Simulation of Ultrasound Produced by the NXN Rectangular Transducer Phased Array to Treat Lung Tumors Using the Angular Spectrum Approach

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Introduction
Pressure generated by ultrasound phased arrays are calculated by superposing the pressure produced by individual transducer sources. There are many methods to get pressure inside biological media such as Rayleigh–Sommerfeld integral, the rectangular radiator method, and the spatial impulse response method. The analytical approaches for those methods are slow due to the large number of calculations [1]. The ASA rapidly calculates the pressure because this approach based on computing the pressure in a sequence of parallel planes. Then decomposes the diffracted wave into plane waves via the two dimensional 2D Fourier transform, propagates these components in the spatial frequency domain, and recovers the pressure in planes parallel to the input plane through the 2D inverse Fourier transform[2]. Section II discuss the soft ware used for analytical approach depending on the ASA. Section III contain the pressure and temperature distributions while section IV contain the conclusion.

Analytical Approach
The software used for the pressure calculation is performed on a 2.4 GHz Pentium 4 PC(1 G byte random access memory) running the Windows XP operating system. All routines are written in the C language, compiled by Microsoft VISUAL C/C__ Version 7.0, and called by MATLAB 7.1 as MEX files.

First the Fast Near field(FNM) Method to calculate the pressure field generated by a small planar array of rectangular transducers under continuous wave excitation will be discussed in the following steps.

General parameters of array in meters
a) width of the element. b) height of the element. c) Number of elements in the x direction. d) Number of elements in the y direction. e) edge-to-edge spacing in the x and y directions. f) Center to center spacing of each element.

Set up our coordinate grid
Now we need to enter the arguments that will be used a) The min and max arguments sets the boundaries of the calculation grid. b) The delta argument is the distance from each points to its nearest neighbor and most be in the form of a matrix.

Define the three layers lung media and its parameters
The layers are a) bolus b) skin. c) muscle. d) tumor.

Finally we calculate the near field pressure using the transducer and coordinate
The pressure can be calculated from [3].

\[ P(r) = P(k_x, k_y, z) = j \delta c U_d(k_x, k_y, z_0) H_0(k_x, k_y, \Delta z) \] (1)

Where \( z_0 \) is the parallel planes distance \( \Delta z = z - z_0 \), \( k_x \) and \( k_y \) are the transverse wave numbers, and \( k_x = \frac{2 \pi}{l_x} k_0 \) and \( k_y = \frac{2 \pi}{l_y} k_0 \). \( P_d(k_x, k_y, z_0) \) is the angular spectrum of the input pressure field \( p_d(x, y, z_0) \); i.e., \( P_d(k_x, k_y, z_0) \) is the 2D Fourier transform of \( p_d(x, y, z_0) \) with respect to \( x \) and \( y \), and \( U_d(k_x, k_y, z_0) \) is the 2D Fourier transform of the normal particle velocity on the radiator surface. \( P(k_x, k_y, z) \) is the angular spectrum of the pressure in a plane parallel to the source plane. The spectral propagator \( H_0(k_x, k_y, \Delta z) \) for an input particle velocity distribution is represented by

\[ H_0(k_x, k_y, \Delta z) = \left\{ \begin{array}{ll} \frac{k}{\sqrt{k_x^2 + k_y^2}} e^{-j(k_x x + k_y y)} & \text{for } k_x^2 + k_y^2 \leq k^2 \\frac{k}{\sqrt{k_x^2 + k_y^2}} e^{-j(k_x x + k_y y)} & \text{for } k_x^2 + k_y^2 > k^2 \end{array} \right. \] (2)

The power deposition due to the pressure field \( p(r) \) is given as

\[ Q_p(x,y,z) = \frac{\alpha}{\delta c} P(r) p^*(r) \] (3)

Where \( \alpha \) is the attenuation coefficients of layers, \( \delta c \) is the layers impedance. and the localized heat transfer in biological media is modeled by the bio heat transfer equation (BHTE),[4-5]

\[ k \nabla^2 T - W_P C_p (T-T_a) + Q_p = 0 \] (4)

where \( T = T(x, y, z, t) \) is the tissue temperature, \( T_a \) is temperature of the arterial blood, \( K \) is the thermal conductivity of tissue, and \( W_P \) and \( C_p \) are the perfusion rate and the specific heat of blood, respectively. Equation (4) is the steady state
BHTE, which models the temperature distribution under equilibrium conditions.

**Pressure and Temperature Distribution**

The numerical analysis using the ASA for 12X12 element 2D planar phased is depicted in this section. The dimension of each element is 0.5 mm(width) and 2 mm(height) rectangular transducer with a 7.5x10^{-4} m (\lambda/2) space between adjacent elements in both x and y directions. The array is located in the xy plane at z=0 cm and centered at the origin of the coordinate system. The z axis is coincident with the normal evaluated at the center of the array as shown in figure 1. The excitation frequency is 1 MHz, the speed of sound is 1500 m/s, and the attenuation coefficient is \alpha = 1 dB/cm/MHz. The array surface should be covered by a bolus layer and the array will be encapsulated by a thin film of the scope material.

The total extent of the array aperture is 1.5 cm width and 3 cm high. This array is put on the bronchoscope and inter the human mouth up to reach in front of lung tissues as shown in figure 2.

The initial evaluations of the pressure field generated by the 12X12 element phased array are performed in the place 1.5cmX3cm with an equal transverse extent in both the x and the y directions. With a sampling interval of 7.5x10^{-4} m (\lambda/2), the computational volume is discretized to a 273x273x161 point grid. Figure (3) shows the reference pressure field generated by this 12X12 element phased array in the y=0 plane. The array elements are phased such that a single focus is produced at (0,0,12) mm.

In addition, the proposed phased array can be used for multi foci points inside the tumor volume at z=12 cm as shown in figure 4. The resulting power deposition given by equation 3 provides the input to BHTE to get the temperature distribution as shown in figure 5 where the boundaries of the computational grid are maintained at 37°C, the blood perfusion is 8 kg/m^3/s, the thermal conductivity is 0.55 W/m °C, and the specific heat of blood is 4000 J/kg °C. The goal of each simulation is to elevate the temperature at the focus to 43°C for hyperthermia cancer therapy [5] or for targeted drug delivery.

**Conclusions**

The pressure and temperature distributions are obtained by the ASA. The computation time is less than that consumed by other method. The NXN phased array discussed in this paper is
used successfully to produce pressure leads to temperature with therapeutic values (45-55°C) efficient to treat tumors in lung. Finally, the phased array can be considered as an efficient non-invasive method for tumor treatments without surgery.

References