Highlighting Volatility and Excess Volatility of Some International Indices against U.S. Benchmarks (DCC GARCH Model)

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Introduction

In finance, volatility is an important concept. It measures the state of returns instability. It is admitted that volatility changes over time and tends to cluster during periods of high volatility and periods of stability. This phenomenon is called heteroscedasticity. We should also consider that volatility is auto-correlated because it depends on its previous value.

As volatility is not observable, then we need a good estimation and prediction model. Among the models that capture the properties mentioned above is the univariate GARCH model developed by Bollerslev (1986). Empirically, this model has been successful in estimating and predicting changes in volatility.

If we consider portfolio optimization model of Black and Litterman (1990), volatility between assets is essential in addition to individual volatility. Symmetric covariance matrix is the tool that can quantify all these components where variance of each individual asset is obtained on the diagonal and covariance of each pair is in other locations of the returns variance-covariance matrix, when i≠j.

It is therefore necessary to extend the univariate GARCH model into a multivariate model that can estimate the covariance matrix of a GARCH model.

When extending the model, some problems should be attended:

• The number of parameters should be reduced, without unduly restricting the flexibility to capture the dynamism in conditional variance, in order to run assessment;
• It is necessary to determine the conditions that ensure positivity of the covariance matrix at each moment (as required by definition) and the conditions of a weak stationary process;
• The parameters should be easily interpretable.

One approach has been proposed which consists in indirectly modelling conditional covariance matrix through conditional correlation matrix. The first such model was the constant conditional correlation (CCC) model proposed by Bollerslev (1990).

Conditional correlation was assumed to be constant and only conditional variances vary through time. Thus, this assumption is not convincing because correlation of several assets changes over time in practice. For this reason, Engle and Sheppard (2001) developed the dynamic conditional correlation model (DCC).

This model has a two-stage algorithm for estimating parameters. Therefore, the model is relatively easy to use in practice. As a first step, conditional variance is estimated by an univariate GARCH for each asset. As a second step, conditional correlation parameters are estimated taking into account the parameters of the first step. This approach was used to estimate covariance of 100 stocks even without a large calculation. Finally, the DCC model has conditions that ensure positivity of the covariance matrix over time and ensure covariance stationarity.

The Dynamic Conditional Correlation Model

To extend hypotheses of returns into a multivariate case, let a portfolio consisting of n assets and the column vector of returns is

\[ r_t = \left( r_{t1}, r_{t2}, ..., r_{tn} \right). \]

In addition, we assume that conditional returns are normally distributed with zero mean and a conditional covariance matrix

\[ H_t = E\left[r_t r_t'|\mathcal{F}_{t-1}\right]. \]

This implies that:

\[ r_t = H_t^{1/2} z_t \quad \text{et} \quad r_t'|\mathcal{F}_{t-1} \sim N(0,H) \]

(1)

Where

\[ z_t = \left( z_{t1}, z_{t2}, ..., z_{tn} \right) \sim N(0,I_n) \]

and \( I_n \) is identity matrix of order n. \( H_t^{1/2} \) can be obtained by a Cholesky factorization of \( H_t \).

In the DCC model, covariance matrix is decomposed into:

\[ H_t = D_t R_t D_t \]

(2)
\( D_t \) is the diagonal matrix of standard deviation that varies over time in a univariate GARCH process:

\[
D_t = \begin{pmatrix}
\sqrt{h_{1t}} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \sqrt{h_{nt}}
\end{pmatrix}
\] (3)

The specification of the elements of the \( D_t \) matrix is not restricted only to GARCH (p, q), but also to any GARCH process with normally distributed errors, which provides suitable stationarity and positivity conditions. The number of lags for each asset and each series is not necessarily the same.

Like Engle and Sheppard (2001), \( R_t \) is conditional correlation matrix of standard errors \( \varepsilon_t^e \):

\[
R_t = \begin{pmatrix}
1 & q_{12,t} & \cdots & q_{1n,t} \\
q_{21,t} & 1 & \cdots & q_{2n,t} \\
\vdots & \vdots & \ddots & \vdots \\
q_{n1,t} & q_{n2,t} & \cdots & 1
\end{pmatrix}
\] (4)

\[
\varepsilon_t = D_t^{-1/2} \varepsilon_t \sim N(0, R_t)
\] (5)

Therefore, conditional correlation is the conditional covariance between standardized errors.

Before further analysing \( R_t \), it should be remembered that \( H_t \) is positive by definition. Since \( H_t \) is a quadratic form based on \( R_t \), it follows the basics of linear algebra such that \( R_t \) should be positive by definition in order for \( H_t \) to be positive by definition. In addition, all the elements of the conditional correlation matrix should be less or equal to unity by definition. To ensure these requirements, \( R_t \) should be broken down as follows:

\[
R_t = Q_t^{-1} Q_t^{* -1}
\] (6)

Where \( Q_t \) is a positive matrix by definition that defines dynamics structure and \( Q_t^{* -1} \), resizes the elements in \( Q_t \) to ensure that \( |q_{ij}| \leq 1 \). In other words, \( Q_t^{* -1} \), is simply the reverse diagonal matrix having the square roots of the diagonal elements of \( Q_t \):

Suppose \( \tilde{Q}_t \) has the following dynamics:

\[
Q_t = (1 - \alpha - \beta) \tilde{Q} + \alpha \varepsilon_{t-1} \varepsilon^e_{t-1} + \beta Q_{t-1}
\] (8)

Where \( \tilde{Q} \) is the conditional variance of standardized errors:

\[
\tilde{Q} = Cov(\varepsilon_t \varepsilon_t^e) = E[\varepsilon_t \varepsilon_t^e]
\] (9)

And \( \alpha \) and \( \beta \) are two scalars.

The proposed dynamic structure seems complicated but, considering the similarity between equation (8) and its counterpart in GARCH (1,1), it is evident that the structure is similar to GARCH (1,1) with a "variance targeting". Indeed, this dynamic structure is the simplest multivariate GARCH, named scalar GARCH. This structure implies that all correlations follow the same structure. This may be considered a drawback of the model. This structure can be generalized as DCC (P, Q):

\[
q_t = (1 - \sum_{j=1}^p \alpha_j - \sum_{j=1}^q \beta_j) \tilde{Q} + \sum_{j=1}^p \alpha_j \varepsilon_{t-j} \varepsilon^e_{t-j} + \sum_{j=1}^q \beta_j Q_{t-j}
\] (10)

1-1 - Constraints of the DCC (1, 1) model

If covariance matrix is not positive then it is impossible to reverse it. Reversal is essential in portfolio optimization. To ensure a positive matrix \( H_t \) whatever \( t \) we should impose simple conditions for all parameters. First, the conditions for univariate GARCH model should be met. Similar conditions for dynamic correlations are also required, including:

\[
\alpha \geq 0 \quad \text{et} \quad \beta \geq 0
\]

\[
\alpha + \beta < 1
\] (11)

And finally, \( Q_0 \) should be positive.

1-2 - Estimation of the DCC (1, 1) model

To estimate the parameters of \( H_t \), Engle and Sheppard (2001) assumed that \( \theta = (\varphi, \varphi) \) and the log-likelihood function, denoted \( l \), which can be used when errors are normally distributed and in a multivariate way:
In this case, the parameters in the DCC (1,1) model can be divided into two groups: \( \varphi = (\omega_1, \delta_1, \gamma_1, ... , \omega_n, \delta_n, \gamma_n) \) and \( \varphi = (\alpha, \beta) \). These parameters will be estimated through the following two steps:

**Step 1**

The matrix in the log-likelihood function (13) is replaced by identity matrix \( I_n \), which gives the following log-likelihood function:

\[
G(\varphi) = \sum_{t=1}^{T} \left[ n \log (2\pi) + \log (|R_t|) + \frac{1}{2} \left( \frac{\varepsilon_t^2}{h_{11}} \right) \right] - \frac{1}{2} \sum_{i=1}^{n} \sum_{t=1}^{T} (\log (2\pi) + \log (h_{ii(t)})) - \frac{1}{2} \sum_{i=1}^{n} \sum_{t=1}^{T} \left( \log (2\pi) + \log (h_{ii(t)}) + \frac{2}{h_{ii(t)}} \right)
\]

When we look at equation (14), it is clear that the log-likelihood function is the sum of log-likelihood functions of univariate GARCH. Therefore, we can estimate the parameters for each univariate GARCH. Since variance \( h_{ii(t)} \) of the stock \( i = 1, ..., n \) is estimated for \( t \in [1, T] \), then the element of the \( D_t \) matrix can also be estimated during the same time interval.

**Step 2**

In the second step, the correctly specified log-likelihood function is used to estimate \( \varphi = (\alpha, \beta) \) by considering the already estimated parameters \( \varphi = ((\omega_1, \delta_1, \gamma_1), ..., (\omega_n, \delta_n, \gamma_n)) \) in the first step.

\[
G(\varphi; \hat{\varphi}) = \sum_{t=1}^{T} \left[ n \log (2\pi) + \log (|R_t|) + \frac{1}{2} \left( \frac{\varepsilon_t^2}{h_{11}} \right) \right] - \frac{1}{2} \sum_{i=1}^{n} \sum_{t=1}^{T} (\log (2\pi) + \log (h_{ii(t)})) - \frac{1}{2} \sum_{i=1}^{n} \sum_{t=1}^{T} \left( \log (2\pi) + \log (h_{ii(t)}) + \frac{2}{h_{ii(t)}} \right)
\]

Considering the fact that the first two terms in the log-likelihood function are constant, the last two terms that contain \( R_{tt} \) are to maximize:

\[
G(\varphi; \hat{\varphi}) = \sum_{t=1}^{T} \left( \log (|R_t|) + \frac{1}{2} \left( \frac{\varepsilon_t^2}{h_{11}} \right) \right)
\]

Standard errors are calculated using equation (5) and \( Q \) is estimated as:

\[
\hat{Q} = \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t^2
\]

Even in this case, variance targeting is used in the dynamic structure, and \( \hat{Q} = \varepsilon_0^2 \). The correlation matrix is the conditional covariance matrix of the standard errors.

2 - Demonstration of volatility and excess volatility of some indices compared to U.S. benchmarks

**2-1 - The Hypotheses**

This empirical validation aims at studying and analyzing the evolution of variance between volatilities of four indices on the one hand and volatilities of four U.S. indices on the other. This study will lead us to check one of these two hypotheses:

• Hypothesis 1: If there is a trend of amplification of returns variance, then there is excess volatility.

• Hypothesis 2: If returns variance is always low or negligible then there is no excess volatility.

**2-2 - Presentation of data**

The data consists of the four U.S. markets of reference are: NYSE, NASDAQ, SP500 and Dow Jones and the four international markets BVSP (Brazil), MERV (Argentina), MXX (Mexico) and HSI (China).

We use daily prices of these indices for the period stretching from 18 January 2000 until 31 December 2010, or 2709 days. These observations are taken from the Yahoo! Finance database.

**2-3 - Empirical results and interpretation**

A-Descriptive statistics of time series
The table below summarizes the characteristics of the returns of the selected four U.S. indices:

**Table 1. Descriptive statistics of US indices returns**

<table>
<thead>
<tr>
<th>Statistics</th>
<th>DJA</th>
<th>NASDAQ</th>
<th>NYSE</th>
<th>SP500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.006737</td>
<td>0.0092756</td>
<td>0.0076743</td>
<td>0.0122346</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.38165</td>
<td>5.756419</td>
<td>5.00557</td>
<td>4.75865</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.878183</td>
<td>-0.3127266</td>
<td>-1.223991</td>
<td>-1.058001</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>68.9188</td>
<td>19.91298</td>
<td>40.85693</td>
<td>32.13651</td>
</tr>
<tr>
<td>Median</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0199997</td>
</tr>
<tr>
<td>Stand. Dev</td>
<td>0.3313897</td>
<td>0.4430291</td>
<td>0.3783568</td>
<td>0.4221651</td>
</tr>
</tbody>
</table>

The table above reports means for the four indexes ranging from 0.006737 (ADI index) to 0.0122346 (SP500 index). The highest values are between 4.38165 (ADI index) and 5.756419 (NASDAQ). However, minimum values are all negative and range from -9.945223 (SP500 index) to -5.230311 (NASDAQ).

Distributions of all U.S. indices extend to the right and tails spread to the left. Kurtosis coefficients are always greater than three therefore returns distributions of the four indices are leptokurtic.

Table (2) summarizes the characteristics of the returns of the four indices of the selected international markets:

**Table 2. Descriptive statistics of the returns of the four international markets**

<table>
<thead>
<tr>
<th>Statistics</th>
<th>BVSP</th>
<th>HSI</th>
<th>MERV</th>
<th>MXX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0225205</td>
<td>0.004473</td>
<td>0.0297005</td>
<td>0.0267008</td>
</tr>
<tr>
<td>Maximum</td>
<td>5.939677</td>
<td>5.822503</td>
<td>6.999315</td>
<td>4.534343</td>
</tr>
<tr>
<td>Minimum</td>
<td>-5.253248</td>
<td>-5.898598</td>
<td>-5.624822</td>
<td>-3.590464</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.08614</td>
<td>-0.045458</td>
<td>-0.0771208</td>
<td>0.0450354</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>6.551229</td>
<td>10.76151</td>
<td>7.851749</td>
<td>7.124008</td>
</tr>
<tr>
<td>Median</td>
<td>0.0524867</td>
<td>0.139656</td>
<td>0.0475758</td>
<td>0.0528936</td>
</tr>
<tr>
<td>Stand. Dev</td>
<td>0.8575317</td>
<td>0.7221079</td>
<td>0.9557759</td>
<td>0.6507112</td>
</tr>
</tbody>
</table>

This table shows that the returns of four different international markets are different from the U.S. markets characteristics. Indeed, we observe very distant means. They vary between 0.004473 (HSI) and 0.0297005 (MERV index). However, maximum values are between 5.939677 (BVSP index) and 6.999315 (MERV index). Minimum values are all negative and range from -5.898598 (HSI) to -3.590464 (MXX index).

We observe two types of distributions: the MXX index has a left-tailed distribution, then a distribution spreading to the left. Other indices have distributions spreading to the right straight and left-tailed distributions. Similar to the U.S. markets, returns distributions of the four indices are leptokurtic.

**B-Results and interpretation**

**B-1-Correlation of international indices volatilities with ADI**

Figure (2-1) reports low correlation during the eleven study years which varies between 0 and 0.4 in terms of absolute values between volatilities of the ADI index and the MERV index. Nevertheless, we notice that at the beginning of 2000 there was fairly average correlation but indices volatilities vary in opposite directions. However, covariance was very low despite the economic crisis of Argentina which started in November 2001. However, both volatilities moderately correlate in 2002 and have evolved in the same direction. However, conditional covariance remained low during the "subprime" financial crisis between 2007 and 2009.

![Figure 1. Evolution of conditional covariance between ADI and MERV indices](image1)

As for the Brazilian BVSP index volatility, conditional covariance was also low with an absolute value that varies between 0 and 0.4. The optimum level of this curve takes place around 2002. This year has been characterized by the Brazilian financial crisis that has affected the bond market. We can then conclude that the Brazilian crisis had a quite average impact on ADI volatility through a contagion phenomenon.

![Figure 2. Evolution of conditional covariance between DJA and BVSP indices](image2)
Concerning the Mexican index volatility, we see generally average correlation with ADI volatility during the study period with the exception of three peaks that exceeded 0.5. The first two peaks took place starting from late 2000 until mid-2001. This period is marked by a "creeping" stock market crash which was characterized by bankruptcy or last minute rescue packages of major U.S. international companies. Similarly, the third peak took place in the middle of 2009, during the "subprime" financial crisis. These three optimum curves indicated a spread of crises between these two indices which generated a remarkable amplification of volatilities covariance.

![Figure 3. Evolution of conditional covariance between ADI and MXX indices](image1.png)

Examining volatility covariance of ADI with that of the Chinese HSI index, we notice generally low values that vary between 0 and 0.5 over the entire period. However, we notice that until 2004, covariance was average and there were two peaks that slightly exceeded the 0.5 threshold. However, during the rest of the study period covariance became very weak with an oscillation around 0. Both peaks show an amplification of the correlation between the two volatilities during the oil crisis of 2003, caused by the war in Iraq.

![Chart 4. Evolution of conditional covariance between ADI and HSI indices](image2.png)

**B-2-Correlations of indices volatilities with NASDAQ**

MERV index volatility had a low correlation with NASDAQ. This conditional covariance has not exceeded the 0.4 threshold only once at the beginning of 2002. As stated before, the year 2002 is characterized by the Argentine economic crisis that exploded in November 2001. The amplification of this correlation shows the existence of contagion between these two indices. However, U.S. crises have not been transferred to the Argentinean market. This volatility contagion is a one-way contagion.

![Figure 5. Evolution of conditional covariance between NASDAQ and MERV indices](image3.png)

Examining the evolution of conditional covariance of the respective volatilities of both NASDAQ and BVSP leads us to detect low covariance values even during periods of U.S. and Brazilian crises. These values vary between -0.4 and 0.3, and they oscillate around 0.

![Chart 6. Evolution of conditional covariance between the NASDAQ and BVSP indices](image4.png)
Generally, Mexican market volatility has no significant correlation with that of NASDAQ. Indeed, the values are between 0 and 0.4 in terms of absolute values with the exception of four optimal curves. The first optimum curve is characterized by a negative covariance close to -0.6 around 2000. Then, there was a considerable increase reaching two peaks exceeding the value of 0.5 at the end of 2000 and mid-2001 respectively. This shows contagion amplification in terms of volatility during crises.

Figure 7. Evolution of conditional covariance between the NASDAQ and MXX indices

Similar to ADI volatility, NASDAQ volatility poorly correlates with HSI volatility except for two observations at the end of 2002 and mid-2003 respectively where covariance exceeded the 0.5 threshold.

Figure 8. Evolution of conditional covariance between NASDAQ and HSI indices

B-3-Correlation of international indices volatilities with NYSE

Conditional covariance between NYSE volatility of the index and MERV volatility is low throughout the study period with the exception of two optimum curves located at the beginning of 2000 and the beginning of the year 2002, respectively, where average covariance exceeded the 0.4 threshold. The first optimum curve is negative while the second is positive and takes place during the Argentinean crisis.

Figure 9. Evolution of conditional covariance between NYSE and MERV indices

As for the Brazilian index, its volatility covariance with the NYSE is still very low with the exception of three small amplifications approaching 0.3. One of these amplifications took place during the Brazilian crisis.

Figure 10. Evolution of conditional covariance between NYSE and BVSP indices

In addition to these three observations, conditional covariance of the volatilities of NYSE and MXX is low. The three exceptions took place during periods of crisis: The first two during the stock market crash of 2001-2002 and the last one during the "subprime" crisis at the end of 2009.
Evolution of volatility covariance of HSI with the of the three U.S. indices ADI, NASDAQ and NYSE volatilities, respectively, has the same trend. Indeed, there are still two peaks during the period from 2002 until 2004. These two peaks indicate a significant covariance amplification which exceeded the 0.5 threshold. However, this covariance is low especially in the second half of the study period.

The shape of volatilities covariance between SP500 and MERV is similar to volatility covariance of ENTERED and MERV on the one hand and volatilities of the three other U.S. Markets indices on the other hand. We notice amplification around early 2002, during the Argentinean crisis. The rest of the period is characterized by low covariance not exceeding 0.4 in terms of absolute values.

The Brazilian index volatility co-varies in the same way with the volatilities of the four U.S. indices. Indeed, there is always a negative average covariance (near -0.4) in early 2000. Next, we notice a relatively large oscillation at the beginning of 2002, i.e. during the Brazilian financial crisis. Generally, covariance is still poor and it varies between 0 and 0.4 in terms of absolute values over the entire period.
As for the Mexican index volatility, it has the same pattern of covariance with the volatilities of the four U.S. indices. More specifically, the period from 2000 to 2002, is characterized by large movements where conditional covariance reached 0.6. This phase is marked by a rampant stock market crash. Similarly, we notice another peak during the end of 2009, which marked the "subprime" crisis.

Figure 15. Evolution of conditional covariance between SP500 and MXX indices

As mentioned before, changes in conditional covariance between HSI volatility on the one hand and U.S. indices volatilities on the other hand are marked by the same rate.

Figure 16. Evolution of conditional covariance between SP500 and HSI indices

Generally, we were able to distinguish conditional covariance between volatilities of the selected four indices on the one hand and the four U.S. benchmarks on the other hand. We found that they have the same characteristics. Covariance is generally low and varies between 0 and 0.4 in terms of absolute values. However, we have noticed significant and sometimes strong amplifications during periods of economic, financial and political crises. This amplification indicates the presence of financial contagion highlighted for the first time by Lexis-Nexis (1997) who indicates that equity volatility on stock markets is not only a function of the stocks' fundamental values but it is a function of several other factors (informational, psychological, political, etc.).

Conclusion

Highlighting these covariance volatilities allows us to accept the first and reject the second hypothesis. Therefore, we can argue that excess volatility exists and market efficiency is no longer accepted because it cannot explain this excess. Our results corroborate those obtained by Shiller (1981), DeBondt and Thaler (1985), Cutler, Poterba and Summers (1989), Lo and MacKinlay (1991), Fama and French (1992), Fama and French (1993), Jagadeesh and Titman (1993) and Shleifer and Vishny (1997).

This paper aimed at examining the different aspects of volatility and excess volatility. Removal of stock price from its fundamental value embodies excess volatility. In the past, researchers explained this phenomenon by several advanced methods. Therefore, market efficiency hypothesis is no longer valid as stipulated mainly by Shiller (1981), DeBondt and Thaler (1985) and Fama and French (1992 and 1993) who have highlighted anomalies and seasonality of stock returns. Efficient market hypothesis models could not explain excess volatility, bubbles and stock market crashes observed on equity markets. For this reason, researchers have changed direction to study the investors’ behaviour.

References