Edge-vertex dominating sets and Edge-vertex domination polynomials of Stars

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ABSTRACT

Let \( G = (V, E) \) be a simple graph. A set \( S \subseteq E(G) \) is an Edge-Vertex dominating set of \( G \) (or simply an ev-Dominating set), if for all vertices \( v \in V(G) \), there exists an edge \( e \in S \) such that \( e \) dominates \( v \). Let \( S_n \) be the Star graph and let \( D_{ev}(S_n, i) \) denote the family of all Edge-Vertex dominating sets of \( S_n \) with cardinality \( i \). Let \( d_{ev}(S_n, i) = \left| D_{ev}(S_n, i) \right| \), be the number of Edge-Vertex dominating sets of \( S_n \) with cardinality \( i \). In this paper, we study the concept of Edge-Vertex domination polynomials of Star graph \( S_n \). The Edge-Vertex Domination polynomial of \( S_n \) is

\[
D_{ev}(S_n, x) = \sum_{i=1}^{n-1} d_{ev}(S_n, i)x^i.
\]

We obtain some properties of \( D_{ev}(S_n, x) \) and its coefficients. Also, we calculate the recursive formula to derive the Edge-Vertex Domination polynomials of Star graph \( S_n \).

1. Introduction

Let \( G = (V, E) \) be a simple graph of order \( |V| = n \). A set \( S \subseteq V(G) \) is a dominating set of \( G \), if every vertex in \( V \setminus S \) is adjacent to at least one vertex in \( S \). For any vertex \( v \in V \), the open neighbourhood of \( v \) is the set \( N(v) = \{ u \in V : uv \in E \} \) and the closed neighbourhood of \( v \) is the set \( N[v] = N(v) \cup \{ v \} \). For a set \( S \subseteq V \), the open neighbourhood of \( S \) is \( N(S) = \bigcup_{v \in S} N(v) \) and the closed neighbourhood of \( S \) is \( N[S] = N(S) \cup S \). For a set \( S \subseteq V \), the open neighbourhood of \( S \) is \( N(S) = \bigcup_{v \in S} N(v) \) and the closed neighbourhood of \( S \) is \( N[S] = N(S) \cup S \). Let \( S_n, n \geq 2 \) be the star with \( n \) vertices \( V(S_n) = \{1\} \) and \( E(S_n) = \{(1, 2), (1, 3), \ldots, (1, n)\} \).

Definition 1.1

For a graph \( G = (V, E) \), an edge \( e = uv \in E(G) \), ev-dominates a vertex \( w \in V(G) \) if

(i) \( u = w \) or \( v = w \) (\( w \) is incident to \( e \)) or
(ii) \( uw \) or \( vw \) is an edge in \( G \) (\( w \) is adjacent to \( u \) or \( v \)).

Definition 1.2

A set \( S \subseteq E(G) \) is an Edge-Vertex dominating set of \( G \) (or simply an ev-dominating set), if for all vertices \( v \in V(G) \), there exist an edge \( e \in S \) such that \( e \) dominates \( v \). The Edge-Vertex domination number of a graph \( G \) is defined as the minimum size of an Edge-Vertex dominating set of edges in \( G \) and it is denoted as \( \gamma_{ev}(G) \).

Definition 1.3

Let \( D_{ev}(S_n, i) \) be the family of Edge-Vertex dominating sets of a Star graph \( S_n \) with cardinality \( i \) and let \( d_{ev}(S_n, i) = |D_{ev}(S_n, i)| \) be the number of Edge-Vertex dominating sets of \( S_n \). We call the polynomial

\[
D_{ev}(S_n, x) = \sum_{i=1}^{n-1} d_{ev}(S_n, i)x^i,
\]

the Edge-Vertex domination polynomial of the graph \( S_n \).
As usual we use \( \lfloor x \rfloor \) for the largest integer less than or equal to \( x \) and \( \lceil x \rceil \) for the smallest integer greater than or equal to \( x \).

Also, we denote the set \( \{e_1, e_2, \ldots, e_n\} \) by \([e_n]\) and the set \( \{1, 2, \ldots, n\} \) by \([n]\), throughout this paper.

2. Edge-Vertex Dominating Sets of Stars

Let \( S_n \), \( n \geq 2 \) be a Star with \( n \) vertices \( V(S_n)=[n] \) and \( E(S_n) = \{e_1, e_2, \ldots, e_{n-1}\} \). Let \( D_{ev}(S_n, i) \) be the family of Edge-Vertex dominating sets of \( S_n \) with cardinality \( i \).

**Lemma 2.1**

The following results hold for all graph \( G \) with \( |V(G)| = n \) vertices and \( |E(G)| = n-1 \) edges.

(i) \( d_{ev}(G,n-1)=1 \),

(ii) \( d_{ev}(G,n-2)=n-1 \),

(iii) \( d_{ev}(G,i)=0 \) if \( i \geq n \),

(iv) \( d_{ev}(G,0)=0 \).

**Proof:**

Let \( G=(V,E) \) be a simple graph of order \( n \) and size \( n-1 \), then

(i) \( D_{ev}(G,n-1)=[G]=\{e_{n-1}\} \), therefore \( |D_{ev}(G,n-1)|=1 \). Therefore, \( d_{ev}(G,n-1)=1 \).

(ii) \( D_{ev}(G,n-2)=[G-\{e_i\} \): \( \forall e_i \in G \} \), therefore \( |D_{ev}(G,n-2)|=n-1 \). Therefore, \( d_{ev}(G,n-2)=n-1 \).

(iii) If \( i \geq n \), there does not exist \( H \subseteq G \) such that \( |E(H)|>|E(G)| \). Therefore, \( d_{ev}(G,i)=0 \).

(iv) For \( i=0 \) there does not exist \( H \subseteq G \) such that \( |E(H)|=0 \), \( \Phi \) is not a Edge-Vertex dominating set of \( G \). Therefore, \( d_{ev}(G,0)=0 \).

**Lemma 2.2**

For all \( n \in \mathbb{Z}^+ \), \( D_{ev}(S_n,i)=\Phi \) if and only if \( i \geq n \) or \( i<0 \).

**Theorem 2.3**

Let \( S_n \) be a Star with vertices \( n \geq 2 \), then

(i) \( d_{ev}(S_n,i)=\begin{pmatrix} n-1 \\ i \end{pmatrix} \) if \( i \leq n-1 \).

(ii) \( d_{ev}(S_n,i)=\begin{pmatrix} d_{ev}(S_{n-1},i)+1, & \text{if } i=1 \\ d_{ev}(S_{n-1},i)+d_{ev}(S_{n-1},i-1), & \text{if } 1 < i \leq n-1 \end{pmatrix} \)

**Proof**

(i) Let \( S_n \) be a star with \( n \) vertices and \( n-1 \) edges and let \( v \in V(S_n) \) be such that \( v \) is the centre of \( S_n \) and let the edges be \( \{e_1, e_2, \ldots, e_{n-1}\} \). Consider an edge \( e_i \). By the definition of Edge-Vertex domination, it covers all the vertices of \( S_n \). Similarly, any other edge of \( S_n \) covers all the vertices of \( S_n \). Therefore, the number of Edge-Vertex dominating sets of cardinality 1 is \( \begin{pmatrix} n-1 \\ 1 \end{pmatrix} \). Consider any two edges of \( S_n \). These edges cover all the remaining vertices of \( S_n \). Therefore, number of Edge-Vertex dominating sets of cardinality 2 is \( \begin{pmatrix} n-1 \\ 2 \end{pmatrix} \). By continuing, we get the number of Edge-Vertex dominating sets of cardinality \( i \) is \( \begin{pmatrix} n-1 \\ i \end{pmatrix} \) \( i \leq n-1 \). Therefore, \( d_{ev}(S_n,i)=\begin{pmatrix} n-1 \\ i \end{pmatrix} \) \( i \leq n-1 \).

From Table 1, we have \( d_{ev}(S_n,i)=d_{ev}(S_{n-1},i)+1 \cdot i=1 \). For \( 1 < i \leq n-1 \), we have \( \begin{pmatrix} n-2 \\ i-1 \end{pmatrix} + \begin{pmatrix} n-2 \\ i \end{pmatrix} = \begin{pmatrix} n-1 \\ i \end{pmatrix} \).

Therefore, \( d_{ev}(S_n,i)=d_{ev}(S_{n-1},i)+d_{ev}(S_{n-1},i-1) \cdot 1 < i \leq n-1 \).

3. Edge-Vertex Domination Polynomials of Stars

In this section, we obtain the Edge-Vertex Domination polynomial \( D_{ev}(S_n,x) \) of the Star graph \( S_n \).
Theorem 3.1
\[ D_{\nu}^n(S_n, x) = (1 + x)^{n-1} - 1. \]
\[ \text{Proof:} \]
Let
\[ D_{\nu}^n(S_n, x) = \sum_{i=1}^{n-1} d_{\nu}^i(S_n, i)x^i \]
\[ = \sum_{i=1}^{n-1} \left( n-1 \right)^i, \text{by theorem 2.3 (i)}. \]
\[ = \left( n-1 \right) \left( 1 + \frac{1}{2} + \ldots + \frac{n-1}{n-1} \right) \]
\[ = 1 + (n-1) + \left( n-1 \right)\left( 1 + \frac{1}{2} + \ldots + \frac{n-1}{n-1} \right) \]
\[ D_{\nu}^n(S_n, x) = (1 + x)^{n-1} - 1. \]

Theorem 3.2
\[ D_{\nu}^n(S_n, x) = (1 + x)D_{\nu}^{n-2}(S_{n-1}, x) + x \] with \( D_{\nu}^n(S_2, x) = x \) for \( n \geq 3 \).
\[ \text{Proof:} \]
\[ D_{\nu}^n(S_n, x) = \sum_{i=1}^{n-1} d_{\nu}^i(S_n, i)x^i \]
\[ = d_{\nu}^1(S_n, 1)x + \sum_{i=2}^{n-1} d_{\nu}^i(S_n, i)x^i \]
\[ = (n-1) + \sum_{i=2}^{n-1} \left[ d_{\nu}^i(S_{n-1}, i) + d_{\nu}^i(S_{n-1}, i - 1) \right]x^i \]
\[ = (n-1) + \sum_{i=2}^{n-1} d_{\nu}^i(S_{n-1}, i)x^i + \sum_{i=2}^{n-1} d_{\nu}^i(S_{n-1}, i - 1)x^i \]
Consider,
\[ \sum_{i=2}^{n-1} d_{\nu}^i(S_{n-1}, i)x^i = d_{\nu}^1(S_{n-1}, 2)x^2 + d_{\nu}^1(S_{n-1}, 3)x^3 + \ldots + d_{\nu}^1(S_{n-1}, n-1)x^{n-1} \]
\[ = d_{\nu}^1(S_{n-1}, 1)x + d_{\nu}^1(S_{n-1}, 2)x^2 + \ldots + d_{\nu}^1(S_{n-1}, n-1)x^{n-1} - \]
\[ d_{\nu}^1(S_{n-1}, 1)x \]
\[ = \sum_{i=1}^{n-1} d_{\nu}^1(S_{n-1}, i)x^i - d_{\nu}^1(S_{n-1}, 1)x \]
\[ = D_{\nu}^1(S_{n-1}, x) - \left( n-2 \right)x \]
\[ = D_{\nu}^1(S_{n-1}, x) - (n-2)x \]
Consider,
\[ \sum_{i=2}^{n-1} d_{\nu}^1(S_{n-1}, i - 1)x^i = x\sum_{i=2}^{n-1} d_{\nu}^1(S_{n-1}, i - 1)x^i \]
\[ = x \left[ d_{\nu}^1(S_{n-1}, 1)x + d_{\nu}^1(S_{n-1}, 2)x^2 + \ldots + d_{\nu}^1(S_{n-1}, n-2)x^{n-2} \right] \]
\[ = x\sum_{i=1}^{n-2} d_{\nu}^1(S_{n-1}, i)x^i \]
\[ = xD_{\nu}^1(S_{n-1}, x) \]
\[ D_{\nu}^n(S_n, x) = (n-1)x + D_{\nu}^n(S_{n-1}, x) - (n-2)x + xD_{\nu}^n(S_{n-1}, x) \]
\[ = nx + xD_{\nu}^n(S_{n-1}, x) - nx + 2x + xD_{\nu}^n(S_{n-1}, x) \]
\[ = (1 + x)D_{\nu}^n(S_{n-1}, x) + x \]
Hence the theorem.
Example for Theorem 3.2
Let \( D_v(S_n, x) \) be the Edge-Vertex domination polynomial of Star graph \( S_n \). Then,

(i) \( D_v(S_3, x) = 2x + x^2 \)

(ii) \( D_v(S_4, x) = 3x + 3x^2 + x^3 \)

(iii) \( D_v(S_5, x) = 4x + 6x^2 + 4x^3 + x^4 \)

(iv) \( D_v(S_6, x) = 5x + 10x^2 + 10x^3 + 5x^4 + x^5 \).

Solution
From Theorem 3.3, \( D_v(S_n, x) = (1 + x)D_v(S_{n-1}, x) + x \) with \( D_v(S_2, x) = x \) for \( n \geq 3 \).

(i) For \( n = 3 \), \( D_v(S_3, x) = (1 + x)D_v(S_2, x) + x \)
    \( = (1 + x)x + x \)
    \( = 2x + x^2 \)

(ii) For \( n = 4 \), \( D_v(S_4, x) = (1 + x)D_v(S_3, x) + x \)
    \( = (1 + x)(2x + x^2) + x \)
    \( = 3x + 3x^2 + x^3 \)

(iii) For \( n = 5 \), \( D_v(S_5, x) = (1 + x)D_v(S_4, x) + x \)
    \( = (1 + x)(3x + 3x^2 + x^3) + x \)
    \( = 4x + 6x^2 + 4x^3 + x^4 \)

(iv) For \( n = 6 \), \( D_v(S_6, x) = (1 + x)D_v(S_5, x) + x \)
    \( = (1 + x)(4x + 6x^2 + 4x^3 + x^4) + x \)
    \( = 5x + 10x^2 + 10x^3 + 5x^4 + x^5 \).

We obtain \( d_v(S_n, i) \) for \( 2 \leq n \leq 13 \) as shown in Table 1.

Table 1. \( d_v(S_n, i) \), the number of Edge-Vertex dominating set of \( S_n \) with cardinality \( i \).

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In the following Theorem, we obtain some properties of \( d_v(S_n, i) \).

Theorem 3.3
The following properties hold for the coefficients of \( D_v(S_n, x) \) \( \forall n \in \mathbb{Z}^+, n \geq 4 \).

(i) \( d_v(S_n, 1) = n - 1 \).

(ii) \( d_v(S_n, n - 1) = 1 \).

(iii) \( d_v(S_n, n - 2) = n - 1 \).

(iv) \( d_v(S_n, i) = 0 \) if \( i \geq n \).

(v) \( \gamma_v(S_n) = 1 \).

(vi) \( d_v(S_n, i) = d_v(S_n, n - 1 - i), 1 \leq i \leq n - 2 \).
Therefore, the result is -1.

(iii) We prove this by the method of induction on “n”. If \( n = 4 \), L.H.S = \( d_v(S_4, 1) = 3 \) (from table 1). R.H.S = 4 - 1 = 3. Therefore, \( d_v(S_{j-1}, 1) = j - 2 \) is true. Now, we have to prove that the result is true for \( n = j \). \( d_v(S_j, 1) = d_v(S_{j-1}, j-1) + 1 = j - 2 + 1 \). Therefore, \( d_v(S_j, 1) = j - 1 \). Therefore, the result is true for \( n = j \). Hence, by the principle of induction, the result is true for all \( n, n \in \mathbb{Z}^+ \).

(ii) We prove this by the method of induction on ‘n’. If \( n = 4 \), L.H.S = \( d_v(S_4, 3) = 1 \) (from table 1). R.H.S = 1. Therefore, the result is true for \( n = 4 \). Assume that the result is true for all \( n < j \). Therefore, \( d_v(S_{j-1}, j - 2) = 1 \) is true. Now, we have to prove that the result is true for \( n = j \). \( d_v(S_j, j - 1) = d_v(S_{j-1}, j - 1) + d_v(S_{j-1}, j - 2) = 0 + 1 = 1 \). Therefore, the result is true for \( n = j \). Hence, by the principle of induction, the result is true for all \( n, n \in \mathbb{Z}^+ \).

(iii) We prove this by the method of induction on ‘n’. If \( n = 4 \), L.H.S = \( d_v(S_4, 2) = 3 \) (from table 1). R.H.S = 4 - 1 = 3. Therefore, the result is true for \( n = 4 \). Assume that the result is true for all \( n < j \). Therefore, \( d_v(S_{j-1}, j - 2) = 1 \) is true. Now, we have to prove that the result is true for \( n = j \). \( d_v(S_j, j - 2) = d_v(S_{j-1}, j - 2) + d_v(S_{j-1}, j - 3) \). Therefore, the result

\[
= 1 + j - 2
= j - 1
\]

is true for \( n = j \). Hence, by the principle of induction, the result is true for all \( n, n \in \mathbb{Z}^+ \).

(iv) From Table 1, we have \( d_v(S_n, i) = 0 \) if \( i \geq n \).

(v) Any edge of \( S_n \) is enough to cover all the vertices and edges of \( S_n \). Therefore, the minimum cardinality of the Edge-Vertex dominating set of \( S_n \) is 1. Therefore, \( \chi_v(S_n) = 1 \).

(vi) L.H.S =

\[
d_v(S_n) = \left(\frac{n-1}{i}\right)
\]

R.H.S =

\[
d_v(S_n, n-1-i) = \left(\frac{n-1}{n-1-i}\right)
\]

\[
= \frac{(n-1)!}{(n-1-i)! (n-1-i+1)!}
\]

\[
= \frac{(n-1)!}{(n-1-i)! i!}
\]

\[
= \left(\frac{n-1}{i}\right)
\]

Therefore, \( d_v(S_n, i) = d_v(S_n, n-1-i), 1 \leq i \leq n-2 \).

**Theorem 3.4**

The Edge-Vertex dominating roots of the Star graph \( S_n \) are

\[
cos \left(\frac{2(k+1)\pi}{n-1}\right) + i \sin \left(\frac{2(k+1)\pi}{n-1}\right), k = 0, \ldots, n-2
\]

**Proof:**

The Edge-Vertex domination polynomial of Star graph \( S_n \) is \( D_v(S_n, x) = (1 + x)^{n-1} - 1 \). To find the Edge-Vertex dominating roots, put \( D_v(S_n, x) = 0 \). Therefore, we get

\[
(1 + x)^{n-1} - 1 = 0
\]

\[
(1 + x)^{n-1} = 1
\]

\[
(1 + x) = (1)^{\frac{1}{n-1}}
\]

\[
= (\cos 2\pi + i \sin 2\pi)^{\frac{1}{n-1}}
\]

\[
= [\cos(2k\pi + 2\pi) + i \sin(2k\pi + 2\pi)]^{\frac{1}{n-1}}, \text{ where } k \text{ is an integer.}
\]
\[
= \left[ \cos 2(k+1)\pi + i \sin 2(k+1)\pi \right]^{n-1},
\]

\[
k = 0,1,\ldots, n-2
\]

\[
(1 + x) = \cos \frac{2(k+1)\pi}{n-1} + i \sin \frac{2(k+1)\pi}{n-1},
\]

\[
k = 0,1,\ldots, n-2
\]

\[
x = \cos \frac{2(k+1)\pi}{n-1} + i \sin \frac{2(k+1)\pi}{n-1} - 1,
\]

\[
k = 0,1,\ldots, n-2
\]

Therefore, the Edge-Vertex dominating roots of the Star graph \( S_n \) are

\[
\cos \frac{2(k+1)\pi}{n-1} + i \sin \frac{2(k+1)\pi}{n-1}, k = 0,1,\ldots, n-2.
\]

**Theorem 3.5**

\[
\frac{d^n}{dx^n} D_{ev}(S_n, x) = (n-1)!.
\]

**Proof:**

The Edge-Vertex domination polynomial of Star graph \( S_n \) is \( D_{ev}(S_n, x) = (1+x)^{n-1} - 1 \).

Differentiating with respect to \( x \) we get,

\[
\frac{d}{dx} [D_{ev}(S_n, x)] = (n-1)(1+x)^{n-2}.
\]

Again differentiating with respect to \( x \) we get,

\[
\frac{d^2}{dx^2} [D_{ev}(S_n, x)] = (n-1)(n-2)(1+x)^{n-3}.
\]

Continuing this way we get \( n \)th derivative,

\[
\frac{d^n}{dx^n} [D_{ev}(S_n, x)] = (n-1)(n-2)...((n-1) - (n-2))(1+x)^{n-n}
\]

\[
= (n-1)(n-2)...(n-1-n+2)(1+x)^0
\]

\[
= (n-1)(n-2)...2.1
\]

\[
= (n-1)!
\]

**Theorem 3.6**

Let \( S_n \) be the Star graph with \( n \) vertices then. \( D_{ev}(S_n, -1) = -1 \).

**Proof:**

The Edge-Vertex domination polynomial of Star graph \( S_n \) is \( D_{ev}(S_n, x) = (1+x)^{n-1} - 1 \).

\[
D_{ev}(S_n, -1) = (1-1)^{n-1} - 1 = 0 - 1 = -1.
\]

**4. Conclusion**

In this paper we obtain the Edge-Vertex dominating sets and Edge-vertex domination polynomial of some specified graphs.

**5. References**


