In this paper, we explain a new type of geometry based on a different way of measuring distance between points, as we explained on different planes like Minkowski and Galilean. Now, we'll give a little background on this unfamiliar geometry. The purpose of this paper is to introduce high school students to taxicab geometry, one type of non-Euclidean geometry in which a new metric to measure distance replaces the usual metric of Euclidean geometry.

In taxicab geometry, we can consider the grid as a net of streets, which a taxi driver navigates through. The crossings are the places, where he can stop. It is remarkable that the taxi driver starting at point A can take different ways, which have the same length, if he is continuously approaching the destination point B (Figure 2).


**Distance Formulas**

Euclidean geometry is based on the Euclidean metric, which is a function that takes any two points as input and gives us the distance between them. In two dimensions, this is just the familiar distance formula between points in the plane as follows:

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

In Galilean geometry, the distance between the two points \( A(x_1,y_1) \) and \( B(x_2,y_2) \) is

\[ d_i = |x_2 - x_1|, \quad \text{if} \quad d_i = 0 \]

then

\[ d_2 = |y_2 - y_1| \].

Obviously, it can be seen that we calculate the distance as projection (Figure 3). If both, \( d_1 = d_2 = 0 \) then we say that these points are coincide, \( A = B \) [4].

**Figure 3. Galilean distance between two points.**

Taxicab geometry is a form of geometry, where the distance between two points is not the length of the line segment as in the Euclidean geometry, but the sum of the absolute differences of their coordinates. In other words, the distance of the distances \( d_1 \) and \( d_2 \) in Galilean plane. So, taxicab distance formula between two points is

\[ d_T = |x_2 - x_1| + |y_2 - y_1| \].

By the way, we can show that taxicab distance formula satisfies metric conditions. Let \( P(x_p,y_p) \cdot Q(x_q,y_q) \) and \( R(x_r,y_r) \) be non-collinear, thus forming a triangle.

\[ d(P,Q) = \sqrt{(x_p - x_q)^2 + (y_p - y_q)^2} \]

\[ d(P, Q) = \frac{1}{a} \left| (x_p - x_q)^2 + (y_p - y_q)^2 - (x_r - x_q)^2 - (y_r - y_q)^2 \right| \]

\[ d(P, Q) = \frac{1}{a} \left| (x_p - x_q)^2 + (y_p - y_q)^2 - (x_r - x_q)^2 - (y_r - y_q)^2 \right| \]

M. Axiom 1: \( d(P, Q) = 0 \) if and only if \( P = Q \)

If \( P = Q \) then \( d(P, Q) = d(P, P) \)

\[ = |x_p - x_q| + |y_p - y_q| \]

\[ = |x_q - y_q| \]

\[ = 0 \]

If \( P \neq Q \) then either \( x_p > x_q \) or \( y_p > y_q \) and \( |x_p - x_q| > 0 \) or \( |y_p - y_q| > 0 \). Suppose that \( x_p > x_q \), then

\[ |x_p - x_q| + |y_p - y_q| > 0 \].

Therefore,

\[ |x_p - x_q| + |y_p - y_q| > 0 \].

M. Axiom 2: \( d(P, Q) = d(Q, P) \)

**Figure 4.**

Example: How many \( t \)-line segments are there (Figure 5) between the points \( A \) and \( B \)?

**Figure 5.**

We can count them simply and get 6 \( t \)-line segments. If we examine the number of \( t \)-lines systematically, we may discover a rule. The number of \( t \)-line segments on a crossing is always the sum of the segments at the foregoing crossings.
This leads to Pascal’s triangle, whose rows are diagonal here. Point \( B \) is on the ninth diagonal and thus on the ninth row of Pascal’s triangle, related to \( A \), and on the fourth place.

There is the number \( 28 + 56 = 84 \) or \( "9 \ choose 3" \) which is \( \binom{28+56}{3} = \frac{9!}{(9-3)!3!} = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{6!3!1!} = 84 \), and these \( 84 \) \( t \)-lines for the case \( "6 \ to \ the \ right, \ 3 \ to \ the \ top" \) to go from \( A \) to \( B \) (Figure 6).

**In summary**

Taxicab geometry takes a more realistic approach in geometry. In Euclidean geometry, the shortest distance between two points is a straight line. In theory, this method works perfectly. However, in real life applications it is not as expected. This is where taxicab geometry comes to play in math. The basic premise of taxicab geometry is that the shortest distance between two points is not always a straight line. We can see the taxicab geometry is a very useful model of urban geography. Only a pigeon would benefit from the knowledge that the distance between two buildings on opposite ends of a city is a straight line. For people, taxicab distance is the “real” distance, and taxicab geometry has many applications also it is relatively easy to explore.

**References**


