The Eccentric Dominating Graph $ED_mG^{abc}(G)$ of a graph $G$

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Abstract

The eccentric dominating graph $ED_mG^{abc}(G)$ of a graph $G$ is obtained from $G$ with vertex set $V' = V \cup S$, where $V = V(G)$ and $S$ is the set of all $\gamma_{ed}$-sets of $G$. Two elements in $V'$ are said to satisfy property 'a' if $u, v \in V$ and are adjacent in $G$. Two elements in $V'$ are said to satisfy property 'b' if $u = D_1, v = D_2 \in S$ and have a common vertex. Two elements in $V'$ are said to satisfy property 'c' if $u \in V, v = D \in S$ such that $u \in D$. Two elements in $V'$ are said to satisfy property 'd' if $u, v \in V$ and there exists $D \in S$ such that $u, v \in D$. A graph having vertex set $V'$ and any two elements in $V'$ are adjacent if and only if they satisfy any one of the property a, b, c is denoted by $ED_mG^{abc}(G)$. In this paper $ED_mG^{abc}(G)$ of some families of graphs and some basic properties of $ED_mG^{abc}(G)$ are studied. Also, we have discussed the eccentricity properties of $ED_mG^{abc}(G)$, and we have characterized graphs $G$ for which $ED_mG^{abc}(G)$ is complete or a tree.

Introduction

Graphs discussed in this paper are undirected and simple. For graph theoretic terminology refer to Harary[5], Buckley and Harary[3]. For a graph, let $V(G)$ and $E(G)$ denotes its vertex and edge set respectively. A graph with $p$ vertices and $q$ edges is called a $(p, q)$ graph.

The concept of distance in graph plays a dominant role in the study of structural properties of graphs in various angles using related concept of eccentricity of vertices in graphs. The length of any shortest path between any two vertices $u$ and $v$ of a connected graph $G$ is called the distance between $u$ and $v$ and it is denoted by $d_G(u, v)$. The distance between two vertices in different components of a disconnected graph is defined to be $\infty$. For a connected graph $G$, the eccentricity $e(v)$ of $v$ is the distance to a vertex farthest from $v$. Thus, $e(v) = \max\{d_G(u, v) : u \in V\}$. The radius $rad(G)$ of a graph $G$ is the minimum eccentricity of the vertices, whereas the diameter $diam(G)$ is the maximum eccentricity. If these two are equal in a graph, that graph is called self-centered graph with radius $r$ and is called an $r$ self-centered graph. For any connected graph $G$, $rad(G) \leq diam(G) \leq 2rad(G)$.

A graph $G$ is connected if every two of its vertices are connected, otherwise $G$ is disconnected. The vertex connectivity or simply connectivity $k(G)$ of a graph $G$ is the minimum number of vertices whose removal from $G$ results in a disconnected or trivial graph. The edge connectivity $\lambda(G)$ of a graph $G$ is the minimum number of edges whose removal from $G$ results in a disconnected or trivial graph. A set $S$ of vertices of $G$ is independent if no two vertices in $S$ are adjacent. The independence number $\beta(G)$ of $G$ is the maximum cardinality of an independent set.

The concept of domination in graphs was introduced by Ore [11]. The concept of domination in graphs is originated from the chess games theory and that paved the way to the development of the study of various domination parameters and its relation to various other graph parameters. For details on $\gamma(G)$, refer to [4, 12].

A set $D \subseteq V$ is said to be a dominating set in $G$, if every vertex in $V\sim D$ is adjacent to some vertex in $D$. The minimum cardinality of a dominating set is called the domination number and is denoted by $\gamma(G)$.

Janakiraman, Bhanumathi and Muthammai [6] introduced and studied the concept of eccentric dominating set. A set $D \subseteq V(G)$ is an eccentric dominating set if $D$ is a dominating set of $G$ and for every $v \in V\sim D$, there exists at least one eccentric vertex of $v$ in $D$. The minimum cardinality of an eccentric dominating set is called the eccentric domination number and is denoted by $\gamma_{ed}(G)$. An eccentric dominating set with cardinality $\gamma_{ed}(G)$ is known as a minimum eccentric dominating set.

If $D$ is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set. An eccentric dominating set $D$ is a minimal eccentric dominating set if no proper subset $D'' \subseteq D$ is an eccentric dominating set.

A partition of $V(G)$ is called eccentric domatic if all its classes are eccentric dominating sets in $G$. The maximum number of classes of an eccentric domatic partition of $V(G)$ is called the eccentric domatic number of $G$ and is denoted by $d_{ed}(G)$.

A vertex $v$ is said to be good [14] if there is a $\gamma$-set of $G$ containing $v$. If there is no $\gamma$-set of $G$ containing $v$, then $v$ is said to be a bad vertex.

In this manner, we define ed-good and ed-bad vertices as follows: Let $u \in \mathcal{V}(G)$. $u$ is said to be ed-good if $u$ is contained in a $\gamma_{ed}$-set of $G$. $u$ is said to be ed-bad if there exists no $\gamma_{ed}$-set of $G$ containing $u$.

In [13], Walikar, Acharya and et al., defined $\gamma_{ed}(G)$ as the total number of minimum dominating sets in a graph $G$.

In [1], we have defined $\gamma_{ed}(G)$ as the total number of minimum eccentric dominating sets in a graph $G$. 

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In [7, 8, 9, 10], Kulli, Janakiram and Niranjan introduced the following concepts in the field of domination theory.

The minimal dominating graph $MD(G)$\cite{8} of a graph $G$ is the intersection graph defined on the family of all minimal dominating sets of vertices of $G$. The vertex minimal dominating graph $M_D(G)$\cite{9} of a graph $G$ with $V(M_D(G)) = V' = V \cup S$, where $S$ is the collection of all minimal dominating sets of $G$ with two vertices $u \neq v \in V'$ are adjacent if either they are adjacent in $G$ or $v = D$ is a minimum dominating set of $G$ containing $u$.

The dominating graph $D(G)$\cite{10} of a graph $G = (V, E)$ is a graph with $V(D(G)) = V \cup S$, where $S$ is the set of all minimal dominating sets of $G$ and with two vertices $u \neq v \in V(D(G))$ are adjacent if $u \in V$ and $v = D$ is a minimal dominating set of $G$ containing $u$.

In [2], we have defined and studied the dominating graph $DG_{abc}(G)$ of a graph $G$.

In this paper, we define $ED_mG_{abc}(G)$ with property $a$, $b$ and $c$. We find $ED_mG_{abc}(G)$ and some basic properties of $ED_mG_{abc}(G)$ are studied. Also, the characterization of $ED_mG_{abc}(G)$ is established.

In section 3, we have defined and studied the properties of $ED_mG_{abc}(G)$.

The following results are needed to study $ED_mG_{abc}(G)$.

**Theorem 1.1**[4] A graph $G$ is Eulerian if and only if every vertex of $G$ is of even degree.

**Theorem 1.2**[1]

(i) $\gamma_{ed}(P_k) = k$.

(ii) $\gamma_{ed}(P_{3k+1}) = 1$.

(iii) $\gamma_{ed}(P_{3k+2}) = \frac{k^2 + 3k + 2}{2}$.

**Theorem 1.3**[13]

(i) $\gamma_{ed}(C_3) = 3$.

(ii) $\gamma_{ed}(C_{3k+1}) = (3k+1)(k+2)/2$.

(iii) $\gamma_{ed}(C_{3k+2}) = 3k+2$.

**Theorem 1.4**[5]

$\gamma_{ed}(K_n) = 1$.

**Theorem 1.5**[5]

$\gamma_{ed}(K_{m, n}) = 2$.

**Theorem 1.6**[5]

$\gamma_{ed}(K_{1, n}) = 2$, $n \geq 2$.

**Theorem 1.7**[5]

$\gamma_{ed}(W_3) = 1$, $\gamma_{ed}(W_4) = 2$, $\gamma_{ed}(W_5) = 3$, $\gamma_{ed}(W_6) = 3$ and $\gamma_{ed}(W_n) = 3$ for $n \geq 7$.

**2. The eccentric dominating graph $ED_mG_{abc}(G)$ of a Graph $G$**

We define a new class of intersection graphs in the field of domination theory as follows.

**Definition 2.1**

Let $G$ be a graph with vertex set $V(G)$ and let $S$ be the set of all $\gamma_{ed}$-sets of $G$. Then two elements in $V'$ are said to satisfy property ‘a’ if $u \neq v \in V$ and are adjacent in $G$. Two elements in $V'$ are said to satisfy property ‘c’ if $u = D_1$, $v = D_2 \in S$ and have a common vertex. Two elements in $V'$ are said to satisfy property ‘e’ if $u \in V(G)$, $v = D \in S$ such that $u \in D$. A graph having vertex set $V' = V \cup S$, where $V = V(G)$ and $S$ is the set of all $\gamma_{ed}$-sets of $G$ and any two elements in $V'$ are adjacent if and only if they satisfy any one of the property $a$, $b$, $c$ is denoted by $ED_mG_{abc}(G)$. Here, the elements of $V(G)$ are called as point vertices and the elements of $S$ are known as set vertices.

**Remark 2.1**

(i) Total number of vertices in $ED_mG_{abc}(G)$ is $p + \gamma_{ed}(G)$.

(ii) Total number of edges in $ED_mG_{abc}(G)$ is $\leq q + \gamma_{ed}(G)$.

(iii) $G$ is an induced sub graph of $ED_mG_{abc}(G)$.

(iv) Number of edges in $ED_mG_{abc}(G) > q$.

(v) $deg_{ED_mG_{abc}(G)}(v) \leq deg_G v + \gamma_{ed}(G)$, $1 \leq i \leq p$. The equality holds when $v \in V(G)$ lie on all $\gamma_{ed}$-sets of $G$.

(vi) $Deg D_1 \leq \gamma_{ed}(G) + \gamma_{ed}(G) - 1$, $1 \leq j \leq n$. The equality holds when $D_1 \cap D_j \neq \phi$.

**Example**

Let $G = K_p$. Then $ED_mG_{abc}(G)$ is $K_{p, p}$ and $ED_mG_{abc}(G)$ is bi-eccentric with radius 2.

**Proof**

When $G = K_p$, each vertex form a $\gamma_{ed}$-set. By the definition, $ED_mG_{abc}(G)$ is $K_{p, p}$. The eccentricity of pendant vertices is 3 and the eccentricity of other vertices is 2. Hence, $ED_mG_{abc}(G)$ is bi-eccentric with radius 2.

**Theorem 2.2**

Let $G = K_{p, p}$, then $ED_mG_{abc}(G)$ is $K_{1, p}$.

**Proof**

When $G = K_{p, p}$, the whole vertex set is a $\gamma_{ed}$-set of $G$. By the definition, $ED_mG_{abc}(G)$ is $K_{1, p}$.

**Lemma 2.1**

(i) If $G = W_3$, then $\gamma_{ed}(G) = 4$.

(ii) If $G = W_4$, then $\gamma_{ed}(G) = 4$.

(iii) If $G = W_p$, then $\gamma_{ed}(G) = 15$ if $p = 5$.

(b) $\gamma_{ed}(G) = 3$ if $p = 6$.

c) $\gamma_{ed}(G) = 28$ if $p = 7$.

d) $\gamma_{ed}(G) = 28$ if $p = 8$.

e) $\gamma_{ed}(G) = 12$ if $p = 9$.

(f) $\gamma_{ed}(G) = p(p-3)/2$ if $p \geq 10$.

**Proof**

Let $G = W_p$. Then $ED_mG_{abc}(G)$ is $C_n + K_1$.

(i) $G = W_3 = K_4$. Hence $\gamma_{ed}(G) = 1$. Then it follows that, $\gamma_{ed}(G) = 4$.

(ii) When $G = W_4$, any two adjacent non-central vertices form $\gamma_{ed}$-set of $G$. Thus, we get four such $\gamma_{ed}$-sets. Hence, $\gamma_{ed}(G) = 4$.

(iii) $\gamma_{ed}(G)$.

Let $u_1$ be the central vertex of $G$. $D_1 = \{x, y, u_1\}$, where $x$ and $y$ are adjacent vertices in $C_n$ and $D_2 = \{x,$
y, z) where x, y and z are three consecutive vertices in C_n and D_1 = [x, y, z] where x and y are adjacent vertices and d(x, z) = d(y, z) = 2 in C_n form γ_d sets of G. Therefore, we get p + p + p = 3p such γ_d-sets of G. Hence, γ_{id}(G) = 3p = 15.

b) When p = 6. Let V = {v_1, v_2, v_3, v_4, v_5, v_6} be the vertices of C_6. D_1 = {v_1, v_4}, D_2 = {v_2, v_3} and D_3 = {v_5, v_6} are γ_d-sets of W_6. Hence, γ_{id}(G) = 3.

c) When p = 7 = 3k + 1. Let u_1 be the central vertex of G. D_1 = {x, y, u_1}, where d(x, y) ≠ 2 in C_n are γ_d-sets of G. Therefore, we get p(p−3)/2 γ_d-sets which contains u_1 and (3k+1)(k+2)/2 γ_d-sets which contain vertices of C_7, since γ_d(C_7) = (3k+1)(k+2)/2. By Theorem 1.3. Therefore, γ_{id}(G) = (3k + 1)(3k + 1 − 3) + (3k + 1)(k + 2) = 2k(3k+1) = 28.

d) When p = 8 = 3k + 2. Let u_1 be the central vertex of G. D_1 = {x, y, u_1}, where d(x, y) ≠ 2 in C_n are γ_d-sets of G. Therefore, we get p(p−3)/2 γ_d-sets which contains u_1 and (3k+2) γ_d-sets which contain vertices of C_8, since γ_d(C_8) = 3k+2, by Theorem 1.3. Therefore, γ_{id}(G) = (3k+2)(3k + 2 − 3)/2 + (3k+2) = (3k + 1)(3k + 2) = 28.

e) When p = 9 = 3k. Let u_1 be the central vertex of G. D_1 = {x, y, u_1}, where d(x, y) ≠ 2 in C_n. Therefore, we get p(p−3)/2 γ_d-sets which contains u_1 and 3 γ_d-sets which contains vertices of C_9, since γ_d(C_9) = 3, by Theorem 1.3. Therefore, γ_{id}(G) = (3k)(3k − 3)/2 + 3(3k − 1)/2 + 3(k − 2)/2 = 12.

f) When p ≥ 10. Let u_1 be the central vertex of G. D_1 = {x, y, u_1}, where d(x, y) ≠ 2 in C_n. Therefore, we get p(p−3)/2 γ_d-sets such that each γ_d-set contains central vertex u_1. Hence, γ_{id}(G) = p(p−3)/2.

**Theorem 2.3**

(i) If G = W_5, then ED_m G_{abc}(G) is K_1. K_1.
(ii) If G = W_6, then ED_m G_{abc}(G) is a 2-self-centered graph.
(iii) If G = W_p, p = 5, 7, 8 and 9, then ED_m G_{abc}(G) is a 2-self-centered graph. If p = 6, then ED_m G_{abc}(G) is bi-eccentric with radius 2.
(iv) If G = W_p, p ≥ 10, then ED_m G_{abc}(G) is of radius 1 and diameter 2.

**Proof**

(i) By Lemma 2.1, γ_{id}(G) = 1 and γ_{id}(G) = 4. Hence, by definition, ED_m G_{abc}(G) is K_1.K_1.
(ii) By Lemma 2.1, γ_{id}(G) = 4.

G:

![Graph](image)

**Fig 2.2**

D_1 = {v_1, v_2}, D_2 = {v_2, v_3}, D_3 = {v_3, v_4} and D_4 = {v_4, v_1} are γ_{id}-sets of G. Thus, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, ED_m G_{abc}(G) is a 2-self-centered graph.

(iii) a) When G = W_5. By Lemma 2.1, γ_{id}(G) = 15. Consider the following cases.

**Case (i)** Suppose u, v ∈ V and d_G(u, v) ≤ 2

Since G is an induced sub graph of G, in ED_m G_{abc}(G), d(u, v) = 1 or 2.

**Case (ii)** Suppose u ∈ V and v = D ∈ S

If u ∈ D, then in ED_m G_{abc}(G), d(u, v) = 1. If u ∉ D, then there exists a vertex u’ ∈ V such that u’ dominates u and u’ ∈ D, then it follows that in ED_m G_{abc}(G), d(u, v) = d(u, u’) + d(u’, D) = 2.

**Case (iii)** Suppose u, v ∈ S

If u and v have a vertex in common, then in ED_m G_{abc}(G), d(u, v) ≤ 1, otherwise d(u, v) = 2. Therefore, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, ED_m G_{abc}(G) is a 2-self-centered graph.

b) When G = W_6. By Lemma 2.1, γ_{id}(G) = 3. Consider the following cases.

**Case (i)** Suppose u, v ∈ V and d_G(u, v) ≤ 2

As in case (i) of (iii) a, d(u, v) ≤ 2 in ED_m G_{abc}(G).

**Case (ii)** Suppose u ∈ V and v = D ∈ S

As in case (ii) of (iii) a, d(u, v) ≤ 2 in ED_m G_{abc}(G).

**Case (iii)** Suppose u, v ∈ S

Set vertices are disjoint. Let u = D_1 and v = D_2 be two γ_{id}-sets of G. There exists some vertices of D_1 is adjacent to some vertices of D_2. Then in ED_m G_{abc}(G), uv, v_2v is a path. Therefore, d(u, v) = 3.

Therefore, eccentricity of point vertices is 2 and eccentricity of set vertices is 3. Hence, ED_m G_{abc}(G) is bi-eccentric with radius 2.

c) When p = 7, 8, 9, G has atleast two disjoint γ_d-sets.

Let u = D_1 and v = D_2 be two γ_d-sets of G. If D_1 and D_2 are disjoint, then there exists a γ_d-set D_3, such that D_3 is adjacent to both D_1 and D_2. In ED_m G_{abc}(G), d(D_1, D_2) = d(D_1, D_3) + d(D_3, D_2) = 2. Therefore, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, ED_m G_{abc}(G) is a 2-self-centered graph.

Therefore, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, ED_m G_{abc}(G) is a 2-self-centered graph.

(iv) When G = W_p, p ≥ 10. By Lemma 2.1, γ_{id}(G) = p(p−3)/2. Let u_1 be the central vertex of G, every γ_{id}-set contains the central vertex. Thus, eccentricity of central vertex is 1 in ED_m G_{abc}(G).
Eccentricity of point vertices except central vertex is 2 and eccentricity of set vertices is also 2. Hence, \( ED_n G^{abc}(G) \) is of radius 1 and diameter 2.

**Theorem 2.4**

If \( G = K_{m,n} \), then \( ED_n G^{abc}(G) \) is a 2-self-centered graph.

**Proof**

\[ G = K_{m,n}, \quad V(G) = V_1 \cup V_2; \quad |V_1| = m \text{ and } |V_2| = n. \]
\[ D = \{u, v\}, \quad u \in V_1 \text{ and } v \in V_2 \text{ are a } \gamma_{sc^{-}} \text{-sets of } G. \]

Then \( u \) and \( v \) are a \( \gamma_{sc^{-}} \)-set of \( G \). If \( D_1 = \{v_1, v_2\} \) and \( D_2 = \{u_1, u_2\} \) are two \( \gamma_{sc^{-}} \)-sets of \( G \). If \( D_1 \) and \( D_2 \) have a common vertex, then, in \( ED_n G^{abc}(G) \), \( d(D_1, D_2) = 1 \). Suppose \( D_1, D_2 \) are disjoint. Then there exists a \( \gamma_{sc^{-}} \)-set \( D_3 \) such that \( D_3 \) is adjacent to both \( D_1 \) and \( D_2 \). Then, in \( ED_n G^{abc}(G) \), \( d(D_1, D_3) = d(D_2, D_3) = 2 \). Thus, the eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence \( ED_n G^{abc}(G) \) is a 2-self-centered graph.

**Theorem 2.5**

If \( G = K_{1,n} \), then, \( K_{1,2n} \) and \( K_{n} \) are edge disjoint sub graphs of \( ED_n G^{abc}(G) \).

**Proof**

\[ G = K_{1,n}, \quad n \geq 3. \]
\[ D = \{u, v\}, \quad \text{where } v \text{ is the central vertex.} \]
\[ \text{The central vertex dominates all vertices in } V - D \text{ and } u \text{ is an eccentric vertex of } V - D. \]
\[ \text{Hence, } D \text{ is a } \gamma_{sc^{-}} \text{-set of } G. \]

We get \( n \) such \( \gamma_{sc^{-}} \)-sets. In \( ED_n G^{abc}(G) \), set vertices form a clique. Central vertex \( v \) is adjacent to \( n \) point vertices and \( n \) set vertices. These edges form \( K_{1,2n} \) and the remaining edges form \( K_{n} \). Hence, \( K_{1,2n} \) and \( K_{n} \) are edge disjoint sub graphs of \( ED_n G^{abc}(G) \).

**Theorem 2.6**

If \( G = P_{p,q} \), then

(i) \( ED_n G^{abc}(G) \) is of radius 2 and diameter 4, when \( p = 3k, 3k+1, k \geq 3 \).

(ii) \( ED_n G^{abc}(G) \) is bi-eccentric with radius 2, when \( p = 3k+2, k \geq 3 \).

(iii) \( ED_n G^{abc}(G) \) is 2-self-centered, when \( p = 3, 4 \) and 5.

**Proof**

(i) \( G = P_{p,q}, \quad p = 3k, 3k+1. \)

Two end vertices of \( G \) are eccentric vertices. Every \( \gamma_{sc^{-}} \)-set contains two end vertices. Hence, set vertices are adjacent to each other in \( ED_n G^{abc}(G) \).

Consider the following sub cases.

**case (i) Suppose } u, v \in V.**

If there exists a \( \gamma_{sc^{-}} \)-set \( D_1 \) such that \( D_1 \) contains \( u \) and \( v \), then in \( ED_n G^{abc}(G) \), \( d(u, v) = 2 \).

If \( u \in D_1 \) and \( v \in D_2 \), then there exists a vertex \( u_1 \) such that \( u_1 \) dominates \( u \) and \( u_1 \in D_2 \). Thus in \( ED_n G^{abc}(G) \), \( u_1D_2v \) is a path. Therefore, \( d(u, v) = 3 \).

If there is no \( \gamma_{sc^{-}} \)-set which contains \( u \) and \( v \), then there exists a \( \gamma_{sc^{-}} \)-set \( D_3 \) such that \( D_3 \) contains \( u \) and \( v \). Then in \( ED_n G^{abc}(G) \), \( uuD_3v \) is a path. Therefore, \( d(u, v) = 4 \).

**case (ii) Suppose } x \in V, y \in S, y = D_4 \text{ is the } \gamma_{sc^{-}} \text{-set of } G.**

If \( x \in D_4 \), then in \( ED_n G^{abc}(G) \), \( d(x, y) = 1 \).

If \( x \in D_4 \), then there exists a vertex \( x' \in V(G) \) such that \( x' \) dominates \( x \) and \( x' \in D_4 \). Then in \( ED_n G^{abc}(G) \), \( d(x', y') = 2 \).

Hence, \( rad(ED_n G^{abc}(G)) = 2 \).

**Case (iii) When } p = 3k+2.**

From Theorem 1.2, \( \gamma_{sc^{-}}(G) = k^2 + 3k + 2 \).

Hence, all vertices are ed-good.

**case (i) Suppose } u, v \in V.**

If there exists a \( \gamma_{sc^{-}} \)-set \( D_1 \) such that \( D_1 \) contains \( u \) and \( v \), then in \( ED_n G^{abc}(G) \), \( d(u, v) = 2 \).

If \( u \in D_1 \) and \( v \in D_2 \), then there exists a vertex \( u_1 \) such that \( u_1 \) dominates \( u \) and \( u_1 \in D_2 \). In \( ED_n G^{abc}(G) \), \( uuD_2v \) is a path. Therefore, \( d(u, v) = 3 \).

**case (ii) Suppose } x \in V, y \in S, y = D_4 \text{ is the } \gamma_{sc^{-}} \text{-set of } G.**

If \( x \in D_4 \), then in \( ED_n G^{abc}(G) \), \( d(x, y) = 1 \).

If \( x \notin D_4 \), then there exists a vertex \( x' \in V(G) \) such that \( x' \) dominates \( x \) and \( x' \in D_4 \). Then in \( ED_n G^{abc}(G) \), \( d(x, y) = d(x, x') + d(x', y) = 2 \).

Hence, \( rad(ED_n G^{abc}(G)) = 2 \).

**Corollary 2.7**

Let \( G = K_{m,n} \), where \( F \) is a 1-factor, \( m \) is even. Then \( ED_n G^{abc}(G) \) is Eulerian if \( n \) is even.

**Proof**

Number of vertices in \( ED_n G^{abc}(G) \) is \( n^2/2 \).

Degree of point vertex in \( ED_n G^{abc}(G) \) is \( n^2/2 \).

Degree of set vertex in \( ED_n G^{abc}(G) \) is \( n^2/2 - 1 \). All vertices have even degree in \( ED_n G^{abc}(G) \), since \( n \) is even. Then by Theorem 1.1, \( ED_n G^{abc}(G) \) is Eulerian.

**Theorem 2.8**

For any graph \( G \), \( ED_n G^{abc}(G) \) is connected.

**Proof**

Case (i) \( G \) is connected.

\( G \) is an induced sub graph of \( ED_n G^{abc}(G) \). If \( D \) is any \( \gamma_{sc^{-}} \)-set, then it is adjacent to some vertices of \( G \). Therefore, \( ED_n G^{abc}(G) \) is connected.
Case (ii) $G$ is disconnected.

Let $D$ be a $\gamma_d$-set of $G$. Then $D$ contains vertices from each component of $G$. In $ED_uG^{abc}(G)$, $D$ is adjacent to those vertices. Therefore, in $ED_uG^{abc}(G)$, any two vertices are connected by a path. Hence, $ED_uG^{abc}(G)$ is connected.

**Theorem 2.9**

$ED_uG^{abc}(G)$ is complete if and only if $G = K_1$.

**Proof**

Suppose $ED_uG^{abc}(G)$ is complete. Then $G$ is complete and each $\gamma_d$ -set contains all the vertices of $G$. That is, $G$ has exactly one $\gamma_d$-set. Hence, $G = K_1$. Conversely, $G = K_1$, by the definition, we get $ED_uG^{abc}(G) = K_2$. This implies that $ED_uG^{abc}(G)$ is complete.

**Theorem 2.10**

For any graph $G$, $ED_uG^{abc}(G)$ is a tree if and only if $G$ is $K_2$ or $K_3$. 

**Proof**

Suppose $ED_uG^{abc}(G)$ is a tree. Then $G$ has no cycle. To prove that $G$ is $K_2$ or $K_3$. On the contrary, suppose $G \neq K_2$ or $K_3$. Consider the following two cases.

Case (i) If $\Delta(G) = p - 1, p \geq 3$, then $G$ is a star. By Theorem 2.5, $ED_uG^{abc}(G)$ has a cycle, a contradiction.

Case (ii) If $\Delta(G) \leq p - 2$. Since $G$ is a tree, then there exists three vertices $u, v$ and $w \in V$ such that $u$ and $v$ are adjacent and $w$ is not adjacent to both $u$ and $v$ and is an eccentric vertex. This implies that, in $ED_uG^{abc}(G)$, $u$ and $v$ are connected by at least two paths, a contradiction. Hence from the above cases, $G = K_2$ or $K_3$. 

Conversely, Suppose $G = K_2$ or $K_3$. $ED_uG^{abc}(G)$ is $K_{3-p}$ or $P_4$. Hence, $ED_uG^{abc}(G)$ is a tree.

**Theorem 2.11**

(i) $\beta_2(ED_uG^{abc}(G)) \geq \max\{\beta_2(D), d_{ed}(G)\}$.

(ii) For any graph $G$, $\kappa(ED_uG^{abc}(G)) \leq \min \{\deg(ED_uG^{abc}(G)) \gamma_d(G) \}

(iii) For any graph $G$, $\lambda(ED_uG^{abc}(G)) \leq \min \{\deg(ED_uG^{abc}(G)) \gamma_d(G) \}

(iv) $\gamma(G) \leq \chi(G) \leq \gamma_d(G)$

**Proof**

(i) Proof is obvious.

(ii) Case (i) Let $v \in V$ and is of minimum degree among the all vertices of $ED_uG^{abc}(G)$. Then by deleting the vertices adjacent to $v$, the resulting graph is disconnected. Thus, $\kappa(ED_uG^{abc}(G)) \leq \min \{\deg(ED_uG^{abc}(G)) \gamma_d(G) \}

Case (ii) Let $S$ be the set of all $\gamma_d$-sets of $G$. Cardinality of each set is $\gamma_d(G)$. Suppose $\gamma_d(G) \leq \delta(G)$. Then by deleting the vertices adjacent to any one $\gamma_d$-set, the resulting graph is disconnected. Hence, $\kappa(ED_uG^{abc}(G)) \leq \min \{\deg(ED_uG^{abc}(G)) \gamma_d(G) \}

(iii) As in ii), $\lambda(ED_uG^{abc}(G)) \leq \min \{\deg(ED_uG^{abc}(G)) \gamma_d(G) \}$

(iv) Proof is obvious.

**Theorem 2.12**

For any graph $G$, distance between any two vertices in $ED_uG^{abc}(G)$ is at most four.

**Theorem 2.13**

Let $G$ be a connected graph with $rad(G) = 1$ and $diam(G) = 2$. Any central vertex lies on all the $\gamma_d$-set if and only if radius of $ED_uG^{abc}(G)$ is one.

**Proof**

Let $G$ be a connected graph with $rad(G) = 1$, $diam(G) = 2$ and let $u$ be any central vertex. Suppose $u$ lies on all the $\gamma_d$-sets of $G$. Then, in $ED_uG^{abc}(G)$, all the $\gamma_d$-sets are adjacent to each other and $\deg(u) = (p-1) + \gamma_d(G)$. Therefore, eccentricity of $u$ in $ED_uG^{abc}(G)$ is one. Since $G$ is connected $\gamma_d(G) \leq p - 1$, implies eccentricity of set vertices is not equal to one. Suppose there exists a vertex $u \in V$ such that $u$ is not in any $\gamma_d$-set, then also eccentricity of $u$ in $ED_uG^{abc}(G)$ is not equal to one. Therefore, $rad(ED_uG^{abc}(G)) = 1$ and if only if there exists $u \in V$ such that $u$ belongs to every $\gamma_d$-set of $G$.

**Theorem 2.14**

Let $G$ be a 2-self-centered graph. If $D_i \cap D_j \neq \phi$ for $i \neq j$, then $ED_uG^{abc}(G)$ is a 2-self-centered graph. Otherwise, $ED_uG^{abc}(G)$ is bi-ecentric with diameter three.

**Proof**

Let $G$ be a 2-self-centered graph. Let $u, v \in V'$. Consider the following cases.

Case (i) Suppose $u, v \in V$. Since $G$ is an induced sub graph of $ED_uG^{abc}(G)$, then, it follows that, in $ED_uG^{abc}(G)$, $d(u, v) = 2$.

Case (ii) Suppose $u \in V$ and $v \notin V$. Then $v = D$ is a $\gamma_d$-set of $G$.

If $u \in D$, then in $ED_uG^{abc}(G)$, $d(u, v) = 1$. If $u \notin D$, then there exists a vertex $w \in V$ such that $w$ dominates $u$ and $w \in D$. 

Proof

Suppose $G$ has at least two vertices then $ED_uG^{abc}(G)$ has at least three vertices. Let $u, v \in V'$. We consider the following cases.

Case (i) Suppose $u, v \in V$.

If $u$ and $v$ are adjacent in $G$, then in $ED_uG^{abc}(G)$, $d(u, v) = 1$. Suppose $u$ and $v$ are not adjacent in $G$.

Sub Case (i) In this case, there exists a $\gamma_d$-set containing $u$ and $v$. This implies that, in $ED_uG^{abc}(G)$, $d(u, v) = 2$.

Sub Case (ii) In this case, there exists a vertex $w$ such that $w$ is adjacent to both $u$ and $v$. Then, in $ED_uG^{abc}(G)$, $d(u, v) = d(u, w) + d(w, v) = 2$.

Sub Case (iii) $y = D$ is a $\gamma_d$-set of $G$. Suppose the vertices $w, x \in D$ are adjacent to $u$ and $v$ respectively, then in $ED_uG^{abc}(G)$, $d(u, v) = d(u, w) + d(w, v) = 2$.

Case (ii) Suppose $u \notin V$ and $v \in S, v = D$ is the $\gamma_d$-set of $G$.

If $u \in D$, then in $ED_uG^{abc}(G)$, $d(u, v) = 1$. If $u \notin D$, then there exists a vertex $w \in D$ adjacent to $u$ and hence in $ED_uG^{abc}(G)$, $d(u, v) = d(u, w) + d(w, v) = 2$.

Case (iii) Suppose $u \in S, u = D$ and $v = D'$ are two $\gamma_d$-sets of $G$.

If $D$ and $D'$ have a vertex in common, then in $ED_uG^{abc}(G)$, $d(u, v) = 1$.

If $D$ and $D'$ are disjoint. Consider the following sub cases.

Sub case (i) If there exists a $\gamma_d$-set $D''$ such that $D''$ is adjacent to both $D$ and $D'$. Thus, in $ED_uG^{abc}(G)$, $d(D, D') = d(D, D'') + d(D'', D') = 2$.

Sub case (ii) every vertex of $w \in D$ is adjacent to some vertex $x \in D'$ and vice versa. Thus, it follows that in $ED_uG^{abc}(G)$, $uvwx$ is a path. Therefore, $d(u, v) \leq 3$.

Hence, from the above cases, distance between any two vertices in $ED_uG^{abc}(G)$ is at most four.
Thus, it follows that, in $ED_nG^{abc}(G)$, $u$-$w$-$v$ is a path, $(d(u, v), d(v, w)) = (d(u, v), d(v, w)) ≤ 2$.

Case (iii) Suppose $u, v \in V$. Then $u = D_1$ and $v = D_2$ are two $\gamma_{ed}$-sets of $G$. If $D_1 \cap D_2 \neq \emptyset$ for $i \neq j$, then any two set vertices have a common vertex. Therefore, there exists a vertex $y \in V$ such that $y \in D_1$ and $D_2$. Thus, it follows that, in $ED_nG^{abc}(G)$, $u$-$y$-$v$ is a path, $(d(u, y), d(y, v)) = (d(u, y), d(y, v)) ≤ 2$.

Therefore, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, $ED_nG^{abc}(G)$ is a 2-self-centered graph.

Suppose $D_1 \cap D_2 = \emptyset$, for $i \neq j$. If $D_1$ and $D_2$ are disjoint. Then each vertex $x \in D_1$ is adjacent to some vertex $z \in D_2$ and vice versa. Thus, it follows that, in $ED_nG^{abc}(G)$, $u$-$x$-$z$-$v$ is a path, $(d(u, v), d(x, v)) = (d(u, x)+d(x, z), d(z, v)) ≤ 3$.

Hence, $ED_nG^{abc}(G)$ is bi-ecentric with diameter three.

**Theorem 2.15**

Let $G$ be a graph with diameter 3. Then $diam(ED_nG^{abc}(G))$ is 2 or 3.

**Proof**

Let $G$ be a graph with diameter 3 and let $u, v \in V'$. We consider the following cases:

1. **Case (i) Suppose $u, v \in V$.**
   
   If $u$ and $v$ are adjacent in $G$, then in $ED_nG^{abc}(G)$, $d(u, v) = 1$, since $ED_nG^{abc}(G)$ contains $\Gamma$. Suppose $u$ and $v$ are not adjacent in $G$.

   **Sub case (i) $d(u, v) = 2$.**
   
   a) There exists a $\gamma_{ed}$-set $D$ such that $D$ contains $u$ and $v$, then in $ED_nG^{abc}(G)$, $d(u, v) = 2$.

   b) There exists no $\gamma_{ed}$-set containing both $u$ and $v$, and $u, v$ belongs to some $\gamma_{ed}$-sets, then in $ED_nG^{abc}(G)$, $d(u, v) = 2$.

   c) $u$ and $v$ are not ed-good vertices, $(d(u, v), d(v, u)) = 2$ in $ED_nG^{abc}(G)$.

   **Sub case (ii) $d(u, v) = 3$.**
   
   a) There exists a $\gamma_{ed}$-set $D_1$ such that $D_1$ contains $u$ and $v$, then in $ED_nG^{abc}(G)$, $d(u, v) = 2$.

   b) $u \in D_1$ and $v \in D_2$. If there exists a vertex $w$ such that $w$ is adjacent to both $u$ and $v$, then in $ED_nG^{abc}(G)$, $d(u, v) = d(u, w)+d(w, v) = 2$.

   c) $u$ and $v$ are not ed-good vertices, $(d(u, v), d(v, u)) = 3$ in $ED_nG^{abc}(G)$.

2. **Case (ii) Suppose $u \in V$ and $v \in S$, $v = D_3$ is the $\gamma_{ed}$-set of $G$.**

   a) If $u \in D_3$, then in $ED_nG^{abc}(G)$, $d(u, v) = 1$. If $u \notin D_3$, then there exists a vertex $u'$ such that $u'$ dominates $u$ and $u' \in D_3$. It follows that, in $ED_nG^{abc}(G)$, $d(u, v) = d(u, u') + d(u', v) = 2$.

3. **Case (iii) Suppose $u, v \in S$, $u = D_4$ and $v = D_5$ are two $\gamma_{ed}$-sets of $G$.**

   a) If $D_1$ and $D_2$ have a common vertex, then in $ED_nG^{abc}(G)$, $d(u, v) = d(D_1, D_2) + d(D_1, D_2) = 1$.

   b) Suppose $D_1$ and $D_2$ are disjoint. Then there exists a $\gamma_{ed}$-set $D_6$ such that $D_6$ is adjacent to both $D_1$ and $D_2$. Then, it follows that, in $ED_nG^{abc}(G)$, $d(D_1, D_2) = d(D_1, D_2) + d(D_1, D_2) = 2$. If there does not exist, then every vertex $w \in D_1$ is adjacent to some vertex $w' \in D_2$ and vice versa. This implies that, in $ED_nG^{abc}(G)$, uwv'w' is a path. Therefore, $(d(u, v), d(v, u)) = 3$.

   Hence, diameter of $ED_nG^{abc}(G)$ is $2$ or $3$.

**Corollary 2.15**

(i) If $G = P_n$, then $ED_nG^{abc}(G)$ is $C_n$ which is 2-self-centered.

(ii) If $G = C_n$, then $ED_nG^{abc}(G)$ is 3-self-centered.

**Theorem 2.16**

Let $G$ be a graph with diameter greater than or equal to 4. Then $diam(ED_nG^{abc}(G)) ≤ 4$.

**Proof**

Let $G$ be a graph with $diam(G) ≥ 4$ and let $u, v \in V'$. Consider the following cases:

1. **Case (i) Suppose $u, v \in V$.**
   
   If $u$ and $v$ are adjacent in $G$, then it follows that, in $ED_nG^{abc}(G)$, $d(u, v) = 1$.

   Suppose $u$ and $v$ are not adjacent in $G$.

2. **Sub case (i) If there exists a vertex $x \in V$ such that $x$ is adjacent to both $u$ and $v$.**
   
   Then in $ED_nG^{abc}(G)$, $d(u, v) = d(u, x)+d(x, v) = 2$.

3. **Sub case (ii) Suppose $u$ and $v$ are eccentric vertices of $G$.**
   
   If there exists an $\gamma_{ed}$-set $D_1$ such that $D_1$ contains $u$ and $v$. Then it follows that, in $ED_nG^{abc}(G)$, $d(u, v) = d(D_1, D_1) + d(D_1, D_1) = 2$.

4. **Sub case (iii) $u \in D_2$, $v \in D_3$.**
   
   In this case, there exists a vertex $y \in V$ such that $y$ dominates both $u$ and $v$. It follows that, in $ED_nG^{abc}(G)$, $d(u, v) ≤ 3$.

5. **Sub case (iv) Suppose $u$ and $v$ are not ed-good vertices.**
   
   In this case, there exists vertices $u', v' \in V$ such that $u'$ dominates $u$, $v'$ dominates $v$ and $D_1$ contains $u'$ and $v'$. Then, it follows that, in $ED_nG^{abc}(G)$, $d(u, v) ≤ 4$.

**Case (ii) Suppose $u \in V$ and $v \in S$, $v = D_4$ is the $\gamma_{ed}$-set of $G$.**

If $u \in D_4$, then in $ED_nG^{abc}(G)$, $d(u, v) = 1$. If $u \notin D_4$, then there exists a vertex $z \in V$ such that $z$ dominates both $u$ and $z \in D_4$. It follows that, in $ED_nG^{abc}(G)$, $d(u, v) = d(u, z)+d(z, v) = 2$.

**Case (iii) Suppose $u, v \in S$, $u = D_5$ and $v = D_6$ are two $\gamma_{ed}$-sets of $G$.**

If $D_1$ and $D_2$ have a common vertex, then in $ED_nG^{abc}(G)$, $d(D_1, D_2) = 1$.

Suppose $D_1$ and $D_2$ are disjoint. Then there exists a $\gamma_{ed}$-set $D_7$ such that $D_7$ is adjacent to both $D_1$ and $D_2$. Then, it follows that, in $ED_nG^{abc}(G)$, $d(D_1, D_2) = d(D_1, D_2) + d(D_1, D_2) = 2$. If $D_1$ and $D_2$ are disjoint, then every vertex $w \in D_1$ is adjacent to some vertex $w' \in D_2$ and vice versa. This implies that, in $ED_nG^{abc}(G)$, uwv'w' is a path. Therefore, $(d(u, v), d(v, u)) = 3$.

So, $diam(ED_nG^{abc}(G)) ≤ 4$.

**Example:**

Consider $G = P_{10}$, then $diam(ED_nG^{abc}(G)) = 4$.

**Fig. 2.3**

**Theorem 2.17**

$ED_nG^{abc}(G)$ is self-centered with diameter 2 if $G$ is any one of the following:

1. **rad(G) = 1, diam(G) = 2 and any central vertex does not lie on any $\gamma_{ed}$-set of $G$.**

2. **G is self-centered with diameter 2 and $D_i \cap D_j \neq \emptyset$ for $i \neq j$.**
(iii) $G = K_m + K_1 + K_1 + K_n$, $m, n \geq 2$.

(iv) $G$ is a wounded spider with $k$ legs, $k > 2$ and one non-wounded leg.

**Proof**

(i) When $\text{rad}(G) = 1$, $\text{diam}(G) = 2$. Consider the following two cases. Let $u$ be the central vertex.

**Case (i) a)** Suppose $u \in D$, where $D$ is a $\gamma_{ed}$-set of $G$.

Let $v \in D$ be the $\gamma_{ed}$-set of $G$. If $u \notin D$, then there exists a vertex $w \in V$ such that $w$ dominates $u$ and $w \in D$. This implies that, in $ED_nG^{abc}(G)$, $d(u, v) = d(u, w) + d(w, v) = 2$.

Let $D_1$ and $D_2$ be two disjoint $\gamma_{ed}$-sets of $G$. Suppose vertices $x \in D_1$ and $x' \in D_2$, and there exists a $\gamma_{ed}$-set $D_3$ containing $x$ and $x'$. Thus, it follows that, $d(D_1, D_2) = d(D_1, D_3) + d(D_3, D_2) = 2$. If $D_1$, $D_2$ are not disjoint, $d(D_1, D_2) = 1$. Also, $\gamma(G) = 1$, $\text{diam}(G) = 2$ implies $\gamma_{ed}(G) < p - 1$. Hence, there exists $u \in V$ such that $d(u, D) = 2$ for any $D$. Hence, $ED_nG^{abc}(G)$ is self-centered with diameter 2.

**Case (ii)** Suppose the central vertex lies on some $\gamma_{ed}$-sets of $G$.

Let $D_1$ be the $\gamma_{ed}$-set of $G$. Suppose $u \notin D_1$, then there exists a $\gamma_{ed}$-set $D_2$ contains $u$ such that $D_1$ and $D_2$ are adjacent. This follows that, in $ED_nG^{abc}(G)$, $d(u, D_1) = d(u, D_2) = d(D_1, D_2) = 2$.

Let $w \in V$ and $D_1$ be the $\gamma_{ed}$-set of $G$. Suppose $w \notin D_1$, then there exists a vertex $w'$ adjacent to $w$ such that $D_1$ contains $w'$. This, it follows that, $d(w, D_1) = d(w, w') + d(w', D_1) = 2$. Hence, $ED_nG^{abc}(G)$ is self-centered with diameter 2.

**Case (iii)** $G$ is self-centered with diameter 2 and $D_1 \cap D_2 \notin \emptyset$ for $i \neq j$.

By Theorem 2.14, $ED_nG^{abc}(G)$ is self-centered with diameter 2.

(iii) When $G = K_m + K_1 + K_1 + K_n$, $m, n \geq 2$.

Let $u, v \in V$, $d_G(u, v) = 3$. Suppose the $\gamma_{ed}$-set $D$ contains $u$ and $v$. Then in $ED_nG^{abc}(G)$, $d(u, v) = d(u, D) + d(D, v) = 2$.

The two central vertices lie on all $\gamma_{ed}$-sets of $G$. Let $w$ be the pendant vertex and $D_1$ be the $\gamma_{ed}$-set of $G$. Suppose $w \notin D_1$, then there exists a $\gamma_{ed}$-set $D_2$ such that $D_1$ contains $w$. This follows that, in $ED_nG^{abc}(G)$, $d(w, D_1) = d(D_1, D_2) + d(D_2, w) = 2$. Hence, $ED_nG^{abc}(G)$ is self-centered with diameter 2.

(iv) Let $G$ be a wounded spider with $k$ legs, $k > 2$ and one non-wounded leg.

Let usv represent the non wounded leg, where $v$ is pendant, $u$ support vertex of the wounded legs. $D_1 = \{u, s, v\}$, $D_2 = \{u, w, v\}$ where $w$ is a pendant vertex of wounded leg are $\gamma_{ed}$-sets of $G$. Let $x, y, v \in V$, $d_G(x, y) = 3$. Then there exists a $\gamma_{ed}$-set $D$ such that $D$ contains $x, y$. Thus, in $ED_nG^{abc}(G)$, $d(x, y) = d(x, D) + d(D, y) = 2$. Every $\gamma_{ed}$-set contains $u$ and $v$. Thus, $\gamma_{ed}$-sets are adjacent to each other. Hence, in $ED_nG^{abc}(G)$, $d(D, D_1) = 1$. Suppose $x \notin D_1$. In $ED_nG^{abc}(G)$, $d(x, D_1) = d(x, u) + d(u, D_1) = 2$. Hence, $ED_nG^{abc}(G)$ is self-centered with diameter 2.

3. The Eccentric dominating graph $EDG^{abc}(G)$ of a graph $G$

**Definition 3.1**

The eccentric dominating graph $EDG^{abc}(G)$ of a graph $G$ is obtained from $G$ with vertex set $V' = V \cup S$, where $V = V(G)$ and $S$ is the set of all minimal eccentric dominating sets of $G$. Then two elements in $V'$ are said to satisfy property ‘$a$’ if $u, v \in V$ and are adjacent in $G$. Two elements in $V'$ are said to satisfy property ‘$b$’ if $u = D_1$, $v = D_2 \in S$ and $S$ is an adjacent in $G$. Two elements in $V'$ are said to satisfy property ‘$c$’ if $u \in V$, $v = D \in S$ such that $u \in D$.

Elements in $V'$ are said to satisfy property ‘$d$’ if $u, v \in V$ and there exists $D \in S$ such that $u, v \in D$. A graph having vertex set $V'$ and any two elements in $V'$ are adjacent if and only if they satisfy any one of the property $a$, $b$, $c$, $d$ is denoted by $EDG^{abc}(G)$.

**Example 3.1**

\[
[1, 3, 6], [1, 4, 6] \text{ and } [1, 2, 5, 6] \text{ are minimal eccentric dominating sets of } G.
\]

**EDG^{abc}(G)**

![Fig. 2.4]

**Remarks: 3.1**

(i) $G$ is an induced sub graph of $EDG^{abc}(G)$.

(ii) $ED_nG^{abc}(G)$ is a sub graph of $EDG^{abc}(G)$.

(iii) Number of vertices in $EDG^{abc}(G)$ is $p+\text{number of minimal eccentric dominating sets of } G$.

(iv) Number of edges in $EDG^{abc}(G)$ is greater than $q$.

(v) $\deg_{EDG^{abc}(G)} v_j = \deg_G v_j + S_j$, $1 \leq j \leq p$, where $S_j$ is the number of minimal eccentric dominating set containing $v_j$.

(vi) $\deg D_i \leq |D_i|S(S_j)-1/2$, $1 \leq i \leq n$, where $S$ is the set of all minimal eccentric dominating sets of $G$.

**Observation: 3.1**

If all $\gamma_{ed}$-sets of $G$ are minimal, then $EDG^{abc}(G) \cong ED_mG^{abc}(G)$.

**Theorem 3.1**

If $G = K_p$, then $EDG^{abc}(G) = ED_mG^{abc}(G)$. That is, $EDG^{abc}(G)$ is $K_p \cong K_1p$ and $EDG^{abc}(G)$ is of radius 2 and diameter 3.

**Proof**

Proof is similar to the proof of Theorem 2.1.

**Theorem 3.2**

If $G = K_{1,p-1}$, then $EDG^{abc}(G)$ is a 2-self-centered graph.

**Proof**

Proof is similar to the proof of Theorem 2.2.

**Theorem 3.3**

If $G = K_{1,p-1}$, then $EDG^{abc}(G)$ is a 2-self-centered graph.

**Proof**

Let $G = K_{1,p-1}, D = \{u, v\}$, where $u$ is the central vertex and $v$ is the non-central vertex of $G$ and all pendant vertices form minimal eccentric dominating sets of $G$. Thus, we get $p$ such minimal eccentric dominating sets of $G$. Let $x, y \in V'$. Consider the following cases.

**Case (i)** Suppose $x, y \in V$.

If $d_G(x, y) \leq 2$, then $G$ is an induced sub graph of $EDG^{abc}(G)$. Then in $EDG^{abc}(G)$, $d(x, y) \leq 2$.

**Case (ii)** $x \in V$ and $y \in S$, $y = D_1$ is the minimal eccentric dominating set of $G$.

If $x \in D_1$, then in $EDG^{abc}(G)$, $d(x, y) = 1$.

If $x \notin D_1$, then there exists a vertex $x' \in V$ such that $x'$ dominates $x$ and $x' \in D_1$. Thus, in $EDG^{abc}(G)$, $d(x, y) = d(x, x') + d(x', y) = 2$. 

**References:**

Case (iii) Suppose $x, y \in S, x = D_2, y = D_3$ are two minimal eccentric dominating sets of $G$.

In this case, set vertices are adjacent to each other. Then, it follows that, in $EDG^{abc}(G), d(x, y) = 1$.

Hence, $EDG^{abc}(G)$ is a 2-self-centered graph.

**Theorem: 3.4**

If $G = K_m$, then $EDG^{abc}(G)$ is a 2-self-centered graph. Here, $EDG^{abc}(G) = ED_mG^{abc}(G)$.

**Proof:**

Proof is similar to the proof of Theorem 2.4.

**Theorem: 3.5**

For any graph $G$, $EDG^{abc}(G)$ is connected.

**Proof:**

Proof is similar to the proof of Theorem 2.8.

**Theorem: 3.6**

$EDG^{abc}(G)$ is complete if and only if $G = K_1$.

**Proof:**

Proof is similar to the proof of Theorem 2.9.

**Theorem: 3.7**

$EDG^{abc}(G)$ is a tree if and only if $G$ is $K_p$ or $K_2$.

**Proof:**

Proof is similar to the proof of Theorem 2.10.

**Theorem: 3.8**

For any graph $G$, distance between any two vertices in $EDG^{abc}(G)$ is at most four.

**Proof:**

Let $u, v \in V$. Consider the following cases:

Case (i) $u, v \in V$.

If $u$ and $v$ are adjacent in $G$, then in $EDG^{abc}(G), d(u, v) = 1$.

Suppose $u$ and $v$ are not adjacent in $G$.

a) There exists a minimal eccentric dominating set containing $u$ and $v$. In $EDG^{abc}(G), d(u, v) = 2$.

b) $y = D$ is a $\text{gcd}$-set of $G$. Suppose the vertices $w, x \in D$ are adjacent to $u$ and $v$ respectively, then in $EDuG^{abc}(G), d(u,v) \leq d(u, w) + d(w, y) + d(y, x) + d(x, v) = 4$.

Case (ii) $u \in V$ and $v \in S$.

In this case, $v \in S$, thus $v = D$ is a minimal eccentric dominating set of $G$. If $u \not\in D$, then in $EDG^{abc}(G), d(u, v) = 1$.

If $u \in D$, then there exists a vertex $w \in D$ dominates $u$ and hence in $EDG^{abc}(G), d(u, v) = d(u, w) + d(w, v) = 2$.

Case (iii) $u \in S$.

In this case, $u = D_1$ and $v = D_2$ are two minimal eccentric dominating sets of $G$. If $D_1$ and $D_2$ are disjoint, then every vertex $z \in D_1$ is adjacent to some vertex in $x \in D_2$ and vice versa. Then it follows that, in $EDG^{abc}(G), uzxv$ is a path. Therefore, $d(u, v) \leq 3$. If there exists a minimal eccentric dominating set $D_3$ such that $D_1$ is adjacent to both $D_2$ and $D_3$. Then it follows that, in $EDG^{abc}(G), d(D_1, D_2) = d(D_1, D_3) = 2$.

If $D_1$ and $D_2$ have a vertex in common, then in $EDG^{abc}(G), d(D_1, D_2) = 1$.

Thus, from all the three cases, distance between any two vertices in $EDG^{abc}(G)$ is at most four.

**Conclusion**

In this paper, we have defined and studied the new eccentric dominating graphs $ED_uG^{abc}(G)$ and $EDG^{abc}(G)$.

**References**


