Certain class of graph with odd and even ratio edge antimagic Labeling

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ABSTRACT
In this paper the existence of odd and even ratio edge antimagic labeling for double triangular snakes ($2\Delta_k$-snake), $2m\Delta_1$-snake, $2m\Delta_2$-snake and $kC_4$-snake are proved.

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Introduction
The graphs considered here are finite, undirected and simple. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph $G$ respectively. Also $p$ and $q$ denote the number of vertices and edges of $G$ respectively. Rosa [7] defined a triangular snake (or $\Delta$-snake) as a connected graph in which all blocks are triangles and the block-cut-point graph is a path. Let $\Delta_k$-snake be a $\Delta$-snake with $k$ blocks while $n\Delta$-snake is a $\Delta$-snake with $k$ blocks and every block has $n$ number of triangles with one common edge.

Max-min edge antimagic labeling was first introduced by J.Jayapriya and D.Muruganandam in the year 2012, seeking application in Welding Technology [4]. In the year 2013[5] Jayapriya showed the existences of Max-min edge antimagic labeling for the graphs path, cycle, star, sunflower etc. Max-min edge antimagic labeling was renamed as ratio edge antimagic labeling [6]. In this paper the existence of odd and even ratio edge antimagic labeling, for double triangular snakes, $2m\Delta_1$-snake and $kC_4$-snake graphs are shown.

Definition 1.1 [4]: Let $G(V, E)$ be a simple graph with $p$ vertices and $q$ edges. A bijective function $f: V(G) \rightarrow \{1, 3, 5, ..., 2p-1\}$ is said to be odd ratio edge antimagic labeling if for every edge $uv$ in $E$, the edge weights

$$\lambda(uv) = \frac{\max\{f(u), f(v)\}}{\min\{f(u), f(v)\}}$$

are distinct.

Definition 1.2 [4]: Let $G(V, E)$ be a simple graph with $p$ vertices and $q$ edges. A bijective function $f: V(G) \rightarrow \{2, 4, 6, ..., 2p\}$ is said to be even ratio edge antimagic labeling if for every edge $uv$ in $E$,

$$\lambda(uv) = \frac{\max\{f(u), f(v)\}}{\min\{f(u), f(v)\}}$$

are distinct.

2. Some Class of Triangular Snake Graph with Odd and Even Ratio Edge Antimagic Labeling
Theorem 2.1: The double triangular snake ($2\Delta_k$-snake) graph admits odd and even ratio edge antimagic labeling.

Proof: Let $G(V, E)$ be a $2\Delta_k$-snake graph. The graph $G$ consists of the vertices $V = \{u_1, u_2, \ldots, u_k\} \cup \{v_1, v_2, \ldots, v_{k+1}\} \cup \{w_1, w_2, \ldots, w_k\}$ and the edges $E = \{u_iv_i : 1 \leq i \leq k\} \cup \{u_iv_{i+1} : 1 \leq i \leq k\} \cup \{v_iw_i : 1 \leq i \leq k\} \cup \{w_iv_{i+1} : 1 \leq i \leq k\}$. Let us consider the function $f: V(G) \rightarrow \{1, 3, 5, \ldots, 2p-1\}$, such that $f(u_i) = 2i-1; 1 \leq i \leq k$. $f(v_i) = 4k + (2i-1); 1 \leq i \leq k+1, f(w_i) = 2k + (2i-1); 1 \leq i \leq k$. Now the edge weights are calculated as follows.
For $1 \leq i \leq k$, $\lambda(u_i v_j) = \frac{4k + 2i - 1}{2i - 1}$. Clearly for $1 \leq i, j \leq k, i \neq j, \lambda(u_i v_i) \neq \lambda(u_j v_j)$, if $\lambda(u_i v_i) = \lambda(u_j v_j)$ then

$$\frac{4k + 2i - 1}{2i - 1} = \frac{4k + 2j - 1}{2j - 1}$$

which implies $i = j$, which is a contradiction. Therefore all edge labels are distinct.

For $1 \leq i \leq k$,

$$\lambda(u_{i+1} v_i) = \frac{4k + (2i + 1)}{2i - 1}.$$ Clearly for $1 \leq i, j \leq k, i \neq j, \lambda(u_i v_{i+1}) \neq \lambda(u_j v_j)$, if $\lambda(u_i v_{i+1}) = \lambda(u_j v_j)$ then

$$\frac{4k + (2i + 1)}{2i - 1} = \frac{4k + (2j + 1)}{2j - 1}$$

which implies $i = j$, which is a contradiction. Therefore all edge labels are distinct. For $1 \leq i \leq k$,

$$\lambda(u_i w_j) = \frac{4k + 2i - 1}{2k + 2i - 1}.$$ Clearly for $1 \leq i, j \leq k, i \neq j, \lambda(u_i w_i) \neq \lambda(u_j w_j)$, if $\lambda(u_i w_i) = \lambda(u_j w_j)$ then

$$\frac{4k + 2i - 1}{2k + 2i - 1} = \frac{4k + 2j - 1}{2k + 2j - 1}$$

which implies $i = j$, which is a contradiction. Therefore all edge labels are distinct.

For $1 \leq i \leq k$,

$$\lambda(v_{i+1} v_i) = \frac{4k + (2i + 1)}{4k + (2i - 1)}.$$ Suppose $\lambda(v_{i+1} v_i) = \lambda(v_j v_j)$ then

$$\frac{4k + (2i + 1)}{4k + (2i - 1)} = \frac{4k + (2j + 1)}{4k + (2j - 1)}$$

which implies $i = j$, which is a contradiction. Therefore all edge labels are distinct. For $1 \leq i \leq k$,

$$\lambda(w_{i+1} v_i) = \frac{4k + 2i + 2}{2i}.$$ Clearly for $1 \leq i, j \leq k, i \neq j, \lambda(w_i v_{i+1}) \neq \lambda(u_j v_{i+1})$, if $\lambda(w_i v_{i+1}) = \lambda(u_j v_{i+1})$ then

$$\frac{4k + 2i + 2}{2i} = \frac{4k + 2j + 2}{2j}$$

which implies $i = j$, which is a contradiction. Therefore all edge labels are distinct.

For $1 \leq i \leq k$,

$$\lambda(w_{i+1} w_i) = \frac{4k + 2i + 2}{2k + 2i}.$$ Clearly for $1 \leq i, j \leq k, i \neq j, \lambda(w_i v_{i+1}) \neq \lambda(v_j w_{i+1})$, if $\lambda(w_i v_{i+1}) = \lambda(v_j w_{i+1})$ then

$$\frac{4k + 2i + 2}{2k + 2i} = \frac{4k + 2j + 2}{2k + 2j}$$

which implies $i = j$, which is a contradiction. Therefore all edge labels are distinct.

For $1 \leq i \leq k$,

$$\lambda(v_{i+1} v_i) = \frac{4k + (2i + 2)}{2k + 2i}.$$ Clearly for $1 \leq i, j \leq k, i \neq j, \lambda(v_i v_{i+1}) \neq \lambda(v_j v_{i+1})$, if $\lambda(v_i v_{i+1}) = \lambda(v_j v_{i+1})$ then

$$\frac{4k + (2i + 2)}{2k + 2i} = \frac{4k + (2j + 2)}{2k + 2j}$$

which implies $i = j$, which is a contradiction. Therefore all edge labels are distinct.
For \(1 \leq i \leq k\), if \(i \neq j\), \(\lambda(v_{i}v_{i+1}) \neq \lambda(v_{j}v_{j+1})\). Suppose \(\lambda(v_{i}v_{i+1}) = \lambda(v_{j}v_{j+1})\) then \(\frac{4k + (2i + 2)}{2k + (2j)} = \frac{4k + (2j + 2)}{2k + (2j)}\) which implies \(i = j\), which is a contradiction. Therefore all edge labels are distinct. Thus the double triangular snakes \(2\Delta_{k}\)-snake graph admits even ratio edge antimagic labeling.

**Theorem 2.2:** \(2m\Delta_{1}\)-snake graph admits odd and even ratio edge antimagic labeling for \(m \geq 1\).

**Proof:** Let \(G(V,E)\) be a \(2m\Delta_{1}\)-snake graph.

The graph \(G\) consists of the vertex \(V = \{u_{1}, u_{2}\} \cup \{v_{1}, v_{2}, \ldots, v_{m}\}\) and the edges \(E = \{u_{1}u_{2}\} \cup \{u_{1}v_{i}^{'}, 1 \leq i \leq m\} \cup \{u_{i}w_{i+1}^{'}, 1 \leq i \leq m\} \cup \{u_{i}w_{i+1}^{'}, 1 \leq i \leq m\}\).

Let us consider the function \(f : V(G) \rightarrow \{1, 3, 5, \ldots, 2km+2k+1\}\) such that \(f(v_{i}^{'}) = 2i-1; 1 \leq i \leq m\).

\[
f(v_{i}^{'}) = 2m + (2i-1); 1 \leq i \leq m.
\]

\[
f(u_{1}) = 4m + 1 \text{ and } f(u_{2}) = 4m+3.
\]

Now the edge weights are calculated as follows.

For \(1 \leq i \leq m\), \(\lambda(u_{1}w_{i}^{'}) = \frac{4m+1}{2i-1}\).

\[
\lambda(u_{2}w_{i}^{'}) = \frac{4m+3}{2i-1}.
\]

\[
\lambda(u_{1}v_{i}^{'}) = \frac{4m+1}{2m + (2i-1)}.
\]

\[
\lambda(u_{2}v_{i}^{'}) = \frac{4m+3}{2m + (2i-1)}.
\]

Thus all edge labels are distinct. Hence \(2m\Delta_{1}\)-snake are odd ratio edge antimagic for \(m \geq 1\).

To prove the existence of even ratio edge antimagic labeling, let us define \(f : V(G) \rightarrow \{2, 4, 6, \ldots, 2p\}\), such that \(f(v_{i}^{'}) = 2i; 1 \leq i \leq m\).

\[
f(v_{i}^{'}) = 2m + 2i; 1 \leq i \leq m.
\]

\[
f(u_{1}) = 4m + 2 \text{ and } f(u_{2}) = 4m+4.
\]

Now the edge weights are calculated as follows.

For \(1 \leq i \leq m\), \(\lambda(u_{1}w_{i}^{'}) = \frac{4m+2}{2i}\).

\[
\lambda(u_{2}w_{i}^{'}) = \frac{4m+4}{2i}.
\]

\[
\lambda(u_{1}v_{i}^{'}) = \frac{4m+2}{2m + 2i}.
\]

\[
\lambda(u_{2}v_{i}^{'}) = \frac{4m+4}{2m + 2i}.
\]

Thus all edge labels are distinct. Hence \(2m\Delta_{1}\)-snake are even ratio edge antimagic for \(m \geq 1\).

**Theorem 2.3:** The \(2m\Delta_{e}\)-snake admits odd and even ratio edge antimagic labeling.

Proof. Let \(G(V,E)\) be a \(2m\Delta_{e}\)-snake graph. The graph \(G(V,E)\) consists of \(k(2m+1)+1\) vertices. Let \(V(G) = \{u_{1}, u_{2}, \ldots, u_{k+1}\} \cup \{v_{1}^{'}, v_{2}^{'}, \ldots, v_{m}^{'}\} \cup \{v_{1}^{''}, v_{2}^{''}, \ldots, v_{m}^{''}\} \cup \{w_{1}^{'}, w_{2}^{'}, \ldots, w_{m}^{'}\} \cup \{w_{1}^{''}, w_{2}^{''}, \ldots, w_{m}^{''}\}\) and edges as \(E(G) = \{v_{i}u_{i}; 1 \leq i \leq m\} \cup \{v_{i}^{'}u_{i+1}; 1 \leq i \leq m, 2 \leq k \leq m\} \cup \{v_{i}^{''}u_{i+1}; 1 \leq i \leq m, 2 \leq k \leq m\} \cup \{w_{i}^{'}u_{i}; 1 \leq i \leq m, 2 \leq k \leq m\} \cup \{w_{i}^{''}u_{i}; 1 \leq i \leq m, 2 \leq k \leq m\}\)

To prove that \(2m\Delta_{e}\)-snake admits odd ratio edge antimagic labeling let us define, \(f : V(G) \rightarrow \{1,3,5, \ldots, 4km+2k+1\}\), such that
\[ f(v_i') = 2m(j-1)+(2j-1); \ 1 \leq j \leq k, \ 1 \leq i \leq m. \]
\[ f(w_i') = 2m(k+j-1)+(2j-1); \ 1 \leq j \leq k, \ 1 \leq i \leq m. \]
\[ f(u_i) = 4km+(2j-1); \ 1 \leq j \leq k+1. \]

The edge weights are calculated as follows:

For \( 1 \leq j \leq k-1, \ 1 \leq i \leq m \), \( \lambda(u_{ij}) = \frac{4km+2j-1}{2m(j-1)+(2i-1)} \).

This implies \( 2 = 0 \), which is a contradiction. Thus edge labels are distinct.

For \( 1 \leq j \leq k, \ 1 \leq i \leq m \), \( \lambda(u_{ij}) = \frac{4km+2j-1}{2m(j-1)+(2i-1)} \).

This implies \( 2 = 0 \), which is a contradiction. Thus edge labels are distinct.

Therefore \( 2m\Delta_{s}-\text{snake graph} \) admits odd ratio edge antimagic labeling.

To prove the existence of even ratio edge antimagic labeling, let us define \( g : V(G) \to \{2, 4, 6, \ldots, 4km+2k+2\} \), such that \( g(v_i') = 2i; \ 1 \leq i \leq m \)
\[ g(v_i') = 2m(j-1)+2i; \ 1 \leq j \leq k, \ 1 \leq i \leq m. \]
\[ g(w_i') = 2m(k+j-1)+2i; \ 1 \leq j \leq k, \ 1 \leq i \leq m. \]
\[ g(u_i) = 4km+2j; \ 1 \leq j \leq k+1. \]

The edge weights are calculated as follows:

For \( 1 \leq j \leq k, \ 1 \leq i \leq m \), \( \lambda(u_{ij}) = \frac{4km+2j}{2m(j-1)+2i} \).

This implies \( 2 = 0 \), which is a contradiction. Thus edge labels are distinct.

For \( 1 \leq j \leq k, \ 1 \leq i \leq m \), \( \lambda(u_{ij}) = \frac{4km+2j}{2m(k+j-1)+2i} \).

This implies \( 2 = 0 \), which is a contradiction. Thus edge labels are distinct.

Therefore \( 2m\Delta_{s}-\text{snake graph} \) admits even ratio edge antimagic labeling.

**Theorem 2.4**: The \( kC_{r}-\text{snake graphs} \) are odd and even ratio edge antimagic labeling.
Proof. Let \( G(V, E) \) be a \( kC_r \)-snake graph where \( k \geq 1 \). This graph has \( 3k+1 \) vertices and \( 4k \) edges. Let 
\[
V(G) = \{w_i; 1 \leq i \leq k+1\} \cup \{u_l; 1 \leq l \leq k\} \cup \{v_i; 1 \leq i \leq k\},
\]
\[
E(G) = \{w_iw_{i+1}; 1 \leq i \leq k\} \cup \{w_iu_l; 1 \leq i \leq k\} \cup \{w_{i+1}v_i; 1 \leq i \leq k\} \cup \{w_{i+1}u_l; 1 \leq i \leq k\}.
\]
To prove that \( kC_r \)-snake admits ratio edge antimagic labeling let us define, 
\[
f : V(G) \rightarrow \{1, 3, 5, \ldots, 6k+2\}
\]
such that 
\[
f(w_{i+1}) = 2i + 1; \quad f(w_i) = 4k + 2i - 1; \quad f(v_i) = 2i - 1; \quad f(u_l) = 2k + 2i - 1.
\]
The edge weights are calculated as follows:
\[
\lambda(w_{i+1}v_i) = 2i + 1,
\]
For \( 1 \leq i \leq k \), clearly \( \lambda(w_{i+1}v_i) \neq \lambda(w_{i}v_{i+1}) \) and \( i \neq j \).
\[
\lambda(w_{i+1}u_l) = 2k + 2i - 1,
\]
For \( 1 \leq i, j \leq k \), clearly \( \lambda(w_{i+1}u_l) \neq \lambda(w_{j+1}u_l) \) and \( i \neq j \).
\[
\lambda(w_{i}v_{i+1}) = \lambda(w_{j}v_{j+1}) \text{ then } \frac{4k + 2i - 1}{2i - 1} = \frac{4k + 2j - 1}{2j - 1}.
\]
This implies \( i = j \), which is a contradiction.
\[
\lambda(w_{i}u_{l}) = \lambda(w_{j}u_{j}) \text{ then } \frac{4k + 2i - 1}{2k + 2i - 1} = \frac{4k + 2j - 1}{2k + 2j - 1}.
\]
This implies \( i = j \), which is a contradiction.
\[
\lambda(w_{i+1}v_{j}) = \lambda(w_{j+1}v_{j}) \text{ then } \frac{4k + 2i + 1}{2k + 2i - 1} = \frac{4k + 2j + 1}{2k + 2j - 1}.
\]
This implies \( i = j \), which is a contradiction. Thus all edge labels are distinct.

To prove the existence of even ratio edge antimagic labeling, let us define 
\[
g : V(G) \rightarrow \{2, 4, 6, \ldots, 6k+2\},
\]
such that 
\[
g(v_i) = 2i; \quad 1 \leq i \leq k,
\]
\[
g(u_l) = 2k + 2i; \quad 1 \leq i \leq k,
\]
This graph has \( 3k+1 \) vertices and \( 4k \) edges. Let 
\[
V(G) = \{w_i; 1 \leq i \leq k+1\} \cup \{u_l; 1 \leq l \leq k\} \cup \{v_i; 1 \leq i \leq k\},
\]
\[
E(G) = \{w_iw_{i+1}; 1 \leq i \leq k\} \cup \{w_iu_l; 1 \leq i \leq k\} \cup \{w_{i+1}v_i; 1 \leq i \leq k\} \cup \{w_{i+1}u_l; 1 \leq i \leq k\}.
\]
The edge weights are calculated as follows:

For $1 \leq i \leq  k$, $\lambda(w_{V_i}) = \frac{4k + 2i}{2i}$.

For $1 \leq i, j \leq k$, clearly $\lambda(w_{V_i}) \neq \lambda(w_{V_j})$ and $i \neq j$.

If $\lambda(w_{V_i}) = \lambda(w_{V_j})$, then $\frac{4k + 2i}{2i} = \frac{4k + 2j}{2j}$. This implies $i = j$.

which is a contradiction. Thus all edge labels are distinct.

For $1 \leq i \leq k$, $\lambda(w_{V_{i+1}}) = \frac{4k + 2i}{2i + 2}$.

For $1 \leq i, j \leq k$, clearly $\lambda(w_{V_{i+1}}) \neq \lambda(w_{V_{j+1}})$.

If $\lambda(w_{V_{i+1}}) = \lambda(w_{V_{j+1}})$ then $\frac{4k + 2i}{2i + 2} = \frac{4k + 2j}{2j + 2}$. This implies $i = j$, which is a contradiction. Thus all edge labels are distinct.

For $1 \leq i \leq k$, $\lambda(w_{U_i}) = \frac{4k + 2i}{2k + 2i}$.

For $1 \leq i, j \leq k$, clearly $\lambda(w_{U_i}) \neq \lambda(w_{U_j})$ and $i \neq j$.

This implies $i = j$, which is a contradiction.

Thus all edge labels are distinct.

For $1 \leq i \leq k$, $\lambda(w_{U_{i+1}}) = \frac{4k + 2i}{2k + 2i + 2}$. Clearly $\lambda(w_{V_i}) \neq \lambda(w_{U_{i+1}})$, if $\lambda(w_{V_i}) = \lambda(w_{V_{i+1}})$ then $\frac{4k + 2i}{2i} = \frac{4k + 2i + 2}{2i}$ implies $2 = 0$.

which leads to a contradiction. Thus edge labels are distinct.

Clearly $\lambda(w_{U_i}) \neq \lambda(w_{V_{i+1}})$, if $\lambda(w_{U_i}) \neq \lambda(w_{V_{i+1}})$ then $\frac{4k + 2i}{2k + 2i} = \frac{4k + 2i + 2}{2k + 2i}$ implies $2=0$, which leads to a contradiction. Thus edge labels are distinct. Therefore $kC_r$-snake graph admits even ratio edge antimagic labeling.

Conclusion

Thus existence of odd and even ratio edge antimagic labeling, for some class of graph is proved.

References


