Steady Plane Couette Flow of Viscous in Compressible Fluid between two Porous Parallel Plates through Porous Medium with Magnetic Field

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ABSTRACT

In this paper we have investigated the steady plane Couette flow of viscous incompressible fluid between two porous parallel plates through porous medium with magnetic field. We have studied the velocity, average velocity, shearing stress, skin frictions, the volumetric flow, drag coefficients and stream lines.

Keywords

Steady Couette flow, Viscous parallel plates, Incompressible fluid, Porous medium, Magnetic field.

Introduction

Nomenclature

\( u = \) Velocity component along \( x \) – axis
\( v = \) Velocity component along \( y \) – axis
\( t = \) the time
\( \rho = \) The density of fluid
\( P = \) the fluid pressure
\( K = \) the thermal conductivity of the fluid
\( \mu = \) Coefficient of viscosity
\( \nu = \) Kinematic viscosity
\( Q = \) the volumetric flow


Formulation of the problem

Let us consider two infinite porous plates AB & CD separated by a distance 2 \( h \). The fluid enters in \( y \) - direction. The velocity component along \( x \) – axis is a function of \( y \) only. The motion of incompressible fluid is in two dimension and is steady then

\[ u = u(y), \quad w = 0, \quad \frac{\partial u}{\partial t} = 0 \]
The equation of continuity for incompressible fluid
\[
0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}
\]
Put \(w = 0\),
\[
0 = \frac{\partial u}{\partial x} \quad \& \quad 0 = \frac{\partial v}{\partial y}
\]
\(v\) is independent of \(y\) but motion along \(y\)-axis. So we can say \(v\) is constant velocity i.e. \(v = v_0\)
or
The fluid enters the flow region through one plate at the same constant velocity \(v_0\).

Also Navier-Stoke's equations for incompressible fluid in the absence of body force when flow is steady:
\[
\begin{align*}
V_0 \frac{du}{dy} &= -\frac{1}{\rho} \frac{dp}{dx} + v \frac{d^2 u}{dy^2} + \frac{v u}{K} + \frac{\sigma B_0^2 v u}{\mu} \quad \text{...... (1)} \\
-\frac{1}{\rho} \frac{dp}{dy} &= 0 \quad \text{.......... (2)}
\end{align*}
\]

Solution of the Problem
Equation (2) shows that the pressure does not depend on \(y\) hence \(p\) is a function of \(x\) only and so (1) reduces to
\[
\begin{align*}
\frac{dp}{dx} &= \rho \left[ v \frac{d^2 u}{dy^2} - v_0 \frac{du}{dy} + \frac{v u}{K} + \frac{\sigma B_0^2 v u}{\mu} \right] \\
\Rightarrow \frac{d^2 u}{dy^2} - \frac{v_0}{v} \frac{du}{dy} + \frac{u}{K} + \frac{\sigma B_0^2 u}{\mu} &= -\frac{P}{\mu} \\
\Rightarrow \left( D^2 - \frac{v_0}{v} D + \frac{1}{K} + \frac{\sigma B_0^2}{\mu} \right) u &= -\frac{P}{\mu}
\end{align*}
\]

\[\text{A.E} \quad m^2 - \frac{v_0}{v} m + 1 + \frac{\sigma B_0^2}{\mu} = 0 \quad \Rightarrow \quad m = \frac{\frac{v_0}{v} \pm \sqrt{\left( \frac{v_0}{v} \right)^2 - 4 \left( 1 + \frac{\sigma B_0^2}{\mu} \right)}}{2} = \frac{v_0}{2v} \pm \sqrt{\left( \frac{v_0}{2v} \right)^2 - \left( 1 + \frac{\sigma B_0^2}{\mu} \right)}
\]

\[\text{C.F.} = e^{\frac{v_0}{2v}} \left[ c_1 \cosh Ay + c_2 \sinh Ay \right] \quad \text{P.I.} = -\frac{P}{B \mu}
\]

Let \(A = \sqrt{\left( \frac{v_0}{2v} \right)^2 - \left( 1 + \frac{\sigma B_0^2}{\mu} \right)}\) and \(B = \frac{1}{K} + \frac{\sigma B_0^2}{\mu}\)

\[u(y) = e^{\frac{v_0}{2v}} \left[ c_1 \cosh Ay + c_2 \sinh Ay \right] - P \quad \frac{B \mu}{\text{...... (3)}}
\]
using boundary conditions:
\[u = 0 \quad \text{at} \quad y = -h \quad \text{and} \quad u = U \quad \text{at} \quad y = h
\]
\[e^{\frac{v_0}{2v}} \left[ c_1 \cosh Ah - c_2 \sinh Ah \right] - \frac{P}{B \mu} = 0 \quad \text{...... (3)}
\]
\[U = e^{\frac{v_0}{2v}h} \left[ c_1 \cosh Ah + c_2 \sinh Ah \right] - \frac{P}{B \mu} \quad \text{...... (4)}
\]
or
\[\frac{P}{B \mu} e^{\frac{v_0}{2v}h} = c_1 \cosh Ah - c_2 \sinh Ah
\]
\[
\left( U + \frac{P}{B\mu} \right) e^{-\frac{v_0}{2v}h} = c_1 \cosh Ah + c_2 \sinh Ah
\]

\[
c_1 = \frac{1}{2\cosh Ah} \left[ \left( U + \frac{P}{B\mu} \right) e^{-\frac{v_0}{2v}h} + \frac{v_0}{B\mu} e^{\frac{v_0}{2v}h} \right]
\]

\[
c_2 = \frac{1}{2\sinh Ah} \left[ \left( U + \frac{P}{B\mu} \right) e^{-\frac{v_0}{2v}h} - \frac{v_0}{B\mu} e^{\frac{v_0}{2v}h} \right]
\]

\[
u(y) = \frac{e^{\frac{v_0}{2v}y} \cosh Ay}{2\cosh Ah} \left[ \left( U + \frac{P}{B\mu} \right) e^{-\frac{v_0}{2v}h} + \frac{v_0}{B\mu} e^{\frac{v_0}{2v}h} \right] + \frac{\sinh Ay}{2\sinh Ah} \left[ \left( U + \frac{P}{B\mu} \right) e^{-\frac{v_0}{2v}h} - \frac{v_0}{B\mu} e^{\frac{v_0}{2v}h} \right] - \frac{P}{B\mu}
\]

Plane Couette flow: In this case \( P = 0 \)

\[
u(y) = \frac{1}{\sinh 2Ah} \left[ \frac{v_0}{2v} \sinh A(y + h) \right] \]

The shearing stress at any point

\[
\sigma_{xy} = \mu \frac{du}{dy} = \frac{\mu U}{\sinh 2Ah} \left[ \frac{v_0}{2v} \sinh A(y + h) + \frac{v_0}{2v} \cosh A(y + h) \right]
\]

\[
= \frac{\mu U e^{\frac{v_0}{2v}(y-h)}}{\sinh 2Ah} \left[ \frac{v_0}{2v} \sinh A(y + h) + ACoshA(y + h) \right] \]

The skin frictions at Lower and Upper plate is given by

\[
\left( \sigma_{xy} \right)_{y=h} = \frac{\mu U e^{\frac{-v_0}{2v}h}}{\sinh 2Ah} \left[ \frac{v_0}{2v} \sinh 2Ah + ACosh2Ah \right] = \mu U \left[ \frac{v_0}{2v} + ACoth2Ah \right] \]

\[
\left( \sigma_{xy} \right)_{y=-h} = \frac{\mu U e^{\frac{v_0}{2v}h}}{\sinh 2Ah} = \frac{\mu U e^{\frac{v_0}{2v}h}}{\sinh 2Ah} \]

The average velocity distribution in plane couette flow:

\[
\left( u \right)_{av} = \frac{1}{2h} \int_{-h}^{h} u(y) dy = \frac{1}{2h} \int_{-h}^{h} U \frac{v_0}{2v} e^{\frac{v_0}{2v}(y-h)} \sinh A(y + h) dy
\]

\[
= \frac{U}{2h \sinh 2Ah} \int_{-h}^{h} e^{\frac{v_0}{2v}(y-h)} \left\{ e^{A(y+h)} - e^{-A(y+h)} \right\} dy
\]

\[
= \frac{U}{4h \sinh 2Ah} \int_{-h}^{h} \left\{ e^{\frac{v_0}{2v}(y-h)+A(y+h)} - e^{\frac{v_0}{2v}(y-h)-A(y+h)} \right\} dy
\]

\[
= \frac{U}{4h \sinh 2Ah} \left[ \frac{v_0}{2v} + A \right] - \frac{v_0}{2v} - A
\]

\[
= \frac{U}{4h \sinh 2Ah} \left[ \frac{v_0}{2v} + A \right] - \frac{v_0}{2v} \left( \frac{v_0}{2v} - A \right)
\]
\[
\frac{U}{4h \sinh 2Ah} \left[ \left( \frac{v_0}{2v} \right)^2 - A^2 \right] = U \left[ \frac{v_0}{2v} \left( e^{2Ah} - e^{-\frac{v_0}{v}} \right) - \left( \frac{v_0}{2v} + A \right) \left( e^{-2Ah} - e^{-\frac{v_0}{v}} \right) \right] \]

Since \( A = \left( \frac{1}{K} + \frac{\sigma B^2}{\mu} \right) \Rightarrow \left( \frac{v_0}{2v} \right)^2 - A^2 = \left( \frac{1}{K} + \frac{\sigma B^2}{\mu} \right) = B \)

\[
(u)_{av} = \frac{U}{4Bh \sinh 2Ah} \left[ \frac{v_0}{2v} \left( e^{2Ah} - e^{-\frac{v_0}{v}} - e^{-2Ah} + e^{\frac{v_0}{v}} \right) - A \left( e^{2Ah} - e^{\frac{v_0}{v}} + e^{-2Ah} - e^{-\frac{v_0}{v}} \right) \right] = \frac{U}{2Bh \sinh 2Ah} \left[ \frac{v_0}{2v} \sinh 2Ah - A \left( \cosh 2Ah - e^{-\frac{v_0}{v}} \right) \right] \quad \text{......... (10)}
\]

The volumetric flow:
\[
Q = 2h u_{av} = U \left[ \frac{v_0}{2v} \sinh 2Ah - A \left( \cosh 2Ah - e^{-\frac{v_0}{v}} \right) \right] \quad \text{......... (11)}
\]

The drag coefficients:
\[
C_f = \left( \frac{\sigma_{xy}}{\mu u^2} \right)_{y=h} = \frac{\mu U}{2 \rho u^2_{av}} \left[ \frac{v_0}{2v} \sinh 2Ah + ACosh 2Ah \right] \frac{8B^2h^2 \mu \sinh 2Ah \left[ \frac{v_0}{2v} \sinh 2Ah + ACosh 2Ah \right]}{\rho U^2 \left[ \frac{v_0}{2v} \sinh 2Ah - ACosh 2Ah + Ae^{-\frac{v_0}{v}} \right]^2} \quad \text{......... (12)}
\]

\[
C'_f = \left( \frac{\sigma_{xy}}{\mu u^2} \right)_{y=-h} = \mu U a e^{-\frac{v_0}{v}} \frac{8B^2h^2 \sinh^2 2Ah}{\rho U^2 \left[ \frac{v_0}{2v} \sinh 2Ah - ACosh 2Ah + Ae^{-\frac{v_0}{v}} \right]^2} \quad \text{......... (13)}
\]

The stream line in the plane couette flow:
\[
\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}
\]

\[
\frac{1}{U} \frac{v_0}{2v} e^{2Ah} \sinh (y+h) = \frac{dy}{v_0} = \frac{dz}{\sigma}
\]
Taking Ist two equations

\[
\begin{align*}
\frac{v_0 \sinh 2Ah}{U} \int dx &= \int e^{\frac{v_0}{2}\left(y-h\right)} \sinh A(y + h) \, dy + C_1 \\
\frac{v_0 \sinh 2Ah}{U} x - \int e^{\frac{v_0}{2}\left(y-h\right)} \left\{ \frac{e^{A(y+h)} - e^{-A(y+h)}}{2} \right\} \, dy &= C_1 \\
x \frac{v_0}{U} \sinh 2Ah &= \frac{1}{2} \int \left\{ \frac{v_0}{2}e^{\frac{v_0}{2}\left(y-h\right)+A(y+h)} - \frac{v_0}{2}e^{\frac{v_0}{2}\left(y-h\right)-A(y+h)} \right\} \, dy = C_1
\end{align*}
\]

\[
\begin{align*}
\frac{v_0}{U} \sinh 2Ah &- \frac{1}{2B} \left[ \left( \frac{v_0}{2} - A \right) e^{\frac{v_0}{2}\left(y-h\right)} e^{A(y+h)} - \left( \frac{v_0}{2} + A \right) e^{\frac{v_0}{2}\left(y-h\right)} e^{-A(y+h)} \right] = C_1 \\
\frac{v_0}{U} \sinh 2Ah &- \frac{e^{\frac{v_0}{2}\left(y-h\right)}}{2B} \left\{ \frac{v_0}{2} e^{A(y+h)} - e^{-A(y+h)} \right\} = C_1 \\
\frac{v_0}{U} \sinh 2Ah &- \frac{e^{\frac{v_0}{2}\left(y-h\right)}}{B} \left\{ \frac{v_0}{2} \sinh A(y + h) - 2A \cosh A(y + h) \right\} = C_1 \\
\frac{v_0}{U} \sinh 2Ah &- \frac{e^{\frac{v_0}{2}\left(y-h\right)}}{B} \left\{ \frac{v_0}{2} \sinh A(y + h) - A \cosh A(y + h) \right\} = C_1 \\
z &= C_2 \quad \text{(15)}
\end{align*}
\]

Now the curl

\[
\begin{align*}
q &= \begin{vmatrix}
  \frac{i}{u} & \frac{j}{v} & \frac{k}{w} \\
  \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
  i & j & k
\end{vmatrix} \\
&= \begin{vmatrix}
  \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
  U e^{\frac{v_0}{2}\left(y-h\right)} \sinh A(y + h) & v_0 & 0
\end{vmatrix}
\end{align*}
\]

\[
\begin{align*}
- \frac{U e^{\frac{v_0}{2}\left(y-h\right)}}{\sinh 2Ah} \left[ \frac{v_0}{2} \sinh A(y + h) + ACosh A(y + h) \right] \hat{k} \neq \bar{0}
\end{align*}
\]

\[
\therefore \text{ Motion of fluid is rotational}
\]

Table for velocity:

\[
U = 6, \quad \mu = .5, \quad h = .5, \quad v_0 = 6, \quad \sqrt{\left( \frac{v_0}{2} \right)^2 - \left( \frac{1}{K} \right)^2 \frac{\sigma B_0^2}{\mu}} = 2, \quad \text{When } \frac{1}{K} = \frac{\sigma B_0^2}{\mu} = 16
\]

\[
\therefore \sqrt{\left( \frac{v_0}{2} \right)^2 - \frac{1}{K}} = \sqrt{20} = A
\]
### Table 1

<table>
<thead>
<tr>
<th>( y )</th>
<th>0</th>
<th>.1</th>
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<td>( \frac{1}{K} = 16 )</td>
<td>( u(y) )</td>
<td>.032</td>
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<td>( \frac{1}{K} + \frac{\sigma B_0^2}{\mu} = 32 )</td>
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</table>

### Table for Skin friction

\[ U = 6, \quad \mu = .5, \quad h = .5, \quad \frac{v_0}{2\nu} = 6, \quad \sqrt{\frac{v_0}{2\nu}^2 - \left( \frac{1}{K} + \frac{\sigma B_0^2}{\mu} \right)} = 2, \quad \text{When} \quad \frac{1}{K} = \frac{\sigma B_0^2}{\mu} = 16 \]

\[ \therefore \sqrt{\frac{v_0}{2\nu}^2} = \frac{1}{K} = \sqrt{20} = A \]

### Table 2

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<td>( \frac{1}{K} = 16 )</td>
<td>( \sigma_{xy} )</td>
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Table for velocity:

\[ U = 6, \quad \mu = .5, \quad h = .5, \quad \frac{v_0}{2v} = 6, \quad \sqrt{\left( \frac{v_0}{2v} \right)^2 - \left( \frac{1}{K} \cdot \frac{\sigma B_0^2}{\mu} \right)} = 2, \quad \text{When} \quad \frac{1}{K} < \frac{\sigma B_0^2}{\mu} \]

\[ \sqrt{\left( \frac{v_0}{2v} \right)^2 - \frac{\sigma B_0^2}{\mu}} = \sqrt{15} = A \]

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Table 3
Table for Skin friction

\[ U = 6, \ \mu = .5, \ h = .5, \ \frac{v_0}{2v} = 6, \ \sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{K} + \frac{\sigma B^2_0}{\mu}\right)} = 2, \ \text{When} \ \frac{1}{K} < \frac{\sigma B^2_0}{\mu} \]

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\[ U = 6, \ \mu = .5, \ h = .5, \ \frac{v_0}{2v} = 6, \ \sqrt{\left(\frac{v_0}{2v}\right)^2 - \left(\frac{1}{K} + \frac{\sigma B^2_0}{\mu}\right)} = 2, \ \text{When} \ \frac{1}{K} > \frac{\sigma B^2_0}{\mu} \]

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\[ U = 6, \quad \mu = 0.5, \quad h = 0.5, \quad \frac{v_0}{2\nu} = 6, \quad \sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \left(\frac{1}{K} + \frac{\sigma B_0^2}{\mu}\right)} = 2, \quad \text{When} \quad \frac{1}{K} \geq \frac{\sigma B_0^2}{\mu} \]

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<tr>
<td>\sigma_{xy} = 79.53</td>
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<tr>
<td>\sigma_{xy} = 0.135</td>
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<td>\sigma_{xy} = 0.405</td>
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<td>\sigma_{xy} = 1.22</td>
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<td>\sigma_{xy} = 3.66</td>
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<td>\sigma_{xy} = 10.98</td>
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<td>\sigma_{xy} = 33</td>
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<td>\sigma_{xy} = 99.14</td>
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<tr>
<td>\sigma_{xy} = 0.417</td>
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<tr>
<td>\sigma_{xy} = 0.95</td>
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<tr>
<td>\sigma_{xy} = 2.15</td>
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<td>\sigma_{xy} = 4.835</td>
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<tr>
<td>\sigma_{xy} = 10.84</td>
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<tr>
<td>\sigma_{xy} = 24.22</td>
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<tr>
<td>\sigma_{xy} = 54.08</td>
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**Result and Discussion**

In this paper, we have investigated the velocity by the graph of table-1 of equation (5). The velocity in porous medium and magnetic field at \( \frac{1}{K} = \frac{\sigma B_0^2}{\mu} = 16 \) is less than the corresponding value of velocity in porous with magnetic field at \( \frac{1}{K} + \frac{\sigma B_0^2}{\mu} = 32 \).
in the interval \(0 \leq y \leq 0.4\) and equal \(\{u(y) = 6\}\) in all mediums at \(y = 0.5\). But the value of velocity in porous medium and magnetic field at \(1 \over K + \frac{\sigma B_0^2}{\mu} = 32\) is greater than the corresponding value of velocity in porous with magnetic field at \(1 \over K + \frac{\sigma B_0^2}{\mu} = 32\) at \(y = 0.6\).

Again from the table-3 the value of the velocity in porous medium at \(\frac{\sigma B^2}{\mu} = 21\) is less than the corresponding value of velocity in magnetic field at \(2 \over \sigma \mu = 11\) and also is less than the corresponding value of velocity in porous medium with magnetic field at \(1 \over K + \frac{\sigma B_0^2}{\mu} = 32\) in the interval \(0 \leq y \leq 0.4\). Velocity is equal \(\{u(y) = 6\}\) in all mediums at \(y = 0.5\) and is greater than the corresponding value of velocity in magnetic field and porous with magnetic field at \(y = 0.6\).

Again from the table-5 the value of the velocity in magnetic field at \(1 \over \mu = 21\) is less than the corresponding value of velocity in porous medium at \(\frac{\sigma B^2}{\mu} = 11\) and also is less than the corresponding value of velocity in porous medium with magnetic field at \(1 \over K + \frac{\sigma B_0^2}{\mu} = 32\) in the interval \(0 \leq y \leq 0.4\). Velocity is equal \(\{u(y) = 6\}\) in all mediums at \(y = 0.5\) and is greater than the corresponding value of velocity in porous medium and porous with magnetic field at \(y = 0.6\).

Again we have investigated the skin friction by the graph of table-2 of equation (8). The skin friction in porous medium and magnetic field at \(1 \over K + \frac{\sigma B_0^2}{\mu} = 32\) is less than the corresponding value of skin friction in porous with magnetic field at \(1 \over K + \frac{\sigma B_0^2}{\mu} = 32\) in the interval \(0 \leq y \leq 0.3\) and the skin friction in porous medium and magnetic field at \(1 \over K + \frac{\sigma B_0^2}{\mu} = 32\) is greater than the corresponding value of skin friction in porous with magnetic field at \(1 \over K + \frac{\sigma B_0^2}{\mu} = 32\) in the interval \(0.4 \leq y \leq 0.6\).

Again from the table-4 the skin friction in porous medium at \(1 \over K = 11\) is less than the corresponding value of skin friction in magnetic field at \(\frac{\sigma B^2}{\mu} = 21\) and also is less than the corresponding value of skin friction in porous with magnetic field at \(1 \over K + \frac{\sigma B_0^2}{\mu} = 32\) in the interval \(0 \leq y \leq 0.3\) and skin friction in porous medium at \(1 \over K = 11\) is greater than the corresponding value of skin friction in magnetic field at \(\frac{\sigma B^2}{\mu} = 21\) and is also greater than the corresponding value of skin friction in porous with magnetic field at \(1 \over K + \frac{\sigma B_0^2}{\mu} = 32\) in the interval \(0.4 \leq y \leq 0.6\).
Again from table-6 the skin friction in magnetic field at 
\[ \frac{\sigma B^2}{\mu} = 21 \]

is less than the corresponding value of skin friction in porous medium at 
\[ \frac{1}{K} = 11 \]

and also is less than the corresponding value of skin friction in porous with magnetic field at 
\[ \frac{1}{K} + \frac{\sigma B^2}{\mu} = 32 \]

in the interval 
[0 ≤ y ≤ 0.3]

and skin friction in magnetic field at 
\[ \frac{\sigma B^2}{\mu} = 21 \]

porous medium at 
\[ \frac{1}{K} = 11 \]

and is also greater than the corresponding value of skin friction in porous with magnetic field at 
\[ \frac{1}{K} + \frac{\sigma B^2}{\mu} = 32 \]

in the interval [0.4 ≤ y ≤ 0.6]. Also we have investigated shearing stress, the volumetric flow, drag coefficients and stream lines by the equations (7), (9), (11), (12), (13), (14) and (15) respectively.

References