1. Introduction

A new class of generalized open sets called b-open sets in topological spaces was defined by Andrijevic [2]. The class of all b open sets generates the same topology as the class of all pre-open sets. In 1986, Maki [11] introduced the concept of generalized $\Lambda$ sets and defined the associated closure operators by using the work of Levine [8] and Dunham [5]. Caldas and Dontchev [3] introduced $\Lambda$-sets, $V_\alpha$-sets and $g\Lambda$-sets and $gV_\alpha$-sets. Ganster and et al. [6] introduced the notion of pre $\Lambda$-sets and pre $V$-sets and obtained new topologies via these sets. M.E. Abd El-Monsef et al. [1] defined $b\Lambda$-sets and $bV$-sets on a topological space and proved that it forms a topology. In 1963 Levine [9] introduced the concept of a simple extension of a topology $\tau$ as $\tau(B) = (B \cap O) \cup O \setminus O \in \tau$ and $B \not\in \tau$. Sr. I. Arockiarani and F. Nirmala Irudayam [12] introduced the concept of $\pi$-genb-open sets in extended topological spaces. Caldas and Jafari[4] introduced the notions of $\pi gb\Lambda$-sets and $\pi gbV$-sets on a topological space and proved that it forms a topology. In 1986, Maki [11] introduced the concept of a simple extension of a topology $\tau$ as $\tau(B) = (B \cap O) \cup O \setminus O \in \tau$ and $B \not\in \tau$. Sr. I. Arockiarani and F. Nirmala Irudayam [13] coined the idea of $\pi gb\Lambda$, $\pi gbV$ sets in simple extended topological spaces.

2. Preliminaries

All through the paper the space $X$ is a SETS in which no separation axioms are assumed unless and otherwise stated.

Definition 2.1

A subset $A$ of a topological space $(X, \tau)$ is said to be,

(i) b-open set[2], if $A \subseteq cl(int(A)) \cup int(cl(A)) \subseteq A$.

(ii) a generalized closed (briefly g-closed) [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

(iii) a generalized b-closed (briefly bg-closed) [6] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

(iv) $gb$-closed[15] if $bcl(A) \subseteq A$ whenever $A \subseteq U$ and $U$ is $\pi$-open in $(X, \tau)$. By $\pi gbc(X, \tau)$ we mean the family of all $gb$-closed subsets of the space $(X, \tau)$.

Definition 2.2[12]: A subset $A$ of a topological space $(X, \tau)$ is said to be,

(i) b-open set if $A \subseteq cl'(int(A)) \cup int(cl'(A)) \subseteq A$.

(ii) a generalized’ closed (briefly g’-closed) if $cl'(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

(iii) a generalized b-closed (briefly bg’-closed) if $bcl'(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

(iv) $gb$’-closed[14] if $bcl'(A) \subseteq A$ whenever $A \subseteq U$ and $U$ is $\pi$-open in $(X, \tau)$. By $\pi gbc' (X, \tau)$ we mean the family of all $gb$’-closed subsets of the space $(X, \tau)$.

Definition 2.3[10]: Let $S$ be a subset of a topological space $(X, \tau)$ we define the sets $\Omega gb(S)$ and $\Omega gb(S)$ as follows,

$\Omega gb(S) = \cap \{G | G \subseteq \pi gbO(X, \tau) \}$ and $\Omega gb(S) = \cup \{F | F \in \pi gbc(X, \tau) \}$

Definition 2.4[14]: A function $f: (X, \tau') \rightarrow (Y, \sigma')$ is called

(i) $\pi'$-irresolute if $f^{-1}(V)$ is $\pi'$-closed in $(X, \tau')$ for every $\pi'$-closed set $V$ of $(Y, \sigma')$.

(ii) $b'$-irresolute if for each $b'$-open set $V$ in $(Y, \sigma')$, $f^{-1}(V)$ is $b'$-open in $(X, \tau')$.

(iii) $b'$-continuous if for each open set $V$ in $(Y, \sigma)$, $f^{-1}(V)$ is $b'$-open in $(X, \tau')$.

3. $\Omega gb$-KERNEL

Definition 3.1: Let $(X, \tau)$ be a topological space, $A \subseteq X$. Then $\Omega gb$-kernel of $A$ is defined by $\Omega gb$-Kernel$(A) = \cap \{G | G \subseteq \Omega gbO(X, \tau) \}$ and $A \subseteq G$.

Definition 3.2: A point $x \in \Omega gb$-kernel of $A$ if for every $\Omega gb$-open set $U$ containing $x$, $A \cap U \neq \phi$.

Let $(X, \tau')$ be a topological space and $A, B$ be subsets of $X$. Let $x, y \in X$ then we have the following lemmas.

Lemma 3.3: $A \subseteq \Omega gb$-kernel$(A)$

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Keywords

Kernel, Separation axiom.
Proof: Let \( x \in \Omega_\beta^s \setminus \ker(A) \) then there exists \( V \in \Omega_\beta^s \setminus \ker(A) \) such that \( x \in V \). Hence \( x \notin A \).

Lemma 3.4: If \( A \subset B \), then \( \Omega_\beta^s \setminus \ker(A) \subset \Omega_\beta^s \setminus \ker(B) \).

Proof: Let \( x \notin \Omega_\beta^s \setminus \ker(B) \). Then there exists \( G \in \Omega_\beta^s \setminus \ker(B) \) such that \( B \cap G \) and \( x \notin G \). Since \( A \subset B \), \( A \cap G \) and hence \( x \notin \Omega_\beta^s \setminus \ker(A) \).

Lemma 3.5: \( \Omega_\beta^s \setminus \ker(A) = \Omega_\beta^s \setminus \ker(B) \).

Proof: Let \( x \in \Omega_\beta^s \setminus \ker(B) \). Then there exists \( G \in \Omega_\beta^s \setminus \ker(B) \) for every \( \Omega_\beta^s \setminus \ker(A) \). \( x \notin \Omega_\beta^s \setminus \ker(A) \).

Lemma 3.6: \( \forall \Omega_\beta^s \setminus \ker(A) \).

Proof: Let \( y \notin \Omega_\beta^s \setminus \ker(A) \) there exists a \( \Omega_\beta^s \)-open set \( V \supseteq \{ x \} \) such that \( y \notin V \). Then \( \forall \Omega_\beta^s \setminus \ker(A) \).

Definition 4.1: \( (X, t^*) \) is \( \Omega_\beta^s \)-T_1 if for each pair of distinct points \( x, y \) of \( X \), there exists a \( \Omega_\beta^s \)-open set containing one of the points but not the other.

Theorem 4.2: \( (X, t^*) \) is \( \Omega_\beta^s \)-T_0 iff for each pair of distinct points \( x, y \in X \), \( \Omega_\beta^s \)-open set containing one of the points but not the other, say \( x \in V \) and \( y \notin V \). Then \( V \) is \( \Omega_\beta^s \)-open set containing \( x \) but not \( y \). But \( \Omega_\beta^s \)-open set containing \( y \) is the smallest \( \Omega_\beta^s \)-open set containing \( y \). Therefore \( \Omega_\beta^s \)-open set containing \( y \) is \( V \). Hence \( x \notin \Omega_\beta^s \)-open set.

Definition 4.3: \( (X, t^*) \) is \( \Omega_\beta^s \)-T_1 for every \( \Omega_\beta^s \)-open set \( U \) in \( X \) such that \( x \in U \) and \( y \notin U \).

Theorem 4.5: In a space \( (X, t^*) \), the following are equivalent

1. \( (X, t^*) \) is \( \Omega_\beta^s \)-T_1
2. \( \forall \Omega_\beta^s \)-open set containing \( x \) in \( X \).
3. \( \forall \Omega_\beta^s \)-open set containing \( x \) in \( X \).

Proof: Let \( x \in X \). Then there exists a \( \Omega_\beta^s \)-open set \( V \) such that \( x \in V \) and \( y \notin V \). If \( x \in \Omega_\beta^s \setminus \ker(A) \) then \( x \in \Omega_\beta^s \)-open set containing \( x \). \( x \in \Omega_\beta^s \)-open set containing \( y \). Therefore \( \forall \Omega_\beta^s \)-open set containing \( y \) is \( V \). Hence \( x \notin \Omega_\beta^s \)-open set containing \( y \). Therefore \( \forall \Omega_\beta^s \)-open set containing \( x \) but not \( y \). Similarly, there exists an \( \Omega_\beta^s \)-open set \( U \) in \( X \) such that \( x \in U \) but \( y \notin U \).

Theorem 4.6: A space \( (X, t^*) \) is \( \Omega_\beta^s \)-T_1 if and only if for every \( \Omega_\beta^s \)-open set \( U \) containing \( x \), \( \forall \Omega_\beta^s \)-open set containing \( x \).

Definition 4.7: \( (X, t^*) \) is \( \Omega_\beta^s \)-T_2 if and only if for every \( \Omega_\beta^s \)-open set \( U \) containing \( x \) but not \( y \). Similarly, there exists an \( \Omega_\beta^s \)-open set \( V \) containing \( y \) but not \( x \).

Remark 4.8: Every \( \Omega_\beta^s \)-open set is \( \Omega_\beta^s \)-open set. But the converse need not be true. For example, let \( X = \{ a, b, c \} \) and \( \tau = \{ \{ a, b, c \} \} \) and \( B = \{ \{ a, b \} \} \). Then \( (X, \tau) \) is \( \Omega_\beta^s \)-open set space but not \( \Omega_\beta^s \)-Ti space.

Theorem 4.9: For a topological space \( (X, \tau) \), the following are equivalent:

1. \( (X, \tau) \) is \( \Omega_\beta^s \)-T_1
2. \( \forall \Omega_\beta^s \)-open set containing \( x \) in \( X \).
3. \( \forall \Omega_\beta^s \)-open set containing \( x \) in \( X \).

Proof: Let \( x \in X \). Then for each \( y \notin X \), there is a \( \Omega_\beta^s \)-open set \( U \) containing \( x \) such that \( y \notin \Omega_\beta^s \)-open set containing \( x \).

Theorem 4.10: Every \( \Omega_\beta^s \)-open set is \( \Omega_\beta^s \)-open set.

5. \( \Omega_\beta^s \)-Continuous and \( \Omega_\beta^s \)-Irresolute Functions

Definition 5.1: A function \( f : (X, t^*) \to (Y, s^*) \) is called \( \Omega_\beta^s \)-continuous if every \( f^{-1}(V) \) is \( \Omega_\beta^s \)-closed in \( (X, t^*) \) for every closed set \( V \in (Y, s^*) \).

Definition 5.2: A function \( f : (X, t^*) \to (Y, s^*) \) is called \( \Omega_\beta^s \)-irresolute if \( f^{-1}(V) \) is \( \Omega_\beta^s \)-closed in \( (X, t^*) \) for every closed set \( V \in (Y, s^*) \).
Definition 5.3: A function $f: X \to Y$ is said to be pre $b^*$-closed if $f(U)$ is $b^*$-closed in $Y$ for each $b^*$-closed set in $X$.

Remark 5.4: Composition of two $\Omega_{gb}^+$-continuous functions need not be $\Omega_{gb}^+$-continuous.

Example 5.5: Let $X=\{a,b,c\}, \tau =\{X, \emptyset, \{a\}, \{a,c\}\}$ and $B=\{c\}, \tau' =\{X, \emptyset, \{a\}, \{a,c\}\}$. $\sigma =\{X, \emptyset, \{a\}, \{a,b\}\}$ and $B'=\{b\}$, $\sigma' =\{X, \Phi, \{a\}, \{a,b\}\}$. $\eta =\{X, \emptyset, \{a\}, \{a\}\}$ and $B'=\{b\}$, $\eta' =\{X, \Phi, \{a\}, \{a\}\}$. Define $f(X, \tau) \to (X, \sigma)$ by $f(a)=a$, $f(b)=b\cup(c(f(b')))$, $f(c)=b$. Define $g(X, \sigma) \to (X, \eta)$ by $g(a)=a$, $g(b)=b$, $g(c)=c$. Then $f$ and $g$ are $\Omega_{gb}^+$-continuous but $gof$ is not $\Omega_{gb}^+$-continuous.

Proposition 5.6: Let $f(X, \tau) \to (Y, \sigma)$ be $\pi^*$-irresolute and pre $b^*$-closed. Then $f(A)$ is $\Omega_{gb}^*$-closed in $Y$ for every $\Omega_{gb}^*$-closed set $A$ of $X$.

Proof: Let $A$ be $\Omega_{gb}^*$-closed in $X$. Let $f(A) \subseteq V$ be $\pi^*$-open in $Y$. Then $A \subseteq f^{-1}(V)$ and $A$ is $\Omega_{gb}^*$-closed in $X$ implies $b^*cl(A) \subseteq f^{-1}(V)$. Hence $f(bcl(A)) \subseteq V$. Since $f$ is pre $b^*$-closed, $b^*cl(f(A)) \subseteq b^*cl(f(bcl(A))) = f(bcl(A)) \subseteq V$. Hence $f(A)$ is $\Omega_{gb}^*$-closed in $Y$.

Definition 5.7: A topological space $X$ is a $\Omega_{gb}^*$-space if every $\Omega_{gb}^*$-closed set is closed.

Proposition 5.8: Every $\Omega_{gb}^*$-space is $\Omega_{gb}^*$-T$_{1/2}$ space.

Theorem 5.9: Let $f(X, \tau) \to (Y, \sigma)$ be a function.

(1) If $f$ is $\Omega_{gb}^*$-irresolute and $X$ is $\Omega_{gb}^*$-T$_{1/2}$ space, then $f$ is $b^*$-irresolute.

(2) If $f$ is $\Omega_{gb}^*$-continuous and $X$ is $\Omega_{gb}^*$-T$_{1/2}$ space, then $f$ is $b^*$-continuous.

Proof: (1) Let $V$ be $b^*$-closed in $Y$. Since $f$ is $\Omega_{gb}^*$-irresolute, $f^{-1}(V)$ is $\Omega_{gb}^*$-closed in $X$. Since $X$ is $\Omega_{gb}^*$-T$_{1/2}$ space, $f^{-1}(V)$ is $b^*$-closed in $X$. Hence $f$ is $b^*$-irresolute.

(2) Let $V$ be $b^*$-closed in $Y$. Since $f$ is $\Omega_{gb}^*$-continuous, $f^{-1}(V)$ is $\Omega_{gb}^*$-closed in $X$. By assumption, it is $b^*$-closed. Hence $f$ is $b^*$-continuous.

Definition 5.10: A function $f: (X, \tau) \to (Y, \sigma)$ is $\pi^*$-open map if $f(U)$ is $\pi^*$-open in $Y$ for every $\pi^*$-open in $X$.

Theorem 5.11: If the bijective $f: (X, \tau) \to (Y, \sigma)$ is $b^*$-irresolute and $\pi^*$-open map, then $f$ is $\Omega_{gb}^*$-irresolute.

Proof: Let $V$ be $\Omega_{gb}^*$-closed in $Y$. Let $f^{-1}(V) \subseteq U$ where $U$ is $\pi^*$-open in $X$. Hence $V \subseteq f(U)$ and $f(U)$ is $\pi^*$-open implies $b^*cl(U) \subseteq f(U)$. Since $f$ is $b^*$-irresolute, $(f^{-1}(b^*cl(U)))$ is $b^*$-closed. Hence $b^*cl(f^{-1}(V)) \subseteq b^*cl(f^{-1}(b^*cl(U))) = f^{-1}(b^*cl(U)) \subseteq U$. Therefore $f$ is $\Omega_{gb}^*$-irresolute.

Theorem 5.12: If $f:X \to Y$ is $\pi^*$-open, $b^*$-irresolute, pre $b^*$-closed surjective function. If $X$ is $\Omega_{gb}^*$-T$_{1/2}$ space, then $Y$ is $\Omega_{gb}^*$-T$_{1/2}$ space.

Proof: Let $F$ be a $\Omega_{gb}^*$-closed set in $Y$. Let $f^{-1}(F) \subseteq U$ where $U$ is $\pi^*$-open in $X$. Then $F \subseteq f(U)$ and $F$ is a $\Omega_{gb}^*$-closed set in $Y$ implies $b^*cl(F) \subseteq f(U)$. Since $f$ is $b^*$-irresolute, $b^*cl(f^{-1}(F)) \subseteq b^*cl(f^{-1}(b^*cl(F))) = f^{-1}(b^*cl(F)) \subseteq U$. Therefore $f^{-1}(F)$ is $\Omega_{gb}^*$-closed in $X$. Since $X$ is $\Omega_{gb}^*$-T$_{1/2}$ space, $f^{-1}(F)$ is $b^*$-closed in $X$. Since $f$ is pre $b^*$-closed, $f(f^{-1}(F)) = F$ is $b^*$-closed in $Y$. Hence $Y$ is $\Omega_{gb}^*$-T$_{1/2}$ space.

References