Soft Rough Sets in approximation spaces

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**Abstract**

A rough set, first described by Zdzisław I. Pawlak, is a formal approximation of a crisp set in terms of a pair of sets which give the lower and the upper approximation of the original set. Soft set theory is a generalization of fuzzy set theory, was proposed by Molodtsova to deal with uncertainties. In this paper, we present concepts of soft rough sets and neutrosophic soft rough sets, and investigate some of its properties. Furthermore, we develop a decision making approach to neutrosophic Soft Rough Sets and a numerical example is illustrated.

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**Keywords**

Rough sets, Neutrosophic sets, Relations.

**Introduction**

The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. There are many approaches to the problem of how to understand and manipulate imperfect knowledge. The most successful approach is based on the fuzzy set notion proposed by L. Zadeh [12]. Rough set theory proposed by Z. Pawlak in [10] presents still another attempt to this problem. Rough sets have been proposed for a very wide variety of applications. In particular, the rough set approach seems to be important for Artificial Intelligence and cognitive sciences, especially in machine learning, knowledge discovery, data mining, expert systems, approximate reasoning and pattern recognition. Inspite, all these theories have their inherent difficulties as pointed out by Molodtsov [6].

To overcome these difficulties, in 1999 Molodtsov introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties. This so-called soft set theory seems to be free from the difficulties affecting the existing methods. The study of hybrid models combining soft sets with other mathematical structures is emerging as an active research topic of soft set theory [2, 4, 5, 8]. But these sets fail when we talk about indeterminate state which exits in the belief system.

One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache [11]. In order to give a new approach to decision making problems, we combine a fuzzy soft relation with neutrosophic rough sets and propose the concept of neutrosophic soft rough sets. Then we can define the upper and lower approximations of any neutrosophic set on parameter set E. Like the traditional neutrosophic rough set models, neutrosophic soft rough sets can also be exploited to extend many practical applications in reality. Therefore, we propose a novel approach to decision making based on neutrosophic soft rough set theory.

**Preliminaries**

**Definition 2.1**: [11]

A neutrosophic set A on the universe of discourse X is defined as

\[ A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X \text{ where } T_A, I_A, F_A : X \rightarrow [0, 1] \text{ and } 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3. \]

**Definition 2.2**: [1]

Let U be the initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a non-empty set A, A ⊆ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U)

**Definition 2.3**: [5]. Let (F, E) be a soft set over U. Then a subset of U × E called a crisp soft relation from U to E is uniquely defined by

\[ T_A(U, x) = \{1, (u, x) \in E \} \text{ and } 0 \leq T_A(U, x) \leq 3. \]

**Definition 2.4**: [4]. Let U be an initial universe set and let E be a universe set of parameters. A pair (F, E) is called a fuzzy soft set over U if F: E → P(U), where F(U) is the set of all fuzzy subsets of U.

**Definition 2.5**: [1] Let U be a nonempty and finite universe of discourse and R ⊆ U × U an arbitrary crisp relation on U. We define a set-valued function R: U → (U) by

\[ R(x) = \{ y \in U \mid (x, y) \in R \}, x \in U. \]

The pair (U, R) is called a crisp approximation space. For any A ⊆ U, the upper and lower approximations of A with respect to (U, R), denoted by \( \bar{R}(A) \) and \( R(A) \), are defined, respectively, as follows:

\[ \bar{R}(A) = \{ x \in U \mid R(x) \cap A \neq \emptyset \} \]

\[ R(A) = \{ x \in U \mid R(x) \subseteq A \} \]

The pair \((A, R(A))\) is referred to as a crisp rough set, and \(R, R : P(U) \rightarrow P(U)\) are, respectively, referred to as upper and lower crisp approximation operators induced from \((U, R)\).

**Example 2.6**. Let U be a universal set, which is denoted by

\[ U = \{ u_1, u_2, u_3, u_4, u_5 \}. \]

Let E be a set of parameters, where \( E = \{ e_1, e_2, e_3, e_4 \} \). Suppose that a soft set over U is defined as Follows

\[ (e_1) = \{ u_1, u_3, u_4 \}, F(e_2) = \{ u_2, u_4 \}, F(e_3) = \emptyset, F(e_4) = U. \]
Then the crisp soft relation on \( U \times E \) is written by
\[
R = \{(u_1, e_1), (u_2, e_1), (u_4, e_1), (u_5, e_2), (u_1, e_2), (u_2, e_2), (u_3, e_2), (u_4, e_3), (u_5, e_3), (u_1, e_3), (u_2, e_3), (u_3, e_3), (u_4, e_3), (u_5, e_3)\}
\]
\( \cup \), \( \cap \), \( \neg \), \( \sim \), \( \subseteq \), \( \supseteq \), \( \cup \) and \( \cap \) is referred as a soft rough neutrosophic set \( \bar{R} \) is given by
\[
\bar{R} = \{\langle u_1, 0, 0.5, 0.5, 0.5 \rangle, \langle u_2, 0.5, 0.4, 0.4 \rangle, \langle u_3, 0.3, 0.4, 0.4 \rangle, \langle u_4, 0.4, 0.4, 0.4 \rangle, \langle u_5, 0.4, 0.4, 0.4 \rangle\}
\]

**Theorem 3.4.** Let \( (U, E) \) be a crisp soft approximation space. Then the upper and lower soft rough neutrosophic approximation operators \( \bar{R} \) and \( \bar{R} \) satisfies the following properties.
\[
\begin{align*}
\text{(SRN1)} & \quad \bar{R}(A) = \bar{R}(A) \sim 1, \\
\text{(SRN2)} & \quad \bar{R}(A \cap B) = \bar{R}(A) \cap \bar{R}(B), \\
\text{(SRN3)} & \quad A \subseteq B \Rightarrow \bar{R}(A) \subseteq \bar{R}(B), \\
\text{(SRN4)} & \quad \bar{R}(A \cup B) \supseteq \bar{R}(A) \cup \bar{R}(B), \\
\text{(SRN5)} & \quad \bar{R}(A) = \bar{R}(A) \sim 1.
\end{align*}
\]

Proof. We only prove properties of the lower soft rough neutrosophic approximation operator \( \bar{R} \). The upper soft rough neutrosophic approximation operator \( \bar{R} \) can be proved similarly.

Then we have
\[
\bar{R}(A) = \{u, \bar{R}(A) (u), 1 - \bar{R}(A) (u), \bar{R}(A) (u) \} \in A \cup E\}
\]
\[
\bar{R}(A \cap B) = \{u, \bar{R}(A \cap B) (u), \bar{R}(A \cap B) (u), \bar{R}(A \cap B) (u) \} \in (A \cap B)
\]
\[
\bar{R}(A \cup B) = \{u, \bar{R}(A \cup B) (u), \bar{R}(A \cup B) (u), \bar{R}(A \cup B) (u) \} \in (A \cup B)
\]
\[
\bar{R}(A) = \{u, \bar{R}(A) (u), 1 - \bar{R}(A) (u), \bar{R}(A) (u) \} \in A
\]

**Remark.3.5.** The properties (1) and (5) show that \( \bar{R} \) and \( \bar{R} \) are dual to each other.
Remark 4.2. Let \((U, E, R)\) be a fuzzy soft approximation space. If \(A \subseteq E\), then neutrosophic soft rough approximation operators \(\overline{R}(A)\) and \(\overline{\overline{R}}(A)\) degenerate to the following forms:

\[
\overline{R}(A) = \{u \mid T_{\overline{R}(A)}(u) \subseteq A\} \\
\overline{\overline{R}}(A) = \{u \mid T_{\overline{\overline{R}}(A)}(u) \subseteq A\}
\]

In this case, neutrosophic soft rough approximation operators \(\overline{R}(A)\) and \(\overline{\overline{R}}(A)\) are identical with the soft fuzzy rough approximation operators. That is, neutrosophic soft rough approximation operators are an extension of the soft fuzzy rough approximation operators.

Example 4.3: Suppose that \(U = \{u_1, u_2, u_3, u_4, u_5\}\) is the set of five SMART PHONES under consideration of a decision maker to purchase. Let \(E = \{e_1, e_2, e_3, e_4\}\) = \{operating system, apps, web access, messaging\}. Mr. X wants to purchase a phone which qualifies with the parameters of \(e\). Assume that Mr. X describes the phones by constructing a fuzzy soft set \((F, E)\) which is a fuzzy soft relation \(R\) from \(U\) to \(E\). And it is presented by a table as in the following form:

<table>
<thead>
<tr>
<th>(u_i)</th>
<th>(e_1)</th>
<th>(e_2)</th>
<th>(e_3)</th>
<th>(e_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_1)</td>
<td>0.70.80.0.28</td>
<td>0.90.80.0.1</td>
<td>0.70.80.0.3</td>
<td>0.60.70.0.2</td>
</tr>
<tr>
<td>(u_2)</td>
<td>0.80.50.0.7</td>
<td>0.60.60.0.2</td>
<td>0.50.40.0.7</td>
<td>0.50.60.0.7</td>
</tr>
<tr>
<td>(u_3)</td>
<td>0.50.60.0.6</td>
<td>0.50.40.0.6</td>
<td>0.60.50.0.5</td>
<td>0.40.50.0.3</td>
</tr>
<tr>
<td>(u_4)</td>
<td>0.90.30.0.3</td>
<td>0.40.20.0.7</td>
<td>0.40.50.0.4</td>
<td>0.30.40.0.6</td>
</tr>
<tr>
<td>(u_5)</td>
<td>0.50.70.0.1</td>
<td>0.30.10.0.7</td>
<td>0.50.60.0.7</td>
<td>0.70.60.0.2</td>
</tr>
</tbody>
</table>

Now suppose that Mr. X gives the optimum normal decision object \(A\) which is an neutrosophic subset defined as follows:

\(A = \{(e_1, 0.7, 0.7, 0.1), (e_2, 0.7, 0.6, 0.5), (e_3, 0.8, 0.9, 0.1), (e_4, 0.7, 0.5, 0.4)\}\)

The Truth, Indeterminacy and False values are given as

\[
\begin{align*}
T_{\overline{R}(A)}(u_1) &= 0.7, \\
T_{\overline{R}(A)}(u_2) &= 0.7, \\
T_{\overline{R}(A)}(u_3) &= 0.6, \\
T_{\overline{R}(A)}(u_4) &= 0.7, \\
T_{\overline{R}(A)}(u_5) &= 0.7
\end{align*}
\]

Theorem 4.4. Let \((U, E, R)\) be a fuzzy soft approximation space. Then the upper and lower neutrosophic soft rough approximation operators \(\overline{R}(A)\) and \(\overline{\overline{R}}(A)\) satisfy the following properties, \(\forall A, B \subseteq E\):

\(\overline{R}(A) \cap \overline{\overline{R}}(B) = \overline{R}(A \cap B)\)

\((\overline{R}(A)) = \overline{\overline{R}}(A)\)

\(\overline{R}(A) \cup \overline{\overline{R}}(B) = \overline{R}(A \cup B)\)

\(\overline{R}(A) \subseteq \overline{\overline{R}}(A)\)

\(\overline{R}(A) = \overline{\overline{R}}(A)\)
Let \( U, E, R \) be a fuzzy soft approximation space, where \( U \) is the universe of the discourse, \( E \) is the parameter set, and \( R \) is a fuzzy soft relation on \( x E \). Then we can give an algorithm based on IF soft rough sets with five steps.

First, according to their own needs, the decision makers can construct a fuzzy soft relation \( R \) from \( U \) to \( E \), or fuzzy soft set \( (F, E) \) over \( U \).

Second, for a certain decision evaluation problem, each person has various opinions on the attributes of the same parameter.

We can compute then neutrosophic soft rough approximation operators \( \tilde{R}(A) \) and \( R(A) \) of the optimum normal decision object \( A \). Thus, we obtain two most close values \( \tilde{R}(A) \) and \( R(A) \) to the decision alternative \( u_j \) of the universe set \( U \).

Fourth two operations on two neutrosophic sets, shown as follows, for all \( A, B \in \mathbb{N}(U) \).

(i) Union operation:

\[
\tilde{A} \cup \tilde{B} = \{x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \}
\]

(ii) Intersection operation:

\[
A \cap B = \{x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \}
\]

(iii) Ring sum operation:

\[
A \oplus B = \{x, T_A(x) + T_B(x) - I_A(x) - I_B(x), F_A(x) + F_B(x) \}
\]

(iv) Ring product operation:

\[
A \otimes B = \{x, T_A(x) T_B(x), I_A(x)I_B(x), F_A(x)F_B(x) \}
\]

We know that (NSR2) holds. Similarly, we can verify that (NSR6) also holds.

In Example 4.4, we can note that \( A \subseteq (A) \). But if \( R \) is referred to as a serial fuzzy soft relation from \( U \) to parameter set \( E \), that is, for each \( u \in U \), there exists \( e \in E \) such that \( R(u, x) = 1 \), we have \( R(A) \subseteq R(A) \).

Theorem 4.6. Let \( U, E, R \) be a fuzzy soft approximation space. If \( R \) is serial, then the upper and lower IF soft rough approximation operators \( \tilde{R}(A) \) and \( R(A) \) satisfy the following properties:

(1) \( \tilde{R}(\Phi) = \Phi \neq \tilde{R}(E) = U \).

(2) \( \tilde{R}(A) \subseteq \tilde{R}(A), \forall A \in \mathbb{N}(E) \).

Proof. It is straightforward.

Application Of Neutrosophic Soft Rough Sets In Decision Making

In the above-mentioned sections, to demonstrate the validity of that new models properties, several examples are carried out. For example, by data validation all upper and lower approximation operators \( \tilde{R} \) and \( R \) in the above examples are dual to each other. By those examples, the models are further understood, laying a good foundation for further study and application.

In this section, we present an approach to the decision making based on neutrosophic soft rough sets.

References


