \mu_A(0) \geq \mu_B(0) \), for all x in G and \( \mu_B(1) \geq \mu_A(1) \), for all x \neq 0 in H, then B is a T-fuzzy subfield of H, where 0, 1 are identity elements of G and H respectively.
2.16 Theorem
Let \( A \times B \) be a T-fuzzy subfield of \( G \times H \) and \( x \) and \( y \) in \( G \). Then \( (x, 0) \) and \( (1, y) \) and \((y, 0)\), \((1, y)\) are in \( G \times H \). Now, using the property if \( \mu_A(x) \leq \mu_B(0) \), for all \( x \) in \( G \) and \( \mu_A(x) \leq \mu_B(1) \), for all \( x \neq 0 \) in \( G \), where 0 and 1 are identity elements of \( G \) and 0 and 1 are identity elements of \( H \), we get, \( \mu_A(x-y) = \mu_B(0)(x-y) = \mu_B(0)(x)+\mu_B(0)(y) \geq T(\mu_B(0), \mu_B(y)) = T(\mu_B(x), \mu_B(y)) \). Therefore \( \mu_A(x-y) \geq T(\mu_B(x), \mu_B(y)) \), for all \( x \) and \( y \neq 0 \) in \( G \). Hence \( A \times B \) is a T-fuzzy subfield of \( G \). Thus (i) is proved.

Now, using the property \( \mu_B(y) \leq \mu_B(0) \) for all \( x \) in \( H \) and \( \mu_B(x) \leq \mu_B(1) \), for all \( x \neq 0 \) in \( H \), we get, \( \mu_A(y-x) = \mu_B(0)(y-x) = \mu_B(0)(y)+\mu_B(0)(x) \geq T(\mu_B(0), \mu_B(x)) = T(\mu_B(y), \mu_B(x)) \). Therefore \( \mu_A(y-x) \geq T(\mu_B(y), \mu_B(x)) \), for all \( x \) and \( y \neq 0 \) in \( H \). Hence \( B \) is a T-fuzzy subfield of \( H \). Thus (ii) is proved. Hence (iii) is clear.

2.17 Theorem
Let \( A \) be a fuzzy subset of a field \( (F, +, \cdot) \) and \( V \) be the strong fuzzy relation of \( F \). Then \( A \) is a T-fuzzy subfield of \( F \) if and only if \( V \) is a T-fuzzy subfield of \( F \times F \).

Proof
Suppose that \( A \) is a T-fuzzy subfield of \( F \). Then for any \( x = (x_1, x_2) \) and \( y = (y_1, y_2) \) in \( F \). We have, \( \mu_A(x_1+y_1, x_2+y_2) = \mu_A(x_1, x_2) \mu_A(y_1, y_2) \geq T(\mu_A(x_1), \mu_A(x_2), \mu_A(y_1), \mu_A(y_2)) \). Therefore \( \mu_A(x) \geq T(\mu_A(x), \mu_A(y)) \), for all \( x \) and \( y \) in \( F \). We have, \( \mu_A(xy) = \mu_A(x, y) \mu_A(xy) \geq T(\mu_A(x), \mu_A(y)) \). Therefore \( \mu_A(xy) \geq T(\mu_A(x), \mu_A(y)) \), for all \( x \) and \( y \neq 0 \) in \( F \). It proves that \( V \) is a T-fuzzy subfield of \( F \). Conversely, assume that \( V \) is a T-fuzzy subfield of \( F \), then for any \( x = (x_1, x_2) \) and \( y = (y_1, y_2) \) in \( F \), we have, \( \mu_V(x_1+y_1, x_2+y_2) = \mu_V(x_1, x_2) \mu_V(y_1, y_2) \geq T(\mu_V(x_1), \mu_V(x_2), \mu_V(y_1), \mu_V(y_2)) \). If we put \( x_1 = 0 \), we get, \( \mu_V(x_1+y_1, x_1, y_1) \geq T(\mu_V(x_1), \mu_V(x_1), \mu_V(y_1)) \). If we put \( y_2 = 1 \), we get, \( \mu_V(x_2+y_2, x_2, y_2) \geq T(\mu_V(x_2), \mu_V(x_2), \mu_V(y_2)) \). If we put \( x_2 = 0 \), we get, \( \mu_V(x_1+y_1, x_1, y_1) \geq T(\mu_V(x_1), \mu_V(x_1), \mu_V(y_1)) \). If we put \( x_1 = 1 \), we get, \( \mu_V(x_2+y_2, x_2, y_2) \geq T(\mu_V(x_2), \mu_V(x_2), \mu_V(y_2)) \). Hence \( A \) is a T-fuzzy subfield of \( F \).

2.18 Theorem
Let \( (F, +, \cdot) \) and \( (F', +, \cdot) \) be any two fields. The homomorphic image of a T-fuzzy subfield of \( F \) is a T-fuzzy subfield of \( F' \).

Proof
Let \( (F, +, \cdot) \) and \( (F', +, \cdot) \) be any two fields and \( f : F \rightarrow F' \) be a homomorphism. That is \( f(x+y) = f(x)+f(y) \) for all \( x \) and \( y \) in \( F \), \( f(xy) = f(x)f(y) \) for all \( x \) and \( y \) in \( F \). Let \( V = f(A) \), where \( V \) is a T-fuzzy subfield of \( F' \). We have to prove that \( V \) is a T-fuzzy subfield of \( F' \). Now, for \( x \) and \( y \) in \( F' \), we have, \( \mu_V(f(x)+f(y)) = \mu_V(f(x+y)) = \mu_V(f(x), f(y)) \). Therefore \( \mu_V(f(x+y)) \geq T(\mu_V(f(x), f(y))) \). Hence \( V \) is a T-fuzzy subfield of \( F' \). We have proved that \( A \) is a T-fuzzy subfield of \( F \). Let \( x \) and \( y \) in \( F \). Then \( \mu_A(x+y) = \mu_A(f(x+y)) = \mu_A(f(x)+f(y)) \). Therefore \( \mu_A(x+y) \geq T(\mu_A(f(x), f(y))) \). Hence \( A \) is a T-fuzzy subfield of \( F \).

2.19 Theorem
A T-fuzzy subfield \( A \) of a field \( (F, +, \cdot) \) is normalized if and only if \( \mu_A(e) = \mu_A(e') = 1 \), where \( e \) and \( e' \) are identity elements of the field \( F \).

Proof
If \( A \) is normalized, then there exists \( x \in F \) such that \( xA = 1 \), but by properties of a T-fuzzy subfield \( A \) of \( F \), \( \mu(x) \leq \mu_A(e) \), for all \( x \) in \( F \) and \( \mu(x) \leq \mu_A(e') \), for all \( x \) in \( F \). Therefore \( \mu_A(e) = \mu_A(e') = 1 \). Conversely, if \( \mu_A(e) = \mu_A(e') = 1 \), then by the definition of normalized fuzzy subset, \( A \) is normalized.

2.20 Theorem
Let \( A \) be a T-fuzzy subfield of a field \( F \) and \( f \) is an isomorphism from a field \( F \) onto \( F \). Then \( A \times f \) is a T-fuzzy subfield of \( F \).

Proof
Let \( x \) and \( y \) in \( F \) and \( A \) be a T-fuzzy subfield of a field \( F \). Then we have, \( \mu_A(f(x)) = f(A)(f(x)) = \mu_A(f(x)+f(y)) = \mu_A(f(x)+f(y)) \geq T(\mu_A(f(x)), \mu_A(f(y))) \). Therefore \( \mu_A(f(x)+f(y)) \geq T(\mu_A(f(x)), \mu_A(f(y))) \). Hence \( A \) is a T-fuzzy subfield of \( F \).
F. And \((\mu_{A^f})(xy^{-1}) = \mu_A(f(x))f(y^{-1}) = \mu_A(f(x))f(y)^{-1}\) \(\geq T(\mu_A(f(x)), \mu_A(f(y))) \geq T(\mu_A^f(x), \mu_A^f(y))\), which implies that \((\mu_{A^f})(xy^{-1}) \geq T(\mu_{A^f}(x), \mu_{A^f}(y))\) for all \(x\) and \(y \neq 0\) in \(F\). Therefore \((A^f)\) is a \(T\)-fuzzy subfield of a field \(F\).

2.21 Theorem

Let 

\(A\) be a \(T\)-fuzzy subfield of a field \((F, +, \cdot)\), then the pseudo \(T\)-fuzzy coset \((aA)^p\) is a \(T\)-fuzzy subfield of a field \(F\), for every \(a \in F\).

Proof

Let 

\(A\) be a \(T\)-fuzzy subfield of a field \((F, +, \cdot)\). For every \(x\) and \(y\) in \(F\), we have, \((a\mu_A)^p(x - y) = p(a)\mu_A(x - y) \geq p(a) T(\mu_A(x), \mu_A(y)) = T(p(a)\mu_A(x), p(a)\mu_A(y)) = T((a\mu_A)^p(x), (a\mu_A)^p(y))\). Therefore \((a\mu_A)^p(x - y) \geq T((a\mu_A)^p(x), (a\mu_A)^p(y))\), for all \(x\), \(y\) in \(F\). And for every \(x\) and \(y \neq 0\) in \(F\), \((a\mu_A)^p(xy^{-1}) = p(a)\mu_A(xy^{-1}) \geq p(a) T(\mu_A(x), \mu_A(y)) = T(p(a)\mu_A(x), p(a)\mu_A(y)) = T((a\mu_A)^p(x), (a\mu_A)^p(y))\). Therefore, \((a\mu_A)^p(xy^{-1}) \geq T((a\mu_A)^p(x), (a\mu_A)^p(y))\), for all \(x\) and \(y \neq 0\) in \(F\). Hence \((aA)^p\) is a \(T\)-fuzzy subfield of a field \(F\).

Reference

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