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ABSTRACT
The objective of this present paper is to analyze the relational thinking process of the Junior High School Students with field-dependent and –independent cognitive styles in solving mathematical problems. Two students were chosen as the research subject, one with dependent- cognitive style, the other, independent cognitive style. The data were collected through in-depth interviews after the subjects solved mathematical problems. All data were video-recorded. The credibility of the data was obtained through meticulous or continuous/consistent observations and time triangulation and analyzed using a flow model consisting of three-path activities occurring simultaneously: data reduction, data presentation and conclusion drawing. The results of the analysis showed that the relational thinking process of the field dependent student in solving mathematical problems is done by making relation among and in the three core elements: understanding the problem, answering the questions and reviewing, meanwhile, the field independent student, besides building relation among and in the three core elements like the field dependent students, chose some strategies.

Introduction
Mathematics solving is one of the objectives in learning mathematics at school. It is due to the fact that in solving a problem, a student obtains the way of thinking, habits to do something carefully, high curiosity and self confidence in any situation and may apply his knowledge and problem solving skills in his daily life in general (Pimta, Tayruakham, Nuangchalerm, 2009; Depdiknas, 2006; NCTM, 2000). Moreover, it is also an integral part of all mathematical learnings (NCTM, 2000) and the base of all mathematical activities (Reys, Lindquist, Lambdin, Smith, &Suydam, 2001).

Skills and capability in solving mathematical problems should be taught to the students from the basic level. Moreover, problem solving skills are dependent upon many factors, among others, problems variation, the problems presentation to the problem solving and individual differences. Individual differences may refer to the differences in cognitive capability or cognitive styles representing the thinking patterns that regulate and control the way an individual processes and rationalizes information (Jonassen & Grabowski, 1993).

Hejný, Jirotková & Kratochvilová (2006) proposes an approach to problem solving using two strategies namely procedural meta-strategy and conceptual meta-strategy. A student who solves a problem using the conceptual meta-strategy is called relational thinking (Molina, Castro and Mason, 2008; Stephens, 2006). In mathematics, relational thinking depends on the students’ ability to think and to use any probability from the variations among the existing numbers in the problems posed. Moreover, the understanding of the mathematical objects (operations and symbols), characteristics and relations between the two are important matters in understanding mathematical structures (Waren, 2001; William & Cooper, 2001). This means that relational thinking plays an important role in mathematics since there are many mathematical basic ideas that contain relations among different representations of and operations among numbers, and other mathematical inter-objects (Molina, Castro and Ambrose, 2005) and also a good foundation to learn the formal algebra (Molina and Ambrose, 2008).


In their studies, they investigated the relational thinking when students are solving problems, the third stage from the four stages of problem solving proposed by Polya (1973) and Posamentier et al (2007). Baiduri (2014a) studied the profile of the relational thinking among elementary school students in solving mathematical problems viewed from gender and mathematical ability. The profile obtained is the relational thinking process of elementary school students in each Polya’s stages, but their relational thinking when solving problems as a whole and their cognitive style have not been explored yet. Characteristics of problem solvers, instead of the types or difficulties of the problems to solve, are the most important factors (McGinn and Boote, 2003; Phonapichat, Wongwanich, and Sujiva, 2014). As a result, it is important to study the process of problem solving on the basis of problem solvers’ cognitive styles. Moreover, from some literature it is necessary to study the field dependent–independent cognitive styles of the students in solving mathematical problems.

Relational Thinking in Mathematical Problem Solving
When solving the problem $34 + x = 36 + 70$, students can employ some strategies: (1) using the computation of addition at the right side, then reducing the results with 34, therefore the value of $x = 72$ is obtained; (2) transforming the right side into
(34 + 2) + 76 and using the associative nature of addition and making use of the relation in both sides where the number 34 exists, then \( x = 72 \) is got; and (3) making use of the relations in both sides at similar or almost similar parts, namely 34 and 26, then students may also relate \( x = 72 \) is found.

In strategy (1), students made some computations without any manipulation or transformation, while in strategies (2) and (3), they made some numerical manipulations and made use of relations among parts in both equation sides. Strategy (1) is called procedural meta-strategy, and strategies (2) and (3) are called conceptual meta-strategy (Hejný, Jirotková & Kratochvílová, 2006). Students solving problems using the conceptual meta-strategy are called relational thinking (Molina, Castro and Mason, 2008; Stephens, 2006; Carpenter and Franke, 2001) or expression analysis (Molina and Ambrose, 2008). Students may be called the ones with relational thinking if they evaluate any problems by making use of the conceptual-meta strategy. Therefore, relational thinking is to build a relation among elements in the problems or characteristics of the arithmetic operations.

Carpenter et al (2003) and Stephens, C.A (2006) state that a student is said to think rationally if: 1) he shows the sign "equal with" as a relation symbol; 2) he may focus the attention on the expression structure and 3) he may give a rationality in using a strategy to solve a problem that involves an operation in number. Baiduri (2014b) states that relational thinking is a process of building a relation among elements of information given (context), prior knowledge on the mathematical characteristics in solving any mathematical problems.

**Field Dependence-Independence Cognitive Styles**

Cognitive style is the basis for distinguishing individuals when they are interacting with their environment and is an important approach to understanding one’s thinking (Sternberg & Williams, 2002). The study of this cognitive style also helps us identify the potency of one’s preparation when planning educational programs and academic guidance (Ates & Catalogu, 2007).

The concepts field dependence and field independence are developed to differentiate two different learning cognitive styles (Witkin&Goodenough, 1979; Witkin, Moore, Goodenough, & Cox, 1977; Witkin, Oltsman, Raskin, & Karp, 1971). According to them, FD-I is defined as how one perceives a part of a field as something that is discreet from the field around it as a whole, instead of being planted in the field; how one sees something analytically. Field dependence–independence (FD-I) is a cognitive style documented in a well and popular fashion (Dillon & Gabbard, 1998). It is considered as one construct of the most heuristic cognitive style (Messick, 1996; Price, 2004; Sternberg, Grigorenko, & Zahn, 2008; Sternberg & Williams, 2002) and has shown its consistency in determining an academic professional (Guisande, Páramo, Soares, & Almeida, 2007), the way a teacher teaches (Evans, 2004) and types of interaction between a teacher and his/her students (Sarcaho, 2000).

Summerville (1999) describes the characteristics of FD-I as a dimension of global style VS articulation that reflects one’s degree in processing information resulting from the contextual field. FD students who are asked to identify a simple form of geometry planted in a complex form will take longer time than FI students, or FD students might not be able to do it anymore. It means that FD students will not respond visually and have greater difficulties in abstracting relevant information from visual (or textual) learning materials that support learning tasks (Liu & Reed, 1994; Lyons - Lawrence, 1994). FD students are more influenced by the prevailing field, so that they often fail to isolate the targeted information, since other information tends to disguise what they are looking for (Jonassen & Grabowski, 1993). FI students more succeed in isolating the information from the complex as a whole, analyzing ideas that become the parts of the composer, and reorganizing ideas into new configurations (Davis, 1991; Snowman & Biehler, 2003).

Whereas, FD students are more globally, factually and traditionally oriented in their minds. FD-I is also very important in science education and mathematical problem solving (Witkin et al. 1977; Witkin & Goodenough 1981; Alamolhodaei, 2002; Azari et al. 2013).

Based on the relational thinking process in problem solving and field dependent-independent cognitive styles, the objective of this present paper is to analyze the relational thinking process of field dependent-independent students in solving mathematical problems. Therefore, the research problems are formulated as follows:

1) How is the relational thinking process of Junior High School Students with field-dependent cognitive style in solving mathematical problems?
2) How is the relational thinking process of Junior High School Students with field-independent cognitive style in solving mathematical problems?

**Method**

The research subject was selected by administrating Group Embedded Figures Test (GEFT) developed by Witkin et al (1977) to determine the cognitive styles of junior high school students. On the basis of the test results, students were divided into two groups, namely an FD cognitive style if 0 ≤ test results < 9 and an FI cognitive style if 9 ≤ test results ≤ 18. Then, based on the cognitive styles, one subject with the low enough score was selected as the FD and the high one as the FI. This is intended to have subjects with significant differences.

There are two types of instruments used to collect the data, namely the main instrument (the researcher himself) and the supporting instrument that consists of 1) audiovisual recorder used to record activities during the research, 2) interview guide, to investigate the relational thinking of junior high school students in solving mathematical problems, 3) student’s work sheet of mathematics (TPM I and II) given to the chosen students to obtain data to answer the research problems. The mathematics tasks had been validated by the experts of mathematics education, of evaluation and education practitioners (teachers of mathematics). The instrument validation dealt with problems construction, materials and language used.

The mechanism of data collection, either in TPM I or II began by asking the subject to solve problems, followed by indepth interviews (semi structured-interviews). The data were video-recorded. To assure the credibility of the obtained data, the researcher made meticulous or continuous/consistent observations and time triangulation (Moleong, 2011; Sugiyono, 2011). Based on the credible data, an analysis using a flow model consisting of three-path activities occurred simultaneously was made: data reduction, data presentation and conclusion drawing (Miles & Huberman, 1992), namely the relational thinking process of junior high school field-dependent-independent students in solving mathematical problems.

**Relational Thinking Process of the Feld Dependent Students**

The relational thinking processes of the FD after being given mathematical problems are praying, understanding the problems, doing the problems, thanking God and rechecking the work. It is known from in-depth interviews after the FD finished doing the problems:
After you received the problems, what did you do?
FD I prayed, tried to understand the problems, did them, thanked God and rechecked the work.

Praying is related to asking directions from Allah SWT to be easy of answering the problems. Thanking God shows thanks from God that has given anything so that the FD may be able to do the work. Understanding the problem was made by reading them many times especially dealing with the question and information of the problem. Information of the problems are anything known (Polya, 1973). Doing the problem means answering it. When answering the problems, FD related it to what is known and the questions in the problem. Moreover, in answering the problem, FD chose an arithmetic operation in accordance with his understanding of it. It is known from the folllowing interview:

R What do you understand the problem?
FD The problem was read many times.

R What do you understand from the problem?
FD Questions a) and b), information in the problem.

R Why did you understand the problem?
FD To be able to do it.

R What did you do?
FD Answering questions.

R When answering questions, what did you do?
FD Scrutinizing the questions and the existing information.

R What else did you do?
FI Multiplying, adding.

Rechecking is related to the answer. Checking the answer is related to the problem and the result. The problem is related to the question and the result to the result of the work by recounting. The result of the work was examined by recounting at the arithmetic operation. It is stated in the following interview:

R After doing the problem, what did you do?
FD Thanking to God and rechecking it.

R What did you check?
FD The answer.

R What did you check from the answer?
FD The problem and the result.

R In the problem, what did you check?
FD The question.

R Oo, in the result, what did you check?
FD Whether the result is in line with the question.

R How did you check the result?
FD Recounting it.

Notes: R: Researcher; FD: Field Dependent subject

Based on the the fact, the relational thinking process of FD in solving problems consist of four main elements, namely the One Supreme God, understanding the problems, answering the questions, and rechecking the answer. Something dealing with God is praying and thanking to God, and two other important things are starting and ending a job. Understanding the problem is related to two important things namely understanding the given data (what is known) and what is asked. Understanding the problem is used to get the answer. Answering the question is related to what is understood from the problem and the selection of the arithmetic operation. It means that understanding the problem and answering the question is a to and fro relation. Recounting is related to some matters namely the problem (question), answer (result) and the used arithmetic operation. The relational thinking of FD in solving problems is presented in Figure 1.

Figure 1. The Relational thinking process of FD in solving problem

Notes: Gy : Given; Q: Question; MS: Scrutinizing the Problem; Job: Answer; Mo: Selecting the arithmetic operation; Oh: Arithmetic Operation; Hk: Recounting; related to

Relational Thinking Process of The Field Independent Students

The relational thinking process of FI after being given a mathematical problem is reading, understanding, and doing the problem and rechecking it. This is obtained from the indepth interview after FI finished doing it:

R After you received the problem, what did you do?
FI Reading and understanding it, reading the question and answering the problem.

R After answering the question, what did you do?
FI Correcting it.

Reading the problem is related to understanding and doing it. Understanding the problem is related to the questions and any information existing in the problem. Meanwhile, doing the problem is related to answering the question. It is explicitly stated from the quotation from the following interview:

R What did you read in the problem?
FI Information in the problem, the questions.

R Why did you read the problem?
FI Understanding and doing it.

R What did you understand in the problem?
FI Questions a) and b), then reading the information in the problem.

R What did you do?
FI Answering the questions.

Answering question is related to understanding the problem (information in the problem), the questions, and the way the arithmetic operation used. It is expressed in the following in-depth interview:

R Before doing the problem, what did you do?
FI understanding the problem, and the questions.

R What do you mean understanding the problem?
FI Reading any information in the problem.

R What else did you do?
FI How.

R How is the way?
FI Multiplying and adding.

R When doing the problem, what did you do?
FI Answering, the way of answering it, counting.

Correcting is related to the problem (questions) and answers. Checking the answer deals with the way of answering and the number (result). The way of correcting is made by
reading the problem, the answer to the question and recounting. It is stated in the following interview:

R After answering to the problem, what did you do?
FI Correcting
R What did you correct?
FI Problem/questions, answer
R What did you scrutinize in the answer?
FI Number, the way
R How did you correct it?
FI Reading the problem, answer to the question, recounting

Notes: R: Researcher; FI: Field Independent subject

On the basis of the fact, the relational thinking process of FI in solving problems consists of four main elements, namely reading the problem, doing the problem and re-checking it. Reading the problem is related to understanding and doing the problem. Understanding the problem deals with the questions and what is known (information in the problem) to answer the problem. Meanwhile doing the problem or answering the problem is related to understanding the problem (information in the problem), the questions, and the way and also the arithmetic operation used. The way means choosing any strategy used to answer the question, correcting deals with the problem (questions) and answers. Checking the answer is related to the the way and the number (result). Procedures of correction are made by reading the problem, answers to the questions and recounting. The relational thinking process of FI in solving problems is presented in Figure 2.

![Figure 2. The Relational thinking process of FI in solving problem](image)

**Discussion**

The relational thinking process of FD in solving problems is related to the One Supreme God, understanding the problem, answering the questions, and rechecking. On the basis of the national curriculum in Indonesia (K13), the main competences a student should possess after completing a course are religion competence (LI-1), social competence (LI-2), knowledge competence (LI-3) and skill competence (KI-4), FD in solving problems is related to KI-1 and KI-3 or K-4 (Kemenedikbud, 2013). KI is seen from praying and thanking activities made by FD after doing his work. Meanwhile, KI-3/4 deals with problem solving stages, namely understanding the problem, planning/choosing strategies, doing the work and rechecking (Polya, 1973; Posamentier at all, 2007). The stage of planning the solution was not done by FD. It is in line with Güçlü (2003) stating that problem solving is cognitive and affective processes covering the planning of various alternative ways and the planning and application of the proper way to solve any uncertainty.

Although in understanding problems FD had been able to determine what to know and to ask, the mental representation of his level of understanding includes in the surface component (Österholm, 2006; Van Dijk and Kintsch, 1983). It is seen from the result of his written work showing his difficulty in understanding important words in the problem so that he was wrong in solving the problem.

While the relational thinking process of FI when solving problems is merely related to the knowledge-skill competences (KI-3/4), namely understanding the problem, answering the question and re-checking the answer. In understanding the problem, FI rightly interpreted important words or phrases in the problem, though he was still difficult in solving the problem. This understanding is included at the textbase level (Österholm, 2006; Van Dijk and Kintsch, 1983). It is in line with Witkin et al. (1997), Threadgill-Sowder & Sowder (1982) and Threadgill-Sowder et al. (1985) stating that the cognitive aspect of FD-I deals with the capability in solving problem, where the FD-I students with high test scores generally showed better results in problem solving than those with low test scores. FD was difficult to separate information from the context given to FI in solving the same tasks (Guisande, Paramo, Tinajero & Almeida, 2007).

In the activities of answering, FI showed special ways or strategies (Posamentier at all, 2007) before selecting operation. It is not the same with FD, meaning that FI has done four stages of solving mathematical problems (Polya, 1973; Posamentier at all, 2007). The relation FD built when rechecking is related to three elements, the answer obtained, problem rechecking and arithmetic operation chosen. Whereas, FI related it to two elements, namely the answer and the problem checking. The arithmetic operation has not become the related element since FI has made a solution plan by choosing a strategy dealing with the selection of the arithmetic operation. Based on Pictures 1 and 2, the relation FI built in rechecking the answer is “richer” than that FD did.

**Conclusion and Future Researches**

Any capability in solving various problems is an important matter for each person to make hi/her to be able to play some roles in this complex and changing society. Solving mathematical problem is the heart in learning mathematics. Some capabilities in solving mathematical problems may be applied in solving any problems in the daily life. The relational thinking process of FD in solving mathematical problems is made by building relation in and among stages of problem solving, besides by relating it to the belief or religion he believes in. but, the stages of problem solving have not followed the stages of mathematical problem solving. Meanwhile, the relational thinking process of FI is made by building some relations in and among the main stages of problem solving, namely understanding the problem, selecting the strategy, doing the strategy and rechecking the answer (Polya, 1973; Posamentier, Jaye & Krulik, 2007).

Viewed from the relation built at the stages of problem solving, the relation built by FI is “richer” than that of FD. On the contrary, from the objective of learning mathematics, the relation built of FD is “richer” than that of FI. This result hopefully may give information to the developers of mathematics curriculum, the authors of mathematics books and learning activities made by the teachers to take into account aspects of religion and attitudes besides those of knowledge and skill.
Mathematical problems employed in this present study is the finding type of problems (Polya, 1973) on algebra and the subject is the junior high school students with field dependent-independent cognitive style. Therefore, any research on the relational thinking process is still open for proving any problems using subjects with different characteristics.

References


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