
U. S. Rajput* and Neetu Kanaujia

Department of Mathematics and Astronomy, University of Lucknow, Lucknow – U.P., India.

ABSTRACT

In the present paper, chemical reaction effect on MHD flow in porous media past an impulsively started oscillating vertical plate with variable mass diffusion and constant wall temperature in the presence of Hall current is studied here. The fluid considered is an electrically conducting. The Laplace transform technique has been used to find the solutions for the velocity, Sherwood number and skin friction. The velocity profile, heat transfer and mass diffusion have been studied for different parameters like Schmidt number, Hall parameter, magnetic parameter, chemical reaction parameter, mass Grashof number, thermal Grashof number, Prandtl number, and time. The effect of various parameters are shown graphically and the values of the skin-friction and Sherwood number for different parameters has been tabulated.

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Introduction

The fluid flows through porous medium are very important especially in the fields of irrigation and Agrology engineering, in petroleum technology to study Petroleum transport and in chemical engineering for clarify and purification. Hossain et al [8] have studied Hall effect on hydro magnetic free-convection flows along a porous flat plate with mass transfer. Attia et al. [3] have studied heat transfer between two parallel porous plates for Couette flow under pressure gradient and Hall current. Seth et al [5] have studied Hall effect on unsteady MHD natural convection flow of a heat absorbing fluid past an accelerated moving vertical plate with ramped wall temperature and by using Laplace transform technique. Ibrahim [1] have studied heat source and chemical effects on MHD convection flow embedded in a porous medium with Soret, viscous and Joules dissipation. Mythreye [4] have studied chemical reaction on unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Maleque and Abdus [6] have studied convective flow due to a rotating disk with Hall effect. Kim [7] has studied heat and mass transfer in MHD micro polar flow over a vertical moving porous plate in a porous medium. Earlier, we [2] have studied MHD flow past a vertical plate with variable temperature and mass diffusion in the presence of Hall current. In this paper, we are considering chemical reaction effect on MHD flow in porous media past an impulsively started oscillating vertical plate with variable mass diffusion and constant wall temperature in the presence of Hall current. The effect of Hall current on the velocity have been analyzed with the help of graphs, and skin friction and Sherwood number have been tabulated.

Mathematical Analysis

The unsteady flow of an electrically conducting, incompressible, viscous fluid past a vertical plate has been considered. The x axis is taken in the direction of the motion and z normal to it. A transverse magnetic field $B_0$ of uniform strength is applied on the flow. Initially it has been considered that the plate as well as the fluid is at the same temperature $T_0$. The species concentration in the fluid is taken as $C$, the temperature of the plate and the concentration of the fluid, respectively are raised to $T$ and $C$. Using the relation $\nabla \cdot B = 0$ for the magnetic field $\vec{B} = (B_x, B_y, B_z)$, we obtain $B_z$ (say $B_0$) $= $ constant, i.e. $B = (0, B_0, 0)$, where $B_0$ is externally applied transverse magnetic field. Figure 1 is physical description of the model.

The fluid model is as under

$$\frac{\partial \bar{u}}{\partial t} = \frac{\partial^2 \bar{u}}{\partial y^2} + g \beta (T - T_0) - \frac{\partial B_0^2}{\rho (l + m^2)} (u + mv) - \frac{\rho u}{K},$$

$$\frac{\partial \bar{w}}{\partial t} = \frac{\partial^2 \bar{w}}{\partial y^2} + \frac{\partial B_0^2}{\rho (l + m^2)} (w - mu) - \frac{\rho w}{K},$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} - K_c (C - C_\infty).$$
The initial and boundary conditions are
\[ \begin{align*}
\tau \leq 0 : & \quad u = 0, w = 0, C = C_0, T = T_0, \text{for all } y \\
\tau > 0 : & \quad u = u_0 \cos \theta, w = 0, T = T_0, C = C_0 + \left( C_a - C_0 \right) \frac{u_0^2}{v} \text{ at } y = 0 \\
 & \quad u = 0, w = 0, C = C_0, T = T_0, \text{ as } y \to \infty.
\end{align*} \] (5)

Here \( u \) and \( w \) are the primary and secondary velocities along \( x \) and \( z \) respectively, \( v \) the kinematic viscosity, \( \rho \) the density, \( C_p \) the specific heat at constant pressure, \( k \) the thermal conductivity of the fluid, \( K \) the permeability parameter, \( \beta \) the mass diffusion coefficient, \( \theta \) the gravitational acceleration, \( \beta \) the volumetric coefficient of thermal expansion, \( t \) time, \( m \) the Hall current parameter, \( T \) temperature of the fluid, \( \beta \) the volumetric coefficient of concentration expansion, \( C \) species concentration in the fluid, \( T_a \) temperature of the plate, \( C_a \) species concentration, \( B_{1\sigma} \) the uniform magnetic field, \( \sigma \) the electrical conductivity.

The following non-dimensional quantities are introduced to transform equations (1), (2), (3) and (4) into dimensionless form:
\[ \begin{align*}
\pi & = \frac{u}{u_0}, \pi = \frac{w}{u_0}, \tau = \frac{y u_0^2}{K}, \frac{u_0}{D}, \beta = \frac{\rho C_p}{K}, \theta = \frac{u_0}{\sqrt{\beta}}, \\
\frac{\partial C}{\partial t} & = \frac{\beta C}{\beta C}, \frac{C_0}{C_0}, \theta = \frac{\beta C}{\theta C}, \frac{\beta C}{\beta C}, \tau = \frac{y}{y_0}, \frac{K_0}{K_0}, \frac{\beta C}{\beta C}.
\end{align*} \] (6)

Symbols used are:
\( \theta \) - the temperature, \( C \) - the concentration, \( Gr \) - thermal Grashof number, \( \beta \) - the primary velocity, \( \beta \) - the secondary velocity, \( \mu \) - the coefficient of viscosity, \( Pr \) - the Prandtl number, \( Sc \) - the Schmidt number, \( K \) - the permeability parameter, \( K_0 \) - chemical reaction, \( \beta \) time, \( Gm \) mass Grashof number, \( Q \) - heat generation parameter, \( M \) - the magnetic parameter.

The dimensionless flow model becomes
\[ \begin{align*}
\frac{\partial \pi}{\partial \tau} & = \frac{\partial^2 \pi}{\partial \xi^2} + Gr \theta - \frac{1}{1 + m^2}, \\
\frac{\partial \pi}{\partial \xi} & = \frac{\partial^2 \pi}{\partial \eta^2} - M (\eta + m \pi) - \frac{1}{1 + m^2} - K u, \\
\frac{\partial C}{\partial \xi} & = \frac{\partial^2 C}{\partial \eta^2} - \frac{1}{1 + m^2} - K_0 C, \\
\frac{\partial \theta}{\partial \xi} & = \frac{\partial^2 \theta}{\partial \eta^2} - Pr \frac{\partial C}{\partial \eta}. \quad \text{(7)}
\end{align*} \]

The corresponding boundary conditions become
\[ \begin{align*}
\tau \leq 0 : & \quad \pi = 0, C = 0, \theta = 0, \frac{\partial C}{\partial \eta} = 0, \text{ for all values of } \eta, \\
\tau > 0 : & \quad \pi = \cos \beta \pi, C = 1, \theta = \pi \text{ at } \eta = 0, \\
 & \quad \pi \to 0, C \to 0, \theta \to 0, \pi \to 0 \text{ as } \eta \to \infty.
\end{align*} \] (11)

Dropping the bars and combining equations (7) and (8), we get
\[ \begin{align*}
\frac{\partial q}{\partial \eta} & = \frac{\partial^2 q}{\partial \eta^2} + Gr \theta - \frac{1}{1 + m^2} - \frac{1}{K} q, \\
\frac{\partial C}{\partial \eta} & = \frac{\partial^2 C}{\partial \eta^2} - K_0 C, \\
\frac{\partial \theta}{\partial \eta} & = \frac{\partial^2 \theta}{\partial \eta^2}, \quad \text{where } q = u + iv,
\end{align*} \] (12)

The boundary conditions become:
\[ \begin{align*}
\tau \leq 0 : & \quad q = 0, \theta = 0, \text{ for all values of } y, \\
\tau > 0 : & \quad q = \cos \beta \theta, w = 0, \theta = 1, \text{ at } y = 0, \\
 & \quad q \to 0, \text{ as } y \to \infty.
\end{align*} \] (15)

The solution of above equations obtained by Laplace transform method, which is an under
\[ \begin{align*}
\theta = Erfc \left[ \frac{\sqrt{Pr}y}{2\sqrt{t}} \right]
\end{align*} \]

\[ C = \frac{1}{4\sqrt{ScK}} e^{-\frac{Erfc \left[ \frac{1}{2\sqrt{ScK}} \right]}{2\sqrt{t}}} \]

\[ \frac{q}{2} = \frac{1}{2} \left( \frac{1}{\sqrt{ScK}} \right) [B(x) - 2\sqrt{ScK}] + e^{-\frac{Erfc \left[ \frac{1}{2\sqrt{ScK}} \right]}{2\sqrt{t}}} [B(1) - 2\sqrt{ScK}] \]

\[ \left( A_0 + e^{-\frac{Erfc \left[ \frac{1}{2\sqrt{ScK}} \right]}{2\sqrt{t}}} \right) \left[ \frac{1}{2} \right] \gamma \left[ \beta \right] \left( A_0 + e^{-\frac{Erfc \left[ \frac{1}{2\sqrt{ScK}} \right]}{2\sqrt{t}}} \right) \frac{1}{2} \gamma \left[ \beta \right] \left( A_0 + e^{-\frac{Erfc \left[ \frac{1}{2\sqrt{ScK}} \right]}{2\sqrt{t}}} \right) \frac{1}{2} \gamma \left[ \beta \right] \left( A_0 + e^{-\frac{Erfc \left[ \frac{1}{2\sqrt{ScK}} \right]}{2\sqrt{t}}} \right)
\]

The dimensionless skin friction at the plate \( y = 0 \) is computed by
\[ \left( \frac{dq}{dy} \right)_{y=0} = \tau_y + i \tau_z \]

The dimensionless Sherwood number at the plate \( y = 0 \) is computed by
\[ Sh = \left( \frac{Sc}{v} \right)_{y=0} \]

\[ Sh = \frac{1}{4\sqrt{ScK}} \left( B_1 \left( \frac{1}{2\sqrt{ScK}} \right) - 2\sqrt{ScK} \right) + e^{-\frac{Erfc \left[ \frac{1}{2\sqrt{ScK}} \right]}{2\sqrt{t}}} \left( B_1 \left( \frac{1}{2\sqrt{ScK}} \right) - 2\sqrt{ScK} \right) + \frac{1}{2\sqrt{ScK}} \int \left( B(x) - 2\sqrt{ScK} \right) + e^{-\frac{Erfc \left[ \frac{1}{2\sqrt{ScK}} \right]}{2\sqrt{t}}} \left( B(x) - 2\sqrt{ScK} \right) + \frac{1}{2\sqrt{ScK}} \int \left( B(x) - 2\sqrt{ScK} \right)
\]

The numerical values of velocity and skin friction are computed for different parameters. The values of the main parameters considered are:
\[ Gr = 10, 20, 30; M = 1, 3, 5; m = 1, 2, 3, K = 0.1, 0.2, 0.3; Pr = 2, 3, 4; Sc = 2, 3, 7; \quad \text{and } \alpha \theta = 30^\circ, 45^\circ, 90^\circ; \quad K_0 = 1, 10, 20; \quad \text{and } Gm = 10, 20, 30; \quad t = 0.1, 0.2, 0.3.
\]

Figures 2, 3, 4, 9 and 10 show that primary velocity increases when \( Gm, Gr, m, K \) and \( t \) are increased. In this case primary velocity increases with \( m \), it means Hall current has increasing effect on the flow of the fluid along the plate. Figures 5, 6, 7, 8, and 11 show that primary velocity decreases when \( M, Pr, Sc, K_0 \) and \( \alpha \theta \) are increased. And figures 12, 13, 15, 19 and 20 show that the secondary velocity increases when \( Gm, Gr, M, K \) and \( t \) are increased.
18 and 21 show that secondary velocity decreases when $m$, $Pr$, $Sc$, $K_0$ and $\omega t$ are increased.

Here secondary velocity decreases when $m$ is increased; this implies that the Hall parameter slows down the transverse velocity. Figures 22 and 23 show that concentration decreases with increase in $Sc$ and $K_0$, and figure 24 shows that it increases with increase in $t$. From table – 2 it is observed that $\tau_z$ decreases with increase in $Pr$, $M$, $Sc$, $K_0$, and it increases with increase in $m$, $Gm$, $K$, $Gr$, and $t$. $\tau_z$ increases with increase in $Gr$, $t$, $Gm$, $K$, and $M$, and it decreases when $Sc$, $Pr$, $K_0$, $\omega t$ and $m$ are increased. From table – 1 it is observed that Sherwood number decreases with increase in $Sc$, $K_0$, and $t$. The results of the model can suitably be applied in the industries and organizations dealing with eclectically conducting fluid in the presence of magnetic field.
Figure 10. Velocity $u$ for different values of $t$.

Figure 11. Velocity $u$ for different values of $\omega t$.

Figure 12. Velocity $w$ for different values of $Gm$.

Figure 13. Velocity $w$ for different values of $Gr$.

Figure 14. Velocity $w$ for different values of $m$.

Figure 15. Velocity $w$ for different values of $M$.

Figure 16. Velocity $w$ for different values of $Pr$.

Figure 17. Velocity $w$ for different values of $Sc$.

Figure 18. Velocity $w$ for different values of $K_0$. 
Conclusions

The conclusions of the study are as under:

- Primary velocity increases with the increase in $Gr$, $m$, $K$, $Gm$ and $t$.
- Primary velocity decreases with the increase in $Pr$, $M$, $Sc$, $K_0$ and $\omega t$.
- Secondary velocity increases with increase in $Gr$, $Gm$, $K$, $t$ and $M$.
- Secondary velocity decreases with the increase in $m$, $Pr$, $Sc$, $K_0$ and $\omega t$.
- Skin fraction $\tau_x$ decreases with increase in $Sc$, $Pr$, $M$, $K_0$.
- It increases with $Gr$, $Gm$, $m$, $K$, $t$, and $\omega t$. $\tau_z$ increases with increase $Gr$, $Gm$, $K$, $t$, and $M$. And it decreases with $Pr$, $m$, $Sc$, $K$, and $\omega t$.
- Sherwood number decreases with increase in $Sc$ and $K$ and it increases with increase in $t$.

Reference

(1) Ibrahim S. M, Suneetha K., Heat source and chemical effects on MHD convection flow embedded in a porous medium with Soret, viscous and Joules dissipation, Ain Shams Engineering Journal (Elsevier), 2016, pp. 001–008.


Appendix

\[ P_1 = e^{-y\sqrt{a+ia}} + e^{y\sqrt{a-ia}} \]
\[ P_2 = e^{-y\sqrt{a+ia-2t\rho}} + e^{y\sqrt{a+ia+2t\rho}} \]
\[ P_3 = e^{-y\sqrt{a+ia}} Erf\left(\frac{y+2t\sqrt{a-ia}}{2\sqrt{t}}\right) \]

\[ P_5 = e^{-y\sqrt{a-2t\rho}} Erf\left(\frac{y-2t\sqrt{a+ia}}{2\sqrt{t}}\right) \]
\[ P_7 = e^{y\sqrt{a+2t\rho}} Erf\left(\frac{y+2t\sqrt{a+ia}}{2\sqrt{t}}\right) \]

\[ A_{01} = 1 + e^{2t\rho} A_{12}, \quad A_{02} = 1 + e^{2t\rho} A_{12}, \quad A_{i1} = Erf\left(\frac{y-2t\sqrt{a+ia}}{2\sqrt{t}}\right) \]
\[ A_{i2} = \frac{a(t+1)}{2}\sqrt{y-x} \]
\[ A_{i3} = \frac{a(t+1)}{2}\sqrt{y-x} \]
\[ A_{i4} = \frac{a(t+1)}{2}\sqrt{y-x} \]
\[ A_{i5} = \frac{a(t+1)}{2}\sqrt{y-x} \]
\[ A_{i6} = \frac{a(t+1)}{2}\sqrt{y-x} \]
\[ A_{i7} = \frac{a(t+1)}{2}\sqrt{y-x} \]
\[ A_{i8} = \frac{a(t+1)}{2}\sqrt{y-x} \]

Table 2. Skin friction for different Parameters.

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<th>M</th>
<th>Gr</th>
<th>Pr</th>
<th>K</th>
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<th>(t)</th>
<th>(Sh)</th>
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Table 1. Sherwood number.

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