Chemical Reaction Effect on Unsteady MHD Flow through Porous Medium Past an Impulsively Started Inclined Plate with Variable Temperature and Mass Diffusion in the presence of Hall current

U. S. Rajput and Gaurav Kumar
Department of Mathematics and Astronomy, University of Lucknow, Lucknow – U.P., India.

ABSTRACT
Chemical reaction effect on unsteady MHD flow through porous medium past an impulsively started inclined plate with variable temperature and mass diffusion in the presence of Hall current is studied here. The fluid taken is electrically conducting. The Governing equations involved in the present analysis are solved by the Laplace-transform technique. The velocity profile is discussed with the help of graphs drawn for different parameters like thermal and mass Grashof Number, Prandtl number, chemical parameter, Hall parameter, permeability parameter, magnetic field parameter and Schmidt number, and the numerical values of skin-friction and Sherwood number have been tabulated.

© 2016 Elixir All rights reserved.

Introduction
The study of magnetohydrodynamic flow with heat and mass transfer problems plays important role in different areas of science and technology like chemical engineering, mechanical engineering, biological science, petroleum engineering, biomechanics, irrigation engineering and aerospace technology. Some applications of MHD flow are worth mentioning. It can be used in magnetic material processing, glass manufacturing control processes and purification of crude oil. Further, study of chemical reaction, mass diffusion and Hall current is important in describing several fluid models. Sahin and Chamkha [12] have analyzed effects of chemical reaction, heat and mass transfer and radiation on the MHD flow along a vertical porous plate in the presence of induced magnetic field. Viscous flow over a nonlinearly stretching sheet in the presence of a chemical reaction and magnetic field was studied by Raptis and Perdikis [10]. Muthucumaraswamy [7] has considered effect of chemical reaction on a moving isothermal vertical surface with suction. Muthucumaraswamy and Ganesan [6] have investigated first order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Effect of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction was studied by Das et al. [4]. Effect of chemical reaction and heat generation or absorption on double-diffusive convection from a vertical truncated cone in porous media with variable viscosity was studied by Mahdy [13]. Rajput and Sahu [6] have investigated combined effects of chemical reactions and heat generation or absorption on unsteady free convection MHD flow between two long vertical parallel plates through a porous medium with constant temperature and mass diffusion. Effects of chemical reaction and radiation on heat and mass transfer past semi-infinite vertical porous plate with constant mass flux and dissipations was analyzed by Uwanta [16]. MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion was considered by Rajput and Kumar [15]. Unsteady MHD flow through porous medium past an impulsively started inclined oscillating plate with variable temperature and mass diffusion in the presence of Hall current was studied considered by Uwanta [16]. MHD flow between two parallel plates with heat transfer was investigated by Attia et al. [5]. Heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of magnetic field was studied by Raptis et al. [2]. Raptis and Kafousias [3] have further studied flow of a viscous fluid through a porous medium bounded by a vertical surface. Attia and Ahmed [9] have studied the Hall effect on unsteady MHD Couette flow and heat transfer of a Bingham fluid with suction and injection. Deka [11] has considered Hall effect on MHD flow past an accelerated plate. Maripala and Naikoti [18] have analyzed Hall Effect on unsteady MHD free convection flow over a stretching sheet with variable viscosity and viscous dissipation. Hall Effect on free and forced convective flow in a rotating channel were studied by Rao et al. [1]. Attia [8] has considered the effect of variable properties on the unsteady Hartmann flow with heat transfer considering the Hall effect. Combined effects of radiation and Hall current on MHD flow past an exponentially accelerated vertical plate in the presence of rotation was studied by Thamizhsudar and Pandurangan [17]. We are considering the unsteady MHD flow through porous medium past an impulsively started inclined plate with variable temperature and mass diffusion in the presence of Hall current and chemical reaction. The results are shown with the help of graphs and table.

Mathematical Analysis
MHD flow between two parallel electrically non conducting plates inclined at an angle α from vertical is considered. x axis is taken along the plane and z normal to it. A transverse magnetic field \( B_0 \) of uniform strength is applied on the flow. Initially it has been considered that the plate as well as the fluid is at the same temperature \( T_e \).
The concentration level \( C_0 \) is same everywhere in stationary condition. At time \( t > 0 \), the plate starts moving with a velocity \( u_0 \) in its own plane, and temperature of the plate is raised to \( T_\infty \). The concentration \( C \) near the plate is raised linearly with respect to time. The flow modal is as under:

\[
\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} + g \beta' \cos \alpha (T - T_\infty) + \frac{g\beta \cos \alpha (C - C_\infty)}{\rho(1 + m^2)} \frac{nu}{K},
\]

\[
\frac{\partial v}{\partial t} = \nu \frac{\partial^2 v}{\partial z^2} + \frac{B_0^2 (mu - v)}{\rho(1 + m^2)} \frac{nu}{K},
\]

\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2} - K_c (C - C_\infty),
\]

\[
\frac{\rho \mu C_p}{\partial t} = k \frac{\partial^2 T}{\partial z^2},
\]

with the corresponding initial and boundary conditions:

\[
t \leq 0: u = 0, \ v = 0, \ T = T_\infty, \ C = C_\infty, \ \text{for all} \ \ z \iff \nu \beta = 0, \ \text{at} \ z = 0,
\]

\[
t > 0: u = u_0, \ v = 0, \ \text{at} \ z = 0,
\]

\[
T = T_\infty + (T_\infty - T_0) \frac{u_0 t}{v},
\]

\[
C = C_\infty + (C_\infty - C_0) \frac{u_0 t}{v},
\]

\[
u \to 0, \ v \to 0, \ T \to T_\infty, \ C \to C_\infty \ \text{as} \ \ z \to \infty.
\]

Here \( u \) is the Primary velocity, \( v \) - the secondary velocity, \( g \) - the acceleration due to gravity, \( \beta' \) - volumetric coefficient of thermal expansion, \( t \) - time, \( m(=\omega_c \tau) \) is the Hall parameter with \( \omega_c \) - cyclotron frequency of electrons and \( \tau \) - electron collision time, \( T \) - temperature of the fluid, \( \kappa \) - the permeability parameter, \( \beta' \) - volumetric coefficient of concentration expansion, \( C \) - species concentration in the fluid, \( \nu \) - the kinematic viscosity, \( \rho \) - the density, \( C_p \) - the concentration coefficient at constant pressure, \( k \) - thermal conductivity of the fluid, \( D \) - the mass diffusion coefficient, \( T_\infty \) - temperature of the plate at \( z = 0 \), \( C_\infty \) - species concentration at the plate \( z = 0 \), \( B_0 \) - the uniform magnetic field, \( K_c \) -chemical reaction parameter, \( \sigma \) - electrically conductivity.

The following non-dimensional quantities are introduced to transform equations (1), (2), (3) and (4) into dimensionless form:

\[
\begin{align*}
\tau &= \frac{z u_0}{v}, \quad \nu = \frac{u}{u_0}, \quad v = \frac{v}{u_0}, \quad S &= \frac{v}{D}, \\
\mu &= \rho u_0, \quad \kappa &= \frac{u_0}{\nu} K, \quad M = \frac{\sigma B_0^2 u_0}{\rho u_0^2}, \quad K_c = \frac{nu K_c}{u_0^2}, \\
P_c &= \frac{\mu C_p}{k}, \quad G_c = \frac{g \beta \nu (T_\infty - T_0)}{u_0^2}, \\
G_m &= \frac{g \beta}{\nu} (C_\infty - C_0), \quad \bar{C} = \frac{(C - C_\infty)}{(C_\infty - C_\infty)}, \\
\bar{u} &= \frac{u_0 u}{nu}, \quad \nu = \frac{(T - T_\infty)}{(T_\infty - T_0)}, \quad \nu = \frac{(T - T_0)}{(T_\infty - T_0)}, \\
\end{align*}
\]

here \( \bar{u} \) is the dimensionless Primary velocity, \( \bar{v} \) - the secondary velocity, \( \bar{t} \) - dimensionless time, \( \theta \) - dimensionless temperature, \( \kappa \) - dimensionless permeability parameter, \( \bar{C} \) - dimensionless concentration, \( G_c \) - thermal Grashof number, \( G_m \) - mass Grashof number, \( \mu \) - the coefficient of viscosity, \( K_c \) - dimensionless chemical reaction parameter, \( P_c \) - the Prandtl number, \( S_c \) - the Schmidt number, \( M \) - the magnetic parameter.

Thus the model becomes

\[
\begin{align*}
\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + G_c \cos \alpha \theta + G_m \cos \alpha \bar{C} - \frac{M (\bar{u} + \bar{m} \bar{v})}{(1 + m^2)} \frac{1}{K},
\end{align*}
\]

\[
\frac{\partial \bar{v}}{\partial \bar{t}} = \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} + \frac{M (\bar{m} \bar{u} - \bar{v})}{(1 + m^2)} \frac{1}{K},
\]

\[
\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{S_c} \frac{\partial^2 \bar{C}}{\partial \bar{z}^2} - K_\bar{C},
\]

\[
\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{P_c} \frac{\partial^2 \theta}{\partial \bar{z}^2},
\]

the boundary conditions become:

\[
t \leq 0: \bar{u} = 0, \ \bar{v} = 0, \ \theta = 0, \ \bar{C} = 0, \ \text{for all} \ \bar{z},
\]

\[
t > 0: \bar{u} = 1, \ \bar{v} = 0, \ \theta = \bar{t}, \ \bar{C} = \bar{t}, \ \text{at} \ \bar{z} = 0,
\]

\[
u \to 0, \ v \to 0, \ T \to T_\infty, \ C \to 0 \ \text{as} \ \bar{z} \to \infty.
\]

Dropping bars in the above equations, we get:

\[
\begin{align*}
\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial z^2} + G_c \cos \alpha \theta + G_m \cos \alpha C - \frac{M (u + m \nu)}{(1 + m^2)} \frac{1}{K},
\end{align*}
\]

\[
\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial z^2} + \frac{M (m u - v)}{(1 + m^2)} \frac{1}{K},
\]

\[
\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2} - K_0 C,
\]

\[
\frac{\partial \theta}{\partial t} = \frac{1}{P_c} \frac{\partial^2 \theta}{\partial z^2},
\]

the boundary conditions become:

\[
t \leq 0: u = 0, \ v = 0, \ \theta = 0, \ \bar{C} = 0, \ \text{for all} \ \bar{z},
\]

\[
t > 0: u = 1, \ v = 0, \ \theta = t, \ C = t, \ \text{at} \ \bar{z} = 0,
\]

\[
u \to 0, \ v \to 0, \ \theta \to 0, \ C \to 0 \ \text{as} \ \bar{z} \to \infty.
\]

Combining equations (12) and (13), The model becomes

\[
\frac{\partial q}{\partial \bar{t}} = \frac{\partial^2 q}{\partial \bar{z}^2} + G_c \cos \alpha \theta + G_m \cos \alpha C - qa,
\]

\[
\frac{\partial C}{\partial \bar{t}} = \frac{1}{S_c} \frac{\partial^2 C}{\partial \bar{z}^2} - K_0 C,
\]
The dimensionless Sherwood number at the plate is obtained as:

\[ S_h = \left( \frac{dC}{dz} \right)_{z=0} = \text{erfc}[-\sqrt{tK_0}]\left(-\frac{1}{4\sqrt{K_0}}\sqrt{S_c} - t\sqrt{S_cK_0} \right) \]

\[ + \sqrt{S_c} \text{erfc}[-\sqrt{tK_0}]\left(-\frac{1}{4\sqrt{K_0}} + t\sqrt{K_0} - \frac{e^{-ik_0\sqrt{tS_cK_0}}}{\sqrt{\pi K_0}} \right). \]

**Result and Discussion**

The velocity profile for different parameters like thermal Grashof number \((Gr)\), mass Grashof number \((Gm)\), magnetic field parameter \((M)\), Hall parameter \((m)\), chemical reaction \((K_0)\), Prandtl number \((Pr)\) and time \((t)\) are shown in figures 1.1 to 2.10. Concentration Profile for different values of \(K_0\), \(Sc\) and time are shown in figures 3.1 to 3.3. It is observed from figures 1.1 and 2.1 that the primary and secondary velocities increase with time (figures 1.2 and 2.2). When the mass Grashof number \(Gm\) is increased then the velocities are increased. From figures 1.3 and 2.3 it is deduced that when thermal Grashof number \(Gr\) is increased then the velocities are increased. If Hall current parameter \(m\) is increased then \(u\) is increased and \(v\) decreased (figures 1.4 and 2.4). It is observed from figures 1.5 and 2.5 that the effect of increasing values of the parameter \(M\) results in decreasing \(u\) and increasing \(v\). When \(K_0\) the chemical reaction parameter is increased then the velocities are decreased (figures 1.6 and 2.6). From figures 1.7 and 2.7, it is deduced that when permeability parameter \(K\) is increased then the velocities are increased.

Further, it is observed that velocities decrease when Prandtl number is increased (figures 1.8 and 2.8). When the Schmidt number is increased then the velocities get decreased (figures 1.9 and 2.9). Further, from figures 1.10 and 2.10 it is observed that velocities increase with time. If reaction parameter, Schmidt number are increased then concentration decreases (figures 3.1 and 3.2), it is observed that velocities increase with time (figures 3.3).

Skin friction is given in table 1. The value of \(\tau_x\) increases with the increase in mass Grashof Number, thermal Grashof number, chemical reaction parameter, Hall currents parameter and \(t\) time, and it decreases with angle of inclination of plate, the magnetic field, permeability parameter, Prandtl number and \(\tau_y\). Schmidt number. The value of \(\tau_x\) decreases when the increase in mass Grashof Number, thermal Grashof number, magnetic field parameter, chemical reaction parameter, permeability parameter, Schmidt number and time, and it decreases with angle of inclination of plate, Hall current parameter and Prandtl number.

Sherwood number is given in table 2. The value of \(S_h\) increases with the increase in chemical reaction parameter, Schmidt number and time.
Table 1. Skin friction for different parameters.

<table>
<thead>
<tr>
<th>α (in degree)</th>
<th>M</th>
<th>m</th>
<th>Pr</th>
<th>Sc</th>
<th>Grm</th>
<th>Gr</th>
<th>Kt</th>
<th>t</th>
<th>K</th>
<th>(τ_x)</th>
<th>(τ_y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2</td>
<td>0.5</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>9.29686</td>
<td>3.20771</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>8.07299</td>
<td>2.89449</td>
</tr>
<tr>
<td>45</td>
<td>2</td>
<td>0.5</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>6.12610</td>
<td>2.39623</td>
</tr>
<tr>
<td>60</td>
<td>2</td>
<td>0.5</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>3.58887</td>
<td>1.74688</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>0.5</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>10.3835</td>
<td>2.08071</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
<td>0.5</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>6.36240</td>
<td>3.15353</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>2</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>10.2273</td>
<td>3.40099</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>3</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>11.4818</td>
<td>3.02510</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>7.94230</td>
<td>2.89094</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>15.5265</td>
<td>8.25026</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>29.4652</td>
<td>28.2906</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>-1.2225</td>
<td>0.45517</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>50</td>
<td>10</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>2.90881</td>
<td>1.53931</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>100</td>
<td>50</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>9.19776</td>
<td>2.91420</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>100</td>
<td>100</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>10.6037</td>
<td>2.93883</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>100</td>
<td>100</td>
<td>2</td>
<td>0.2</td>
<td>0.2</td>
<td>30.1426</td>
<td>29.0702</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>100</td>
<td>100</td>
<td>1</td>
<td>0.3</td>
<td>0.2</td>
<td>14.0360</td>
<td>4.29476</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>100</td>
<td>100</td>
<td>1</td>
<td>0.4</td>
<td>0.2</td>
<td>20.1464</td>
<td>5.70506</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.5</td>
<td>7.00</td>
<td>2.01</td>
<td>100</td>
<td>100</td>
<td>1</td>
<td>0.2</td>
<td>1</td>
<td>-39.024</td>
<td>28.9358</td>
</tr>
</tbody>
</table>

Table 2. Sherwood number for different Parameters.

<table>
<thead>
<tr>
<th>(K_0)</th>
<th>(S_c)</th>
<th>(t)</th>
<th>(S_b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.01</td>
<td>0.2</td>
<td>-0.762200</td>
</tr>
<tr>
<td>5</td>
<td>2.01</td>
<td>0.2</td>
<td>-0.933049</td>
</tr>
<tr>
<td>10</td>
<td>2.01</td>
<td>0.2</td>
<td>-1.118240</td>
</tr>
<tr>
<td>1</td>
<td>3.00</td>
<td>0.2</td>
<td>-0.931175</td>
</tr>
<tr>
<td>1</td>
<td>4.00</td>
<td>0.2</td>
<td>-1.075230</td>
</tr>
<tr>
<td>1</td>
<td>2.01</td>
<td>0.3</td>
<td>-0.961323</td>
</tr>
<tr>
<td>1</td>
<td>2.01</td>
<td>0.4</td>
<td>-1.141570</td>
</tr>
</tbody>
</table>

Figure 1.1. Velocity u for different values of α

Figure 1.2. Velocity u for different values of \(Gm\)

Figure 1.3. Velocity u for different values of Gr

Figure 1.4. Velocity u for different values of m

Figure 1.5. Velocity u for different values of M
Figure 1.6. Velocity $u$ for different values of $K_0$

Figure 1.7. Velocity $u$ for different values of $K$

Figure 1.8. Velocity $u$ for different values of $Pr$

Figure 1.9. Velocity $u$ for different values of $Sc$

Figure 1.10: Velocity $u$ for different values of $t$

Figure 2.1. Velocity $v$ for different values of $\alpha$

Figure 2.2. Velocity $v$ for different values of $Gm$

Figure 2.3. Velocity $v$ for different values of $Gr$

Figure 2.4. Velocity $v$ for different values of $m$
Figure 2.5: Velocity $v$ for different values of $M$

Figure 2.6: Velocity $v$ for different values of $K_0$

Figure 2.7: Velocity $v$ for different values of $K$

Figure 2.8: Velocity $v$ for different values of $Pr$

Figure 2.9: Velocity $v$ for different values of $Sc$

Figure 2.10: Velocity $v$ for different values of $t$

Figure 3.1: Concentration $c$ for different values of $K_0$

Figure 3.2: Concentration $c$ for different values of $Sc$

Figure 3.3: Concentration $c$ for different values of $t$

Conclusion

The conclusions of the study are as follows:

- Primary velocity increases with the increase in thermal Grashof number, mass Grashof Number, Hall current parameter, permeability parameter and time.
- Primary velocity decreases with the angle of inclination of plate, the magnetic field, chemical reaction parameter, Prandtl number and Schmidt number.
- Secondary velocity increases with the increase in thermal Grashof number, mass Grashof Number, the magnetic field, permeability parameter and time.
- Secondary velocity decreases with the angle of inclination of plate, Hall currents parameter, chemical reaction parameter, Prandtl number and Schmidt number.
- Concentration near the plate increase with time, and it decreases with $K_0$ and $Sc$. 
\( \tau_x \) increases with the increase in \( Gm, Gr, K_0, m, Sc \) and \( t \), and it decreases with angle of inclination of plate, \( M, K \) and \( Pr \).

\( \tau_y \) increases with the increase in \( Gm, Gr, K_0, K, Sc \) and \( t \), and it decreases with angle of inclination of plate, \( m \) and \( Pr \).

\( S_b \) decreases with the increase in \( K_0, Sc \) and \( t \).

Appendix

\[ A_1 = 1 + A_{16} + e^{2z\sqrt{t}}(1 - A_{17}), \]
\[ A_2 = -A_1, \]
\[ A_3 = A_{16} - A_1, \]
\[ A_4 = -1 + A_{22} + A_{18}(A_{31} - 1), \]
\[ A_5 = -1 - A_{24} + A_{19}(A_{25} - 1), \]
\[ A_6 = -1 - A_{26} + A_{19}(A_{33} - 1), \]
\[ A_7 = -A_{18}, \]
\[ A_8 = -1 - A_{20} + A_{30}(A_{21} - 1), \]
\[ A_9 = A_8 + 2(A_{21} + 1), \]
\[ A_{10} = -1 - A_{28} + A_{19}(A_{29} - 1), \]
\[ A_{11} = e^{2z\sqrt{t}}(2A_{1} + 2atA_{2} + \sqrt{\pi}A_{12}), \]
\[ A_{12} = -1 + erf\left(\frac{z\sqrt{a}}{2\sqrt{t}}\right) \]
\[ A_{13} = e^{at}\frac{1}{1 + S_0}, \]
\[ A_{14} = e^{at}\frac{1}{1 + S_0}, \]
\[ A_{15} = 1 + A_{16} + e^{2z\sqrt{t}}A_{17}, \]
\[ A_{16} = erf\left(\frac{2\sqrt{a}t - z}{2\sqrt{t}}\right), \]
\[ A_{17} = erf\left(\frac{2\sqrt{a}t + z}{2\sqrt{t}}\right), \]
\[ A_{18} = e^{-2z\sqrt{a}t}, \]
\[ A_{19} = e^{-2z\sqrt{a}t}, \]
\[ A_{20} = erf\left(\sqrt{K_0} - \frac{z\sqrt{S}}{2\sqrt{t}}\right) \]
\[ A_{21} = erf\left(\sqrt{K_0} + \frac{z\sqrt{S}}{2\sqrt{t}}\right), \]
\[ A_{22} = erf\left(\frac{2at}{2\sqrt{t}}\right) - \frac{ab}{2\sqrt{t}}, \]
\[ A_{23} = erf\left(\frac{2at}{2\sqrt{t}} - 1\right) \]
\[ A_{24} = erf\left(\frac{2at}{2\sqrt{t}} - 1\right), \]
\[ A_{25} = erf\left(\frac{2at}{2\sqrt{t}} + 1\right), \]

References


