Optimum Bit Error Rate for Linear Multiuser Detection in Wireless Communications
Ch.PapaRao, P.Sumithabhashini, P.Chandra Sekhar Reddy and Jagadeesh Thati
Department of ECE Tirumala Engineering College, NRT, Andhra Pradesh, India.

ARTICLE INFO
Article history:
Received: 6 May 2016;
Received in revised form: 25 June 2016;
Accepted: 30 June 2016;

Keywords
DS-CDMA, 
Multiple Access Interference, 
BER performance.

ABSTRACT
Direct sequence code division multiple access (DS-CDMA) is a popular wireless technology. This system suffers from Multiple Access Interference (MAI) caused by Direct Sequence users and Near –far effect. Multi-User Detection schemes are used to detect the users’ data in presence of MAI and Near- far problem. In this dissertation, we present comparative study between linear multiuser detectors and conventional single user matched filter in DS-CDMA system. Analysis and simulations are conducted in synchronous AWGN channel, and Gold sequence and Kasami sequence are used as the spreading codes. Simulation results depict the performance of three detectors, conventional detector, Decorrelating detector and MMSE (Minimum Mean Square Error) detector. It shows that the performance of these detectors depends on the length of PN code used and Number of users. Linear multiuser detectors perform better than the conventional matched filter in terms of BER performance. All the simulations have been performed on MATLAB 7.0.

© 2016 Elixir All rights reserved.

I. Introduction
The communication system has challenge of accommodating many users in a small area. The wireless domain is the current area of interest. The conventional systems used either frequency spectrum sharing or timesharing and hence there was the limitation on the capacity. With the advent of spread spectrum and hence CDMA, fixed bandwidth was used to accommodate many users by making use of certain coding properties over the bandwidth. But this system suffers from MAI (Multiple Access Interference) caused by direct sequence users. Multiuser Detection Technique is going to be the key to this problem. These detection schemes were introduced to detect the users’ data in the presence of Multiple Access Interference (MAI), Inter Symbol Interference and noise. Spread spectrum CDMA systems (DS/CDMA) are becoming widely accepted and promise to play a key role in the future of wireless communications applications because of their efficient use of the channel and there allowance for nonscheduled user transmissions. Hence recent interests are in techniques, which can improve the capacity of CDMA systems.

The focus of most current research is on Wideband CDMA (W-CDMA) or NG (next generation) CDMA. In W-CDMA, the multimedia wireless network will become feasible. Not only voice, but also images, video and data can be transmitted by mobile phones or other portable devices. Achieving a higher data rate and higher capacity are two major goals for W-CDMA, which makes the multiuser interference problem more and more crucial. As Mobile communication systems based on CDMA are inherently subject to Multiple-Access Interference (MAI), since it is impossible to maintain orthogonal spreading codes in mobile environments. MAI (Multiple-Access Interference) limits the capacity of Conventional detectors and brings on strict power control requirements to alleviate the Near-Far problem.

Direct sequence code division multiple access (DS-CDMA) system is well known wireless technology. In DS-CDMA system, all of the users signals overlap in time and frequency cause mutual interference. This system suffers from Multiple Access Interference (MAI) caused by Direct Sequence users and near –far effect. The general structure of these detectors consists of a bank of matched filters. The detection is done on the basis of a filter matched to the pseudo-random sequence of the user. We refer to this detector as the conventional matched filter detector. Since the conventional matched filter was designed for orthogonal signature waveforms, it suffers from many drawbacks due to the MAI term which it does not take into account. Multi-user Detector (MUD) techniques exploit the character of the MAI by removal of the Multi-User Interference from each user’s received signal before making data decision, and thus offer significant gains in capacity and Near-Far Resistance over the conventional receiver. Verdu’s work shows that optimum Maximum-Likelihood Sequence Detector can completely eliminate MAI, thus greatly increase CDMA system capacity. However, the complexity of the Optimum detector is exponential in number of users, which is too complicated for practical implementation. There have been great interests in finding sub optimum detectors with acceptable complexity and marginal performance degradation compared with the optimum detector. Sub optimum detectors can be classified into two linear multi-user detectors and subtractive interference canceller. Two of the most cited linear multi-user detectors are Decorrelating detector and MMSE detector. This work presents comparative study between linear multiuser detectors, and conventional single user matched filter in DS-CDMA system. Analysis and simulations are conducted in synchronous AWGN channel, and Gold sequence and kasami sequence are used as the spreading codes.

Teile:
E-mail address: paparao.challagundla@gmail.com
© 2016 Elixir All rights reserved
Multiuser detectors derivation is presented for synchronous DS-CDMA systems. The synchronous assumption considerably simplifies exposition and analysis and often permits the derivation of closed form expressions for desired performance measures. These are useful since similar trends are found in the analysis of the more complex asynchronous case. Furthermore, every asynchronous system is viewed as equivalent synchronous system with larger effective user population.

The probability of error of bit error rate (BER), as a function of the signal-to–Noise Ratio (SNR), is a common and essential figure of merit for a communication system, indicating the feasibility of reliable data transfer across the channel. The BER can be used as a metric to compare different communication systems. Throughout this work we utilize the BER as a measure of performance for the multiuser detection schemes.

II. Literature Survey

Multiple access Techniques

An important issue in wireless communication systems is multiple random access: communication links can be activated at any moment while several links can be active simultaneously. As multi-access and random-access are properties mainly determined by the chosen data communication technique it is important to keep these requirements in mind from the very beginning. Three possible concepts to realize multi-access communication systems are in use.

FDMA

Frequency Division Multiple Access commonly used in conventional telephone systems: every user gets a certain frequency band assigned and can use this part of the spectrum to perform its communication. If only a small number of users are active, not the whole resource (frequency spectrum) is used. Assignment of the channels can be done centrally or by carrier sensing in a mobile. The later possibility enables random access.

TDMA

Time Division Multiple Access applied nowadays in mobile phone systems: every user is assigned a set of time slots. Transmission of data is only possible during this time slot, after that the transmitter has to wait until it gets another time slot. Synchronization of all users is an important issue in this concept. Consequently, there must be a central unit (base station) that controls the synchronization and the assignment of time slots. This means that this technique is difficult to apply in random access systems.

CDMA

Code Division Multiple Access (spread spectrum): a unique code is assigned to each user. This code is used to ‘code’ the data message. As codes are selected for the cross correlation properties, all users can transmit simultaneously in the same frequency channel while a receiver is still capable of recovering the desired signal. Synchronization between links is not strictly and so random access is possible. A practical application at the moment is the cellular CDMA phone system.

Spread Spectrum Modulation

Spread spectrum techniques originated in answer to the needs of military communications. They are based on signaling schemes which greatly expand the transmitted spectrum relative to the data rate.

A transmission technique in which a pseudorandom code, independent of the data, is employed as a modulation wave form to spread the signal energy over a band width much greater than the information signal band width is called SSM. This group of modulation techniques is characterized by its wide frequency spectra. The modulated output signals occupy a much greater band width than the signals base band information band width. To qualify has a spread spectrum signal, two criteria should be met.

1. The transmitted signal band width is much greater than the information band width.
2. Some function other than the information being transmitted is employed to determine the resultant transmitted band width.

The processing gain is the ratio of the bandwidth of the spread signal to the bandwidth of information bits:

\[ PG = \frac{B_s}{B_i} \]

where \( B_s \) is the bandwidth of the spread signal and \( B_i \) is the bandwidth of the information bit.

Direct-Sequence Spread Spectrum (DS-SS)

The DS-SS technique is one of the most popular forms of spread spectrum. This is probably due to the simplicity with which direct sequencing can be implemented. Figure 3.2 shows the basic model and the key characteristics that make up the DS-SS communications system. In this form of modulation, a pseudo-random noise generator creates a spreading code or better known as the pseudo-noise (PN) code sequence. Each bit of the original input data is directly modulated with this PN sequence and is represented by multiple bits in the transmitted signal. On the receiving end, only the same PN sequence is capable of demodulating the spread spectrum signal to successfully recover the input data.

![Fig 1. Basic model of the direct-sequence spread spectrum communications system.](image)

The bandwidth of the transmitted signal is directly proportional to the number of bits used for the PN sequence. A 7-bit code sequence spreads the signal across a wider frequency band that is seven times greater than a 1-bit code sequence, otherwise termed as having a processing gain of seven. Figure 2 illustrates the generation of a DS-SS signal using an exclusive-OR (XOR) operation. The XOR obeys the following rules:

\[ 0 \oplus 0 = 0 \quad 0 \oplus 1 = 1 \quad 1 \oplus 0 = 1 \quad 1 \oplus 1 = 0 \]

![Fig 2. Generation of a DS-SS signal with processing gain = 7](image)
Note that an input data bit of zero causes the PN sequence coding bits to be transmitted without inversion, while an input data bit of one inverts the coding bits. Rather than to represent the binary data with bits 0’s and 1’s, the input data and PN sequence are converted into a bipolar waveform with amplitude values of ±1.

### III. Spreading Codes

The DS-CDMA system uses two general categories of spreading sequences: PN sequences and orthogonal codes

#### PN Sequence

The PN sequence is produced by the pseudo-random noise generator that is simply a binary linear feedback shift register, consisting of XOR gates and a shift register. This PN generator has the ability to generate an identical sequence for both the transmitter and the receiver, and yet retaining the desirable properties of a noise-like randomness bit sequence. A PN sequence has many characteristics such as having a nearly equal number of zeros and ones, very low correlation between shifted versions of the sequence and very low cross correlation with any other signals such as interference and noise. However, it is able to correlate very well with itself and its inverse. Another important aspect is the autocorrelation of the sequence as it decides the ability to synchronize and lock the spreading code to the received signal. This effectively combats the effects of multipath interference and improves the SNR. M-sequences, Gold codes and Kasami sequences are examples of this class of sequences.

#### Multiuser Direct Sequence Spread Spectrum Systems

Spread spectrum can also be used as a mechanism for many users to share the same spectrum. Using spreading code properties to support multiple users within the same spread bandwidth is also called spread-spectrum multiple access (SSMA), which is a special case of code-division multiple access (CDMA). In multiuser spread spectrum, each user is assigned a unique spreading code or hopping pattern, which is used to modulate their data signal. The transmitted signal for all users is superimposed in time and in frequency. The spreading codes or hopping patterns can be orthogonal, in which case users do not interfere with each other under ideal propagation conditions, or they can be non-orthogonal, in which case there is interference between users, but this interference is reduced by the spreading code properties. Thus, while spread spectrum for single-user systems is spectrally inefficient, as it uses more bandwidth than the minimum needed to convey the information signal, spread spectrum multiuser systems can support an equal or larger number of users in a given bandwidth than other forms of spectral sharing such as time-division or frequency-division. However, if the spreading mechanisms are non-orthogonal either by design or through channel distortion, users interferer with each other. If there is too much interference between users, the performance of all users degrades.

Performance of multiuser spread spectrum also depends on whether the multiuser system is a downlink channel (one transmitter to many receivers) or an uplink channel (many transmitters to one receiver). These channel models are illustrated in Fig 3 the downlink channel is also called a broadcast channel or forward link, and the uplink channel is also called a multiple access channel or reverse link. The performance differences of DSSS in uplink and downlink channels result from the fact that in the downlink, all transmitted signals are typically synchronous, since they originate from the same transmitter. Moreover, both the desired signal and interference signals pass through the same channel before reaching the desired receiver. In contrast, users in the uplink channel are typically asynchronous, since they originate from transmitters at different locations, and the transmitted signals of the users travel through different channels before reaching the receiver.

#### III. Spreading Codes

### For Multiuser DSSS

Multiuser DSSS is accomplished by assigning each user a unique spreading code sequence $S_i(t)$. The autocorrelation function of the spreading code determines its multipath rejection properties. The cross-correlation properties of different spreading codes determine the amount of interference between users modulated with these codes. For asynchronous users, their signals arrive at the receiver with arbitrary relative delay, and the cross-correlation between the codes assigned to user $i$ and user $j$ over one symbol time with this delay is given by

$$
\rho_{ij}(\tau) = \frac{1}{T} \int_{0}^{T} S_i(t) S_j(t - \tau) dt = \frac{1}{N} \sum_{n=1}^{N} S_i(nT_c) S_j(nT_c - \tau)
$$

For synchronous users, their signals arrive at the receiver aligned in time, so $\tau = 0$ and the cross-correlation becomes

$$
\rho_{jj}(0) = \frac{1}{T} \int_{0}^{T} S_i(t) S_i(t) dt = \frac{1}{N} \sum_{n=1}^{N} S_i(nT_c) S_i(nT_c)
$$

Ideally, since interference between users is dictated by the cross-correlation of the spreading code, we would like $\rho_{ij}(\tau) = 0 \forall \tau$, $i \neq j$ for asynchronous users and $\rho_{jj}(0) = 0, i \neq j$ for synchronous users to eliminate interference between users. A set of spreading codes for asynchronous users with $\rho_{ij}(\tau) = 0 \forall \tau$, $i \neq j$ or for synchronous users with $\rho_{jj}(0) = 0, i \neq j$ is called an orthogonal code set. A set of spreading codes that does not satisfy this cross-correlation property is called a non-orthogonal code set. It is not possible to obtain orthogonal codes for asynchronous users, and for synchronous users there is only a finite number of spreading codes that are orthogonal within any given bandwidth. Thus, an orthogonality requirement restricts the number of different spreading codes (and the corresponding number of users) in a synchronous DSSS multiuser system. We now describe the most common chip sequences and their associated spreading codes that are used in multiuser DSSS systems.

#### Maximal Length Sequences

Maximal length sequences or m-sequences are the most widely recognized and used pseudo noise (PN) sequences; they can be generated by two methods by using a linear feedback shift register (LFSR). The first using simple LFSR and the other use modular LFSR. Each of the LFSR, either simple or modular, can be represented by means of a polynomial. A sequence, generated by an LFSR with m registers, is said to be a maximal length sequence or an m-sequence if its length is $L = 2^m - 1$. An m-sequence is generated when the LFSR structure represents a primitive polynomial. The length of the m-sequence is the possible...
number of states an LFSR can take, 10 except for an all zero state. For an LFSR, an m-sequence of length L provides the best autocorrelation properties, as follows:

$$R(n) = \begin{cases} L & n = 0, L \ 1 & \text{otherwise} \end{cases}$$

(3)

The constructions in Fig. 4(a) and Fig. 4(b) are equivalent: they generate the same m-sequence, represent the same polynomial $1 + x^2 + x^5$, and implement the same difference equation $x[i] = x[i-2] \oplus x[i-5]$.

![Fig 4(a). Simple LFSR. Simple linear feedback shift register realization of a polynomial given by the difference equation $x[i] = x[i-2] \oplus x[i-5]$.](image)

![Fig 4(b). Modular LFSR. Modular linear feedback shift register realization of a polynomial given by the difference equation $x[i] = x[i-2] \oplus x[i-5]$.](image)

**Gold Sequences**

The m-sequences have excellent autocorrelation properties but their cross-correlation properties do not follow any particular rules and typically exhibit undesirably high values. Furthermore, the number of m-sequences for a given number of registers in an LFSR is limited. Gold sequences address these problems, and are derived by combining the m-sequences from two LFSRs. In comparison to m-sequences, Gold sequences provide larger sets of sequences and exhibit better cross-correlation properties. Gold sequences are generated from two equal length m-sequences that form a so-called preferred pair. The cross-correlation of two m-sequences that form a preferred pair is tri-valued and it takes the values from the set $\{-1, 0, 1\}$, where $t(m) = 1 + 2 \frac{m^2}{2}$, and m is the number of binary shift registers in the LFSR. A requirement for the generation of Gold sequences is that $m$ should be equal to 2 Modulo 4.

The preferred sequences are chosen so that Gold codes have a three-valued cross-correlation with values:

$$\rho_{ij}(\tau) = \begin{cases} -1/N & \text{if } |t(i)/n| \leq 1/2 \\ s(n)/N & \text{if } |t(i)/n| > 1/2 \end{cases}$$

(4)

where $t(n) =$

$$\begin{cases} 2^{g_1(n)/2} + 1 & n \text{ odd} \\ 2^{g_0(n)/2} + 1 & n \text{ even} \end{cases}$$

The autocorrelation takes on the same three values. Gold codes take advantage of the fact that if two distinct m-sequences with time shifts $\tau_1$ and $\tau_2$ are modulo-2 added together, the resulting sequence is unique for every unique value of $\tau_1$ or $\tau_2$.

**Kasami Sequences**

Kasami sequences also address the two undesirable properties of the m-sequences: smaller sets of sequences and potentially higher cross-correlation values. Kasami sequences can be generated either as a small set or as a large set. The small set has better cross-correlation properties, while the large set provides more sequences to choose from. Generation of Kasami sequences involves a method similar to the one used to generate the Gold sequences, as given below. Small Set To generate the small set Kasami sequences, an m-sequence, denoted as $u$, is first generated. Let the length of the sequence $u$ be $L$, and $m$ be the number of binary registers in the LFSR, such that $L = 2^m - 1$. One period of the sequence $u$ is decimated by $2^{m/2} + 1$ to generate a sequence $w$ of length $(2^m - 1)(2^{m/2} + 1)$, the length of $w$ simplifies to $2^{m/2} - 1$. Next, $2^{m/2} + 1$ repetitions of $w$ are concatenated to form a sequence $v$ of length $L$. The small set Kasami sequences are then given by the set $\{u, u \oplus T^i(u)\}$, where $T^i(u)$ indicates the right (or left) shift of sequence $v$ by $i$ bits, and $i$ varies from 0 to $2^{m/2} - 1$. The preferred sequences are chosen so that Kasami codes have a three-valued cross-correlation with values:

$$\rho_{ij}(\tau) = \begin{cases} -1/N & \text{if } |t(i)/n| \leq 1/2 \\ -s(n)/N & \text{if } |t(i)/n| > 1/2 \end{cases}$$

(5)

Since $|s(n)| < |t(n)|$, Kasami codes have better autocorrelation and cross-correlation than Gold codes. The large set Kasami sequences are generated either from the Gold set or from the Gold-like set. Large set Kasami sequences include the Gold set and the small set Kasami sequences as the subsets.

**Walsh-Hadamard Codes**

Walsh-Hadamard codes of length $N = T_s/T_c$ that are synchronized in time are orthogonal over a symbol time, so that the cross-correlation of any two sequences is zero. Thus, synchronous users modulated with Walsh-Hadamard codes can be separated out at the receiver with no interference between them, as long as the channel does not corrupt the orthogonality of the codes (Delayed multipath components are not synchronous with the LOS paths, and thus the multipath components associated with different users will cause interference between users. The loss of orthogonality can be quantized by the orthogonality factor). While it is possible to synchronize users on the downlink, where all signals originate from the same transmitter, it is more challenging to synchronize users in the uplink, since they are not co-located. Hence, Walsh-Hadamard codes are rarely used for DS-SS uplink channels. Walsh-Hadamard sequences of length N are obtained from the rows of an $N \times N$ Hadamard matrix N. For $N=2$ the Hadamard matrix is

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Larger Hadamard matrices are obtained using 2 and the recursion

$$H_{2N} = \begin{bmatrix} H_N & H_{2N} \\ H_N & -H_{2N} \end{bmatrix}$$

(6)

Each row of $H_N$ specifies the chip sequence associated with a different sequence, so the number of spreading codes in a Walsh-Hadamard code is $N$. Thus, DS-SS with Walsh-Hadamard sequences can support at most $N = T_s/T_c$ users. Since DS-SS uses roughly $N$ times more bandwidth than required for the information signal, approximately the same number of users could be supported by dividing up the total system bandwidth into $N$ non-overlapping channels.
Similarly, the same number of users can be supported by dividing time up into $N$ orthogonal timeslots (time-division) where each user operates over the entire system bandwidth during his timeslot. Hence, any multiuser technique that assigns orthogonal channels to the users such that they do not interfere with each other accommodates approximately the same number of users.

The performance of a DSSS multiuser system depends both on the spreading code properties as well as the channel over which the system operates.

**IV. Synchronous DS-CDMA System Transmitter Model**

Consider a DS-CDMA communication system with $K$ users. Assuming Binary Phase Shift Keying (BPSK) signaling, at the transmitter, the signal for the $k$th user can be written as

\[ r(t) = \sum_{i=1}^{K} A_k b_k s_k(t - iT_b) + n(t) \quad i T_b \leq t < (i + 1)T_b \]  

where, $A_k$ is gain of the channel and $n(t)$ is the AWGN noise process with zero mean and variance $\sigma_n^2$. Assuming that the receiver is interested in the data of all users (e.g. in the case of uplink communication, this receiver can be the base station), the objective of the receiver is to estimate the vector $\mathbf{b}(i) = [b_1(i), \ldots, b_K(i)]$ of transmitted symbols for all time intervals $i$.

**V. Synchronous DS-CDMA System Receiver Model.**

Synchronous DS-CDMA system Receiver has a bank of $K$ matched filters. The received signal is the noisy sum of all users’ signals

\[ r(t) = \sum_{i=1}^{K} A_k b_k s_k(t - iT_b) + n(t) \quad i T_b \leq t < (i + 1)T_b \]  

The bank of matched filters consists of $K$ filters matched to the individual spreading codes. This detector is a matched filter to the desired signal. Other users’ signals are treated as noise (self noise). These self-noise limit the systems capacity and can jam out all communications in the presence of a strong near by signal (Near-Far Problem). The out of the $k$th user matched filter is

\[ y_j = \int_0^{T_b} r(t) s_j(t) \, dt \]  

The first term is desired information. The second term is interference from other users.

**VI. Multiuser Detection Receivers**

There are two types of receivers: 1. Optimal receivers 2. Suboptimal receivers

Optimal detector or maximum likelihood sequence estimation detector proposed by verdu this detector is too complex for practical DS-CDMA systems.
There are two categories of the most proposed detectors: linear multiuser detectors and non-linear detectors. In linear multiuser detection, a linear mapping (transformation) is applied to the Soft outputs of the conventional detector to produce a new set of outputs, which hopefully provide better performance. In non-linear detection, estimates of the interference are generated and subtracted out.

\[
\begin{align*}
\Omega(b) &= -2y^T Ab + y^T A R A b \\
\end{align*}
\]

Figure 8 shows the general structure of multiuser detection systems for detecting each K user’s transmitted symbols from the received signal, which consists of a matched filter bank that converts the received continuous-time signal to the discrete-time statistics sampled at chip rate without masking any transmitted information relevant to demodulation. This is followed by applying multiuser detection algorithm for optimality conditions to produce the soft output statistics. The soft outputs are passed to the single user decoders. With the statistic \( \{ y_1, \ldots, y_K \} \) at the output of the matched filter, an estimate for the transmitted bits \( \{ b_1, \ldots, b_k \} \) that minimizes the probability of error can be found.

**Maximum-Likelihood (ML) sequence detection**

The ML criterion is based on selecting the input bit that minimizes the Euclidean distance between the transmitted symbol (corresponding to the input bit) and the received symbol. In the case of multi-user detection, the Euclidean distance between a transmitted symbol vector corresponding to the input bit-vector \( b \) and the received symbol vector is given by

\[
d(b) = \sum_{n=1}^{N} \left[ y(n) - \sum_{k=1}^{M} A_k b_k s_k(n) \right]^2
\]

Expanding the above expression, we get:

\[
d(b) = \sum_{n=1}^{N} |y(n)|^2 - 2 \sum_{k=1}^{M} \sum_{n=1}^{N} A_k b_k s_k(n) + \sum_{n=1}^{N} \left( \sum_{k=1}^{M} A_k b_k s_k(n) \right)^2
\]

The first term in the expression is independent of \( b \) and so it can be removed from the minimization process (instead we define a likelihood function \( b \) that differs from \( d(b) \) by a constant). Using the definitions of \( y \) and using the definitions of \( A \) and \( b \), the above expression can be simplified as:

\[
\Omega(b) = -2y^T Ab + y^T A R A b
\]

Again, removing the common factor \( N \) and using the fact that maximizing the negative of a function is same as minimizing the function, the problem of optimal multiuser detection can be stated as:

\[
\text{Maximize} \quad \Omega(b) = 2b^T A y - b^T A R A b
\]

Subject to, \( b \in \{+1, -1\}^M \)

The maximization problem stated above is a combinatorial optimization problem, since the variables of the optimization problem are basically limited to a finite set. The straight-forward method for solving such combinatorial optimization problem is an exhaustive search over all the possibilities. In the above case, since \( b \in \{+1, -1\}^M \), there are \( 2^M \) possibilities. Thus the search space increases in a geometric fashion with the number of users. In other words, the complexity required for decoding \( M \) bits of data is \( O(2^M) \). It has been shown by Verdu that no other algorithm whose computational complexity is a polynomial in the number of users exists to solve this combinatorial optimization problem.

The problem with MLS approach is that here there are \( 2^R \) possible \( d \) vectors; an exhaustive search is clearly impractical for typical message sizes and numbers of users.

**Linear Multiuser Detectors**

These class of algorithms involve applying a linear transformation to the matched filter (single user detector) outputs. The output of the matched filter can be written in matrix form as

\[
\mathbf{Y}_{\mathbf{R}} = \mathbf{R} \mathbf{A} \mathbf{b} + \mathbf{n}
\]

**Decorrelating Detector**

The Decorrelating receiver applies the inverse of the correlation matrix to the output of the matched filter in order to decouple the data.

\[
\text{In the synchronous channel}
\]

Consider the output of the bank of \( K \) matched filters

\[
y = \mathbf{R} \mathbf{A} \mathbf{b} + \mathbf{n}
\]

where \( \mathbf{n} \) is a Gaussian random vector with zero mean and covariance matrix \( \sigma^2 \mathbf{R} \). If we process the output vector as

\[
\mathbf{R}^{-1} y = \mathbf{A} \mathbf{b} + \mathbf{R}^{-1} \mathbf{n}
\]

Clearly the \( k \)th component of vector \( \mathbf{R}^{-1} y \) is free from interference caused by any other users for any \( k \) (since \( \mathbf{A} \) is diagonal). Note that the cross correlation matrix \( \mathbf{R} \) is invertible if signature sequences are linear independent. If the background noise is vanishing, that is, \( \sigma = 0 \), then

\[
\mathbf{b}_k = \text{Sgn} (\mathbf{R}^{-1} y)_k = \text{Sgn} ((\mathbf{A} \mathbf{b})_k)
\]
Hence, in absence of background noise, we get error free performance. In the presence of the background noise, decision is affected only by the background noise, that is, 
\[ \tilde{y} = \text{sgn}(R^{-1}y) = \text{sgn}(\tilde{A}b + R^{-1}n_k) \] 
(24)

This is why the detector is called the decorrelating detector. Decorrelating detector can achieve any given performance level in the multiuser environment regardless of the multiuser interference, provided that the desired user is supplied enough power. Thus, it provides a substantial performance or capacity gains over the conventional detector under most conditions. The Decorrelating detector corresponds to the maximum likelihood sequence detector when the energies of all users are not known at the receiver. In other words, it yields the joint maximum likelihood sequence estimation of the transmitted bits and their received amplitudes.

For the case of 2 users, the correlation matrix is 

\[ R = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \]

\[ R^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix} \]

The output of decorrelator given by

\[ R^{-1}y = \begin{bmatrix} \tilde{A}b_1 \\ \tilde{A}b_2 \end{bmatrix} \begin{bmatrix} n_1 - \rho n_2 \\ n_2 - \rho n_1 \end{bmatrix} \]

This detector,
1. Completely eliminates the MAI, hence is near-far resistant
2. Does not require estimates of the channel parameters
3. Enhances the noise,(in two user case noise is enhanced by more significant disadvantage of this detector is that the computations needed to invert the matrix R are difficult to perform in real time. For synchronous systems, the problem is somewhat simplified. We can decorrelate one bit at a time. In other words we can apply the inverse of a K*K correlation Matrix Minimum Mean-Squared Error (MMSE) Detector

The MMSE implements the linear mapping which minimizes the mean-squared error between the actual data and the soft output of the conventional detector. At this stage, the MMSE detector applies a modified inverse of the correlation matrix to the matched filter bank outputs, and takes into account the background noise and utilizes knowledge of the received signal powers.

The amount of modification is directly proportional to the background noise; the higher the noise level, the less complete an inversion of R can be done without noise enhancement causing performance degradation. Thus, the MMSE detector balances the desire to decouple the users (and completely eliminate MAI) with the desire to not enhance the background noise. The algorithm presented in is summarized in Figure 11.

Algorithm

The MMSE detector implements a linear mapping L which minimizes the mean squared error \( E[(b_i - Ly_i)^2] \). The detection scheme can be written as

\[ \tilde{b} = \text{sgn}(Ly) \]

(23)

Fig 11. Minimum Mean-Squared Error (MMSE) detector.

Fig 12. Algorithm for the Minimum Mean-Squared Error (MMSE) detector.

The approach here is to turn linear multi-user detection problem into a linear estimation problem.

VII. Simulation Results

Detectors that are simulated include conventional single user matched filter (MF), Decorrelating and Minimum mean-squared error (MMSE). First of all, the BER performance comparison between the conventional detector and two suboptimal linear multiuser detectors is conducted. The performance evaluation with increasing number of active users is carried out. These simulations are done with the assumption that all active users have equal power. Simulations are carried out considering Conventional detector, Decorrelating detector and MMSE (Minimum Mean Square Error) detector. AWGN channel is considered and there is perfect power control. To simplify the discussion, we make assumptions that all carrier phases are equal to zero. This enables us to use baseband notation while working only with real signals. We also assume that each transmitted signal arrives at the receiver over a single path.

<table>
<thead>
<tr>
<th>SNR</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional(BER)</td>
<td>0.2259</td>
<td>0.2089</td>
<td>0.191</td>
<td>0.17</td>
<td>0.176</td>
<td>0.165</td>
<td>0.17</td>
<td>0.16</td>
</tr>
<tr>
<td>decorrelator</td>
<td>0.0744</td>
<td>0.0559</td>
<td>0.031</td>
<td>0.01</td>
<td>0.008</td>
<td>0.003</td>
<td>0.001</td>
<td>0.0004</td>
</tr>
<tr>
<td>MMSE</td>
<td>0.0737</td>
<td>0.0546</td>
<td>0.031</td>
<td>0.017</td>
<td>0.008</td>
<td>0.003</td>
<td>0.001</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
Performance Analysis

Case 1: Gold sequence of length 31 and 2 users
Two users synchronously transmitting the 5000 bits through an AWGN channel. For spreading gold sequence of length \( L_c = 31 \) is used. SNR is varying from 1 dB to 8 dB. Here \( K \) = Number of users and \( L_c \) = PN sequence length

The number of user is 2 the three detectors performance is almost similar. if number of user are increasing then the effect of MAI also increase that influence the detection of data.

Fig 11.1 BER performance of the three detectors for \( K = 2 \), \( L_c = 31 \)

Case 2: Gold sequence of length 31 and 4 users
Four users synchronously transmitting the 5000 bits through a AWGN channel. For spreading gold sequence of length \( L_c = 31 \) is used. SNR is varying from 1 dB to 8 dB

Fig 11.2 BER performance of the three detectors for \( K = 4 \), \( L_c = 31 \)

Table 1: SNR VS BER for \( K = 4, L_c = 31 \)

Case 3: Gold sequence of length 31 and 8 users.
Eight users synchronously transmitting the 5000 bits through an AWGN channel. For spreading gold sequence of length \( L_c = 31 \) is used. SNR is varying from 1 dB to 8 dB

Fig 11.3. BER performance of the three detectors for \( K = 8 \), \( L_c = 31 \).

The Linear multiuser detectors has less bit error rate (\( \sim 10^{-5} \)) compare to the conventional detector (\( 10^{-1} \)). Linear multiuser detectors are outperforming the Conventional detectors.

Fig 11.4. BER performance of the three detectors for \( K = 8 \), \( L_c = 63 \)

The performance comparison of the three detection schemes can be done by varying the length of the Gold code used (63). The increase in the length of the Gold codes leads to a significant rise of the non-orthogonality of the signature sequences. This leads to a considerable degradation in the system performance shown in above figure 11.4.

Case 4: Gold sequence of length 63 and 8 users
Eight users synchronously transmitting the 5000 bits through an AWGN channel. For spreading gold sequence of length \( L_c = 63 \) is used. SNR is varying from 1 dB to 8 dB.

Fig 11.5. BER performance of the three detectors for \( K = 8 \), \( L_c = 63 \) with near-far effect

An important disadvantage of this detector is that, unlike the Decorrelating detector, it requires estimation of the received amplitudes. Another disadvantage is that its performance depends on the powers of the interfering users. Therefore, there is some loss of resistance to the near-far problem shown in above fig as compared to the Decorrelating detector.

Case 5: Performance comparison with near-far effect
Eight users synchronously transmitting the 5000 bits through a AWGN channel. For spreading gold sequence of length \( L_c = 31 \) is used. SNR is varying from 1 dB to 8 dB. The signal strength is different for different users.

Fig 11.6. BER performance of the three detectors for \( K = 4 \), \( L_c = 31 \)

Eight users synchronously transmitting the 5000 bits through a AWGN channel. For spreading gold sequence of length \( L_c = 63 \) is used. SNR is varying from 1 dB to 8 dB.

Table 4: SNR VS BER for \( K = 8, L_c = 31 \)

<table>
<thead>
<tr>
<th>SNR</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional(BER)</td>
<td>0.304</td>
<td>0.262</td>
<td>0.22</td>
<td>0.203</td>
<td>0.015</td>
<td>0.170</td>
<td>0.168</td>
<td>0.164</td>
</tr>
<tr>
<td>decorrelator</td>
<td>0.1529</td>
<td>0.1055</td>
<td>0.063</td>
<td>0.037</td>
<td>0.017</td>
<td>0.007</td>
<td>0.002</td>
<td>0.000</td>
</tr>
<tr>
<td>MMSE</td>
<td>0.1487</td>
<td>0.1020</td>
<td>0.057</td>
<td>0.034</td>
<td>0.015</td>
<td>0.006</td>
<td>0.002</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Case 7. Comparison of multi user detection of DS CDMA system using orthogonal codes.

![Figure 11.7](image)

Fig 11.7. BER performance of the three detectors for Walsh code $L_c=16$.

From Fig 11.7 we can say that if we use orthogonal spreading codes, the effect of MAI is zero. So the detectors are having the same performance.

Using kasami sequence

Case 8: Two users synchronously transmitting the 5000 bits through an AWGN channel. For spreading kasami sequence of length $L_c=63$ is used. SNR is varying from 1 dB to 8 dB.

![Figure 11.8](image)

Fig 11.8. BER performance of the three detectors for kasami code $L_c=63$, $K=2$.

Case 9: Four users synchronously transmitting the 5000 bits through an AWGN channel. For spreading kasami sequence of length $L_c=63$ is used. SNR is varying from 1 dB to 8 dB.

![Figure 11.9](image)

Fig 11.9. BER performance of the three detectors for kasami code $L_c=63$, $K=4$.

Case 10: Eight users synchronously transmitting the 5000 bits through an AWGN channel. For spreading kasami sequence of length $L_c=63$ is used. SNR is varying from 1 dB to 8 dB.

![Figure 11.10](image)

Fig 11.10. BER performance of the three detectors for kasami code $L_c=63$, $K=8$.

Case 11: Two users synchronously transmitting the 5000 bits through an AWGN channel. For spreading kasami sequence of length $L_c=255$ is used. SNR is varying from 1 dB to 8 dB.

![Figure 11.11](image)

Fig 11.11. BER performance of the three detectors for kasami code $L_c=255$, $K=2$.

Case 12: Four users synchronously transmitting the 5000 bits through an AWGN channel. For spreading kasami sequence of length $L_c=255$ is used. SNR is varying from 1 dB to 8 dB.

![Figure 11.12](image)

Fig 11.12. BER performance of the three detectors for kasami code $L_c=255$, $K=8$.

Case 13: BER performances of the detectors for increasing number of active users

![Figure 11.13](image)

Fig 11.13. BER performances of the detectors for increasing number of active users at SNR=4 dB and Gold sequence $L_c=31$.

![Figure 11.14](image)

Fig 11.14. BER performances of the detectors for increasing number of active users at SNR=8 dB.

Figure 11.13 and 11.14 shows the BER performances of the detectors are investigated for increasing number of active users in the same channel. All interfering users, from $K=1$ through $K=8$ are signaling at SNR=4 dB & SNR=8 dB. The performance of the conventional detector degrades sharply than the linear detectors as the number of active users' increases.

VIII. CONCLUSION

The optimal multiuser detector performs better than the conventional matched filter and the linear multiuser detectors.
However, this detector is too complex for practical DS-CDMA system. MMSE detector generally performs better than the Decorrelating detector because it takes the background noise into account. With increasing in the number of users, the performance of all detectors will degrade as well. This is because as the number of interfering users increases, the amount of MAI becomes greater as well. Thus there is a trade of between the performance measures (BER vs SNR) and the practicality measure (complexity and detection delay). Depending on the situations, a suboptimum receiver satisfying the implementation constrains can be chosen.

Multiuser detection holds promise for improving DS-CDMA performance and capacity. Although multiuser detection is currently in the research stage, efforts to commercialize multiuser detectors are expected in the coming years as DS-CDMA systems are more widely deployed. The success of these efforts will depend on the outcome of careful performance and cost analysis for the realistic environment.

References