Input-Output Linearization of an Induction Motor Using SVM Method

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ABSTRACT
This paper presents the non-linear control of an induction motor (IM). The objective of nonlinear control is to control separately flux and the speed, several techniques of control are used for (IM), The technique of control oriented flux (FOC) which permits the decoupling between input and output variables, so (IM) is assimilate to continuous current motor, this method has a problem is how exactly oriented the axis d on the flux. However, feedback linearization amounts to cancelling the nonlinearities in a nonlinear system so that the closed-loop (CL) dynamics is in a linear form. A goal of feedback linearization is to control separately flux and the speed, the motor model is strongly nonlinear then it’s composed to the autonomous and mono-variables too under systems so every under system presented an independent loop of control for each variables is given. The space vector modulation [SVM] method gives a good tracking for the nonlinear control, SVM became a standard for the switching power converters and important research effort has been dedicated, tens of papers, research reports and patents were developed in the theory of space vector modulation.

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Introduction
Feedback linearization amounts to cancelling the nonlinearities in a nonlinear system so that the closed-loop (CL) dynamics is in a linear form. A goal of nonlinear control is to control separately flux and the speed, the motor model is strongly nonlinear then it’s composed to the autonomous and mono-variables too under systems, so every under system presented an independent loop of control for each variables is given.

Then the induction motors constitute a theoretically interesting and practically important class of non-linear systems. The control task is further complicated by the fact that induction motors are subject to unknown load disturbances and change in values of parameters during its operation. The control engineering community is faced then with the challenging problem of controlling a highly nonlinear system with varying parameters, where the regulated outputs, besides some of them being not measurable, are perturbed by an unknown disturbance signal [1-3]. The roots of vectorial representation-phase systems are presented in the research contributions of park [4]. They provided both mathematical treatment and a physical description and understanding of the drive transients even in the cases when machines are fed through electronic converters [5].

In early seventies, space vector theory was already widely used by industry and presents in numerous books. Stepina [6] and Serrano –libarnegaray [7] suggested that the correct designation for the analytical tool to analyzing electrical machines has to be space phasor instead of space vector. Space phasor concepts now days mainly used for current and flux analysis of electrical machines.

The paper is organized as follows: in section 2, we give the mathematical input-output model for the induction motor by differentiating the outputs with Lie derivative [8-11] and expressing all states and inputs in terms of these outputs [12], in section 3, we represented the feedback linearization of IM and in the section 4 we give the control of flux and speed of linear system, finally we give the results simulation and the conclusion. Simulation was given by the classic PWM, after by the SVM method.

Model of the Induction Motor
The state equations in the stationary reference frame of an induction motor can be writing as [12]:

\[ \begin{align*}
    X &= F(X) + GU \\
    Y &= H(X)
\end{align*} \]

\[ X = [i_a \beta, i_d \beta, \phi_a \beta, \phi_d \beta, \Omega] \]
The variables, which are controlled, are the flux $\phi_r$ and the speed $\Omega$.

$$\begin{align*}
Y(X) &= \begin{bmatrix} y_1(X) \\ y_2(X) \end{bmatrix} = \begin{bmatrix} h_1(X) \\ h_2(X) \end{bmatrix} = \begin{bmatrix} x_3^2 + x_4^2 \\ \Omega \end{bmatrix} = \begin{bmatrix} \phi_r^2 \\ \Omega \end{bmatrix}
\end{align*}$$

Feedback linearization of IM

Relative degree of the flux

$$h_1(x) = (\Phi_r^2 + \Phi_m^2)$$

$$l_1 h_1 = \frac{2}{T_r} \left[ M(\Phi_{ra} \Phi_{rb} + \Phi_{rb} \Phi_{ra}) - (\Phi_{ra}^2 + \Phi_{rb}^2) \right]$$

$$l_2 h_1 = \frac{4}{T_r^2} M \left( \Phi_{ra}^2 + \Phi_{rb}^2 \right) - \left( \frac{6M}{T_r} + \frac{2y}{T_r} \right) \left( \Phi_{ra}^2 \Phi_{sa} + \Phi_{rb}^2 \Phi_{sb} \right) + \frac{2M_0}{T_r} \left( \Phi_{ra} \Phi_{rb} \Phi_{sa} + \Phi_{rb} \Phi_{ra} \Phi_{sb} \right) + \frac{2M^2}{T_r} \left( i_{sa} + i_{sb} \right)$$

$$l_{g2} h_1 = 2R_s K_{\Phi r}$$

The degree of $h_1(x)$ is $r_1=2$.

Relative degree of speed

$$h_2(x) = \Omega$$

$$l_1 h_2 = -p M \left( \Phi_{ra} \Phi_{rb} - \Phi_{rb} \Phi_{ra} \right) - \frac{1}{T_r} \left( C_r - f \Omega \right)$$

$$l_{g1} l_1 h_2 = -p M \Phi_{rb}$$

$$l_{g1} l_2 h_2 = \left[ l_{g2} h_2 \Phi_{ra} - \Phi_{rb} \right]$$

$$l_{g2} l_2 h_2 = \frac{K}{T_r} \Phi_{ra}$$

The degree of $h_2(x)$ is $r_2=2$.

Global relative degree

The global relative degree is lower than the order $n$ of the system $r = r_1 + r_2 = 4 < n = 5$.

The system is siding partly linearized [13-14]

Decoupling matrix

The matrix defines a relation between the input (U) and the output (Y(X)) is given by the Expression (13).

$$\begin{bmatrix} \frac{d\phi_r}{dt} \\ \frac{d\phi_m}{dt} \end{bmatrix} = \begin{bmatrix} A(X) + D(X) \end{bmatrix} \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix}$$

Where

$$A(X) = \begin{bmatrix} L_1 h_1 & L_2 h_2 \end{bmatrix}^T$$

The decoupling matrix is:
The nonlinear feedback provide to the system a linear comportment input/output
\[
\begin{bmatrix}
u_s \\
u_p 
\end{bmatrix} = D(X)^{-1} \left[ -A(X) + \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \right]
\]  
(14)

Where
\[
\begin{bmatrix}
V_1 \\
V_2 
\end{bmatrix} = \begin{bmatrix}
\frac{d\hat{\theta}_f}{dt} \\
\frac{d\hat{\Omega}}{dt} 
\end{bmatrix}
\]

Control Flux and Speed of Linear System

The internal outputs \((V_1, V_2)\) are definite:
\[
\begin{align}
V_1 &= \frac{d^2\Phi_f}{dt^2} = -K_{11}(\Phi_f^2 - \Phi_{ref}^2) - K_{12}\left(\frac{d}{dt}\Phi_f^2 - \frac{d}{dt}\Phi_{ref}^2\right) + \frac{d^2\Phi_{ref}^2}{dt^2} \\
V_2 &= \frac{d^2\Omega}{dt^2} = -K_{22}(\Omega - \Omega_{ref}) - K_{12}\left(\frac{d}{dt}\Omega - \frac{d}{dt}\Omega_{ref}\right) + \frac{d^2\Omega_{ref}^2}{dt^2}
\end{align}
\]  
(15)  
(16)

The error of the track in \((CL)\) are:
\[
\begin{align}
\hat{e}_1 &= K_{12}e_1 + K_{11}e_1 = 0 \\
\hat{e}_2 &= K_{21}e_2 + K_{22}e_2 = 0
\end{align}
\]  
(17)  
(18)

With:
\[
\begin{align}
e_1 &= \Phi_f^2 - \Phi_{ref}^2 \\
e_{21} &= \Omega - \Omega_{ref}
\end{align}
\]

The coefficients \(K_{11}, K_{12}, K_{21}, K_{22}\) are choosing to satisfy asymptotic stability and excellent tracking.
\[
\begin{align}
V_1 &= -K_{11}(\Phi_f^2 - \Phi_{ref}^2) - K_{12}\frac{d}{dt}\Phi_f^2 \\
V_2 &= -K_{22}(\Omega - \Omega_{ref}) - K_{21}\frac{d}{dt}\Omega
\end{align}
\]  
(19)  
(20)

\[
\begin{bmatrix}
u_s \\
u_p 
\end{bmatrix} = D(X)^{-1} \left[ -A(X) + (-K_{11}e_1 - K_{12}\frac{d}{dt}\Phi_f^2 - K_{22}e_2 - K_{21}(C_r + f\Omega)) \right]
\]  
(21)

**Figure 1. Schema block of Non-Linear control**

**SVM Method**

Any three-phase system (defined by \(a_x(t), a_y(t), a_z(t)\)) can be represented uniquely by a rotating vector as:
\[
\begin{bmatrix} a_x(t) \\
a_y(t) \\
a_z(t) \end{bmatrix} = \begin{bmatrix} L_{x1} \\
L_{x2} \\
L_{x3} \end{bmatrix}
\]

where
\[
\begin{align}
a &= e^{\frac{2\pi}{3}t} \\
a^2 &= e^{\frac{4\pi}{3}t}
\end{align}
\]

Given a three-phase system, the vectorial representation is achieved by the following 3/2 transformation:
\[
\begin{bmatrix} A_x \\
A_y \\
A_z \end{bmatrix} = \begin{bmatrix} 1 & -1/2 & -1/2 \\
\sqrt{3} & -\sqrt{3} & 2 \\
0 & -\sqrt{3} & -2 \end{bmatrix} \begin{bmatrix} a_x \\
a_y \\
a_z \end{bmatrix}
\]
Table 1. Reality table of the inverter

<table>
<thead>
<tr>
<th>Vector</th>
<th>Sa</th>
<th>Sb</th>
<th>Sc</th>
<th>V_sα</th>
<th>V_sβ</th>
<th>V_sγ</th>
<th>Vector V_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>V_1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>V_2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-E/3</td>
<td>E/3</td>
<td>2E/3</td>
<td>- \sqrt{1/6}E</td>
</tr>
<tr>
<td>V_3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-E/3</td>
<td>2E/3</td>
<td>-E/3</td>
<td>- \sqrt{1/6}E</td>
</tr>
<tr>
<td>V_4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-2E/3</td>
<td>E/3</td>
<td>E/3</td>
<td>- \sqrt{2/3}E</td>
</tr>
<tr>
<td>V_5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2E/3</td>
<td>-E/3</td>
<td>-E/3</td>
<td>- \sqrt{2/3}E</td>
</tr>
<tr>
<td>V_6</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>E/3</td>
<td>E/3</td>
<td>E/3</td>
<td>\sqrt{2/3}E</td>
</tr>
<tr>
<td>V_7</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>E/3</td>
<td>-2E/3</td>
<td>-2E/3</td>
<td>\sqrt{2/3}E</td>
</tr>
</tbody>
</table>

Figure 3. Switching vectors corresponding to the unmodulated operation of the inverter

Simulation results
Simulation results without SVM

Figure 4. Stator current [A]

Figure 5. Speed rotor [rad/s]
Figure 6. Electromagnetic Torque [Nm]

Figure 7. Rotor flux [Wb]

Figure 8. Speed rotor [Rad/sec]

Figure 9. Stator current [A]

Figure 10. Stator current

Simulation results with SVM
Simulations Results

To confirm the performances of the proposed control with SVM, we present a series of simulations; The results are showing in figure. (4-9) represented the simulation without SVM, so response of speed for an echelon of 150(rad/s) is given in figure.4 and we can varied in the speed this is shown in figure.8. This variation give the variation in the current and the flux, the norm of flux remnants to 1[Wb] with several undulation and the same for the torque and the stator current. With associated SVM method we remarked that the undulation in current and flux lesser, a good response for the stator voltage, the stator current, and the flux rotor.

Conclusion

Non linear control gives an excellent decoupling between the flow and speed, is represented a very effective control with respect to the vector control, however the use of a natural PWM does not give correct answers to the flow velocity and power and even for the stator voltage, so the usefulness of SVM and improve these results after the decoupling between the flow and the speed is, assures, the results show that the SVM is very efficient. Then input-output control gives a good tracking for the speed with basing of its static and dynamic properties. The results show that the decoupling between the parameters of IM is excellent.

Specifications of the induction motor

1.1KW, 220/380V, 50Hz, 1500 rpm
Parameters of the induction motor
Rr=3.6Ω , J=0.015Kgm²,
Rs=8.0Ω , f = 0.005Nms
$L_p = 0.47H \quad P=2$

References


