Steady plane poiseuille flow of viscous incompressible fluid between two porous parallel plates through porous medium
Anand Swrup Sharma
Department of Applied Sciences, Ideal Institute of Technology, Ghaziabad.

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Nomenclature:
\( u \) = Velocity component along \( x \)-axis
\( v \) = Velocity component along \( y \)-axis
\( t \) = the time
\( \rho \) = The density of fluid
\( P \) = the fluid pressure
\( K \) = the thermal conductivity of the fluid
\( \mu \) = Coefficient of viscosity
\( \nu \) = Kinematic viscosity
\( Q \) = the volumetric flow

Introduction:

Formulation of problem:
Let us consider two infinite porous plates AB & CD separated by a distance \( 2h \). The fluid enters in y-direction. The velocity component along x-axis is a function of y only. The motion of incompressible fluid is in two dimension and is steady then
\[
u = u (y), \quad w = 0 \quad \text{and} \quad \frac{\partial}{\partial t} = 0
\]
The equation of continuity for incompressible fluid
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{Put } w = 0,
\]
\[
\frac{\partial u}{\partial x} = 0 \quad \& \quad \Rightarrow \frac{\partial v}{\partial y} = 0
\]
v is independent of \(y\) but motion along \(y\)-axis. So we can say \(v\) is constant velocity i.e. \(v = v_0\)

or The fluid enters the flow region through one plate at the same constant velocity \(v_0\)

Also Navier-Stoke's equations for incompressible fluid in the absence of body force when flow is steady
\[
v_0 \frac{du}{dy} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{d^2 u}{dy^2} + \frac{v u}{\nu} \right) \quad \text{............ (1)}
\]
\[
-\frac{1}{\rho} \frac{\partial p}{\partial y} = 0 \quad \text{............ (2)}
\]

Solution of the problem:

Equation (2) Shows that the pressure does not depend on \(y\) hence \(p\) is a function of \(x\) only and so (1) reduces to

\[
\frac{dp}{dx} = \rho \left[ \nu \frac{d^2 u}{dy^2} + \frac{v u}{\nu} \right] \quad \text{Where } \frac{dp}{dx} = \text{Constant} = -P
\]

\[
\Rightarrow \frac{d^2 u}{dy^2} + \frac{v_0}{\nu} \frac{du}{dy} + \frac{u}{K} = \frac{P}{\rho \nu}
\]

\[
\Rightarrow \left( \frac{d^2}{dy^2} - \frac{v_0}{\nu} D + \frac{1}{K} \right) u = -\frac{P}{\rho \nu}
\]

A.E. \(m^2 - \frac{v_0}{\nu} m + \frac{1}{K} = 0\)

\[
\Rightarrow m = \frac{-\frac{v_0}{\nu} \pm \sqrt{\left(\frac{v_0}{\nu}\right)^2 - 4 \frac{1}{K}}}{2}
\]

Let \(\sqrt{\left(\frac{v_0}{2\nu}\right)^2 - \frac{1}{K}} = A \quad \& \quad \frac{1}{K} = B\)

\[
C.F. = e^{\frac{v_0}{2\nu} y} \left[ C_1 \text{Cosh } Ay + C_2 \text{Sinh } Ay \right] \quad \text{P.I.} = -\frac{PK}{\mu}
\]

\[
u(y) = e^{\frac{v_0}{2\nu} y} \left[ C_1 \text{Cosh } Ay + C_2 \text{Sinh } Ay \right] - \frac{PK}{\mu}
\]

using boundary conditions: \(u = 0\) at \(y = -h\) and \(u = U\) at \(y = h\)

\[
e^{-\frac{v_0}{2\nu} h} \left[ C_1 \text{Cosh } Ah - C_2 \text{Sinh } Ah \right] - \frac{PK}{\mu} = 0 \quad \text{............ (3)}
\]

\[
U = e^{\frac{v_0}{2\nu} h} \left[ C_1 \text{Cosh } Ah + C_2 \text{Sinh } Ah \right] - \frac{PK}{\mu} \quad \text{............ (4)}
\]

or \(\frac{PK}{\mu} e^{\frac{v_0}{2\nu} h} = C_1 \text{Cosh } Ah - C_2 \text{Sinh } Ah\)

\[
\left( U + \frac{PK}{\mu} \right) e^{-\frac{V_y h}{2v}} = C_1 \cosh(A h) + C_2 \sinh(A h)
\]

\[
C_1 = \frac{1}{2 \cosh(A h)} \left[ \left( U + \frac{PK}{\mu} \right) e^{\frac{V_y h}{2v}} + \frac{PK}{\mu} e^{\frac{V_y h}{2v}} \right]
\]

\[
C_2 = \frac{1}{2 \sinh(A h)} \left[ \left( U + \frac{PK}{\mu} \right) e^{\frac{V_y h}{2v}} - \frac{PK}{\mu} e^{\frac{V_y h}{2v}} \right]
\]

\[
u(y) = \frac{e^{2v y}}{2 \cosh(A h)} \left( U + \frac{PK}{\mu} \right) e^{-\frac{V_y h}{2v}} + \frac{PK}{\mu}
\]

\[
\nu(y) = \frac{e^{2v y}}{2 \sinh(A h)} \left( U + \frac{\rho K}{\mu} \right) e^{-\frac{V_y h}{2v}} - \frac{PK}{\mu}
\]

\[
u(y) = \frac{1}{\sinh(2 A h)} \left[ \left( U + \frac{PK}{\mu} \right) e^{\frac{V_y (y-h)}{2v}} \sinh(A (y+h)) - \frac{PK}{\mu} e^{\frac{V_y (y+h)}{2v}} \sinh(A (y-h)) \right] - \frac{PK}{\mu}
\]

Plane Poiseuille flow: In this case both plates are at rest so \( U = 0 \)

\[
\therefore \nu(y) = \frac{1}{\sinh(2 A h)} \left[ \frac{PK}{\mu} e^{\frac{V_y (y-h)}{2v}} \sinh(A (y+h)) - \frac{PK}{\mu} e^{\frac{V_y (y+h)}{2v}} \sinh(A (y-h)) \right] - \frac{PK}{\mu}
\]

Shearing stress at any point

\[
\sigma_{xy} = \mu \frac{du}{dy} = \frac{\mu PK}{\mu \sinh(2 A h)} \left[ e^{\frac{V_y (y-h)}{2v}} \sinh(A (y+h)) + A e^{\frac{V_y (y-h)}{2v}} \cosh(A (y+h)) \right]
\]

\[
- \frac{V_y}{2v} e^{\frac{V_y (y+h)}{2v}} \sinh(A (y-h)) - Ae^{\frac{V_y (y+h)}{2v}} \cosh(A (y-h))
\]

\[
= \frac{PK}{\sinh(2 A h)} \left[ \frac{V_y}{2v} \sinh(A (y+h)) - e^{\frac{V_y (y+h)}{2v}} \sinh(A (y-h)) \right] + A \left[ \frac{V_y}{2v} \cosh(A (y+h)) - e^{\frac{V_y (y+h)}{2v}} \cosh(A (y-h)) \right]
\]

Skin friction at lower & upper plates

\[
\left( \sigma_{xy} \right)_{y=h} = \frac{PK}{\sinh(2 A h)} \left[ \frac{V_y}{2v} \left\{ \sinh(2 A h) + A \left\{ \cosh(2 A h) - e^{\frac{V_y h}{2v}} \right\} \right\} \right]
\]

\[
\left( \sigma_{xy} \right)_{y=-h} = \frac{PK}{\sinh(2 A h)} \left[ \frac{V_y}{2v} \sinh(2 A h) + ACosh(2 A h) - e^{\frac{V_y h}{2v}} \right]
\]

\[
\left( \sigma_{xy} \right)_{y=-h} = \frac{PK}{\sinh(2 A h)} \left[ \frac{V_y}{2v} \sinh(2 A h) + A \left\{ e^{\frac{V_y h}{2v}} - \cosh(2 A h) \right\} \right]
\]

\[
\left( \sigma_{xy} \right)_{y=-h} = \frac{PK}{\sinh(2 A h)} \left[ \frac{V_y}{2v} \sinh(2 A h) - ACosh(2 A h) + e^{\frac{V_y h}{2v}} \right]
\]
The average velocity distribution in poiseuille flow:

\[ u_{av} = \frac{1}{2h} \int_{-h}^{h} u(y) dy \]

\[ = \frac{PK}{2\mu h \operatorname{Sinh} 2Ah} \int_{-h}^{h} \left[ e^{\frac{y}{2\nu}(y-h)} \operatorname{Sinh} A(y+h) - e^{\frac{-y}{2\nu}(y+h)} \operatorname{Sinh} A(y-h) - \operatorname{Sinh} 2Ah \right] dy \]

Now Let \[ I_1 = \int_{-h}^{h} e^{\frac{y}{2\nu}(y-h)} \operatorname{Sinh} A(y+h) dy \]

\[ = \frac{1}{2} \int_{-h}^{h} \left[ e^{\frac{y}{2\nu}(y-h)+A(y+h)} - e^{\frac{y}{2\nu}(y-h)-A(y+h)} \right] dy \]

\[ = \frac{K}{2} \left[ \frac{\nu_0}{2\nu} - A \left( e^{2Ah} - e^{-\frac{\nu_0 h}{\nu}} \right) - \left( \frac{\nu_0}{2\nu} + A \right) \left( e^{-2Ah} - e^{-\frac{\nu_0 h}{\nu}} \right) \right] \]

\[ = \frac{K}{2} \left[ \frac{\nu_0}{2\nu} \left( e^{2Ah} - e^{-\frac{\nu_0 h}{\nu}} - e^{-2Ah} + e^{-\frac{\nu_0 h}{\nu}} \right) - A \left( e^{2Ah} - e^{-\frac{\nu_0 h}{\nu}} + e^{-2Ah} - e^{-\frac{\nu_0 h}{\nu}} \right) \right] \]

\[ I_1 = K \left[ \frac{\nu_0}{2\nu} \operatorname{Sinh} 2Ah - ACosh 2Ah + A e^{-\frac{\nu_0 h}{\nu}} \right] \]

\[ I_2 = \int_{-h}^{h} e^{\frac{y}{2\nu}(y-h)} \operatorname{Sinh} A(y-h) dy = K \left[ \frac{\nu_0}{2\nu} \operatorname{Sinh} 2Ah + ACosh 2Ah - A e^{-\frac{\nu_0 h}{\nu}} \right] \]

\[ I_3 = \int_{-h}^{h} \operatorname{Sinh} 2Ah dy = 2h \operatorname{Sinh} 2Ah \]

\[ \therefore u_{av} = \frac{PK}{2\mu h \operatorname{Sinh} 2Ah} \left[ I_1 - I_2 - I_3 \right] \]

\[ = \frac{PK}{2\mu h \operatorname{Sinh} 2Ah} \left[ \frac{K\nu_0}{2\nu} \operatorname{Sinh} 2Ah - KA Cosh 2Ah + KA e^{-\frac{\nu_0 h}{\nu}} - \frac{K\nu_0}{2\nu} \operatorname{Sinh} 2Ah - K A Cosh 2Ah + KA e^{-\frac{\nu_0 h}{\nu}} - 2h \operatorname{Sinh} 2Ah \right] \]

\[ u_{av} = \frac{PK}{\mu h \operatorname{Sinh} 2Ah} \left[ AK \left( \operatorname{Cosh} \frac{\nu_0}{\nu} h - \operatorname{Cosh} 2Ah \right) - h \operatorname{Sinh} 2Ah \right] \quad \cdots \cdots \text{(10)} \]

The volumetric flow \( Q = 2h u_{av} \)

\[ = \frac{2PK}{\mu \operatorname{Sinh} 2Ah} \left[ AK \left( \operatorname{Cosh} \frac{\nu_0}{\nu} h - \operatorname{Cosh} 2Ah \right) - h \operatorname{Sinh} 2Ah \right] \quad \cdots \cdots \text{(11)} \]

The Drug coefficients: \( C_f \) & \( C'_f \) at \( y = h \) & \( y = -h \)

\[ C_f = \frac{\sigma_{xy}}{2h^2 \nu^2} = \frac{1}{2} \frac{\rho(u_{av})^2}{PK} \left[ \frac{\nu_0}{2\nu} \operatorname{Sinh} 2Ah + ACosh 2Ah - A e^{-\frac{\nu_0 h}{\nu}} \right] \quad \cdots \cdots \text{(12)} \]
\[ C_f = \frac{1}{2} \rho u^2 h^2 \sinh 2Ah \left( \frac{\nu_0 \sin 2Ah - ACosh 2Ah + Ae^{-\frac{v_0}{2d}}}{PK} \right) \]

\[ \frac{PK}{\mu \sinh 2Ah} \left\{ e^{\frac{\nu_0}{2d}(y-h)} \sinh A(y + h) - e^{\frac{\nu_0}{2d}(y+h)} \sinh A(y - h) - \sinh 2Ah \right\} = \frac{dy}{v_0} = \frac{dz}{0} \]

The stream line in the plane poiseuille flow:

\[ \frac{PK}{\mu \sinh 2Ah} \left\{ e^{\frac{\nu_0}{2d}(y-h)} \sinh A(y + h) - e^{\frac{\nu_0}{2d}(y+h)} \sinh A(y - h) - \sinh 2Ah \right\} = \frac{dy}{v_0} = \frac{dz}{0} \]

Taking Ist two

\[ \frac{v_0 \mu \sinh 2Ah}{PK} x = \int \left\{ e^{\frac{\nu_0}{2d}(y-h)} \sinh A(y + h) - e^{\frac{\nu_0}{2d}(y+h)} \sinh A(y - h) - \sinh 2Ah \right\} dy = C_1 \]

\[ \begin{align*}
I_1 &= \frac{1}{2} \left[ \left( \frac{v_0}{2d} + A \right) - \left( \frac{v_0}{2d} - A \right) \right] \\
&= K e^{\frac{\nu_0}{2d}(y-h)} \left[ \frac{v_0}{2d} \sinh A(y + h) - ACosh A(y + h) \right] \\
I_2 &= \int e^{\frac{\nu_0}{2d}(y+h)} \sinh A(y - h) . \ dy = K e^{\frac{\nu_0}{2d}(y+h)} \left[ \frac{v_0}{2d} \sinh A(y - h) - ACosh A(y - h) \right] \\
I_3 &= \int \sinh 2Ah . \ dy = y. \sinh 2Ah
\end{align*} \]

\[ \therefore \text{Ist stream line}. \]

\[ \frac{v_0 \mu \sinh 2Ah}{PK} x = \{ I_1 - I_2 - I_3 \} = C_1 \]

\[ \frac{v_0 \mu \sinh 2Ah}{PK} x = K e^{\frac{\nu_0}{2d}(y-h)} \left\{ \frac{v_0}{2d} \sinh A(y + h) - ACosh A(y + h) \right\} \\
+ K e^{\frac{\nu_0}{2d}(y+h)} \left\{ \frac{v_0}{2d} \sinh A(y - h) - ACosh A(y - h) \right\} + y \sinh 2Ah = C_1 \ldots \ldots \ (14) \]

Second stream line

\[ z = c_2 \ldots \ldots (15) \]

Clearly the curl \( \frac{q}{q} \neq 0 \) \quad \therefore \text{the fluid is Rotational}

Table for velocity:

\[ P = 9, \mu = .5, \frac{v_0}{2d} = 2, k = \frac{1}{3}, A = \sqrt{\frac{v_0}{2d}^2 - \frac{1}{k}} = \sqrt{4 - 3} = 1 \text{ are same for all but } h \text{ change} \]

<table>
<thead>
<tr>
<th>h</th>
<th>y</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>u(y)</td>
<td>0.804</td>
<td>0.818</td>
<td>0.587</td>
<td>0</td>
<td>-1.096</td>
<td>-2.912</td>
<td>-5.734</td>
</tr>
<tr>
<td>0.4</td>
<td>u(y)</td>
<td>1.423</td>
<td>1.524</td>
<td>1.34</td>
<td>0.94</td>
<td>0</td>
<td>-1.623</td>
<td>-4.21</td>
</tr>
<tr>
<td>0.5</td>
<td>u(y)</td>
<td>2.21</td>
<td>2.42</td>
<td>2.43</td>
<td>2.131</td>
<td>1.385</td>
<td>0</td>
<td>-2.29</td>
</tr>
<tr>
<td>0.6</td>
<td>u(y)</td>
<td>3.164</td>
<td>3.51</td>
<td>3.67</td>
<td>3.56</td>
<td>3.045</td>
<td>1.94</td>
<td>0</td>
</tr>
<tr>
<td>0.7</td>
<td>u(y)</td>
<td>4.282</td>
<td>4.778</td>
<td>5.122</td>
<td>5.226</td>
<td>4.969</td>
<td>4.181</td>
<td>2.63</td>
</tr>
</tbody>
</table>
Velocity Profile

Table for skin friction:

\[ P = \frac{9}{2}, \mu = 0.5, \frac{v_0}{2u} = 2, k = \frac{1}{3}, A = \sqrt{\frac{v_0}{2u}} - \frac{1}{k} = \sqrt{4 - \frac{3}{k}} = 1 \] are same for all but \( h \) change

<table>
<thead>
<tr>
<th>( h )</th>
<th>( y )</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>( \sigma )</td>
<td>0.532</td>
<td>-0.465</td>
<td>-1.936</td>
<td>-4.059</td>
<td>-7.076</td>
<td>-11.318</td>
<td>-17.229</td>
</tr>
<tr>
<td>0.4</td>
<td>( \sigma )</td>
<td>0.936</td>
<td>0.016</td>
<td>-1.356</td>
<td>-3.36</td>
<td>-6.21</td>
<td>-10.25</td>
<td>-15.89</td>
</tr>
<tr>
<td>0.5</td>
<td>( \sigma )</td>
<td>1.445</td>
<td>0.62</td>
<td>-0.632</td>
<td>-2.48</td>
<td>-5.14</td>
<td>-8.92</td>
<td>-14.24</td>
</tr>
<tr>
<td>0.6</td>
<td>( \sigma )</td>
<td>2.05</td>
<td>1.34</td>
<td>0.225</td>
<td>-1.44</td>
<td>-3.88</td>
<td>-7.37</td>
<td>-12.31</td>
</tr>
<tr>
<td>0.7</td>
<td>( \sigma )</td>
<td>2.75</td>
<td>2.163</td>
<td>1.21</td>
<td>-0.26</td>
<td>-2.45</td>
<td>-5.62</td>
<td>-10.14</td>
</tr>
</tbody>
</table>
Table for velocity: when y & A are vary and other are fixed

\[
\begin{align*}
\text{let } & P = 9, \quad \mu = .5, \quad \frac{v_0}{2v} = 6, \quad h = .5, \quad \text{& } \sqrt{\frac{v_0}{2v}} - \frac{1}{K} = A \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>A</th>
<th>y</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>u(y)</td>
<td>4.08</td>
<td>6.09</td>
<td>8.43</td>
<td>10.31</td>
<td>9.44</td>
<td>0</td>
<td>-31.67</td>
</tr>
<tr>
<td>2</td>
<td>u(y)</td>
<td>3.11</td>
<td>4.5</td>
<td>6.07</td>
<td>7.29</td>
<td>6.6</td>
<td>0</td>
<td>-22.26</td>
</tr>
<tr>
<td>3</td>
<td>u(y)</td>
<td>2.19</td>
<td>3.03</td>
<td>3.93</td>
<td>4.59</td>
<td>4.09</td>
<td>0</td>
<td>-13.92</td>
</tr>
<tr>
<td>4</td>
<td>u(y)</td>
<td>1.51</td>
<td>1.984</td>
<td>2.464</td>
<td>2.78</td>
<td>2.43</td>
<td>0</td>
<td>-8.41</td>
</tr>
<tr>
<td>5</td>
<td>u(y)</td>
<td>1.05</td>
<td>1.31</td>
<td>1.56</td>
<td>1.69</td>
<td>1.45</td>
<td>0</td>
<td>-5.178</td>
</tr>
</tbody>
</table>
let $P = 9, \ \mu = 0.5, \ \frac{v_0}{2\nu} = 6, h = 0.5, \ \& \ \sqrt{\frac{v_0}{2\nu}} - \frac{1}{K} = A$

Table 4

<table>
<thead>
<tr>
<th>A</th>
<th>y</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\sigma_{xy}$</td>
<td>8.83</td>
<td>11.18</td>
<td>11.62</td>
<td>5.44</td>
<td>-18.66</td>
<td>-86.39</td>
<td>-254.45</td>
</tr>
<tr>
<td>2</td>
<td>$\sigma_{xy}$</td>
<td>6.215</td>
<td>7.62</td>
<td>7.695</td>
<td>3.30</td>
<td>-13.27</td>
<td>-60.3</td>
<td>-180.99</td>
</tr>
<tr>
<td>3</td>
<td>$\sigma_{xy}$</td>
<td>3.855</td>
<td>4.496</td>
<td>4.328</td>
<td>1.56</td>
<td>-8.422</td>
<td>-37.26</td>
<td>-114</td>
</tr>
<tr>
<td>4</td>
<td>$\sigma_{xy}$</td>
<td>2.253</td>
<td>2.462</td>
<td>2.219</td>
<td>0.567</td>
<td>-5.134</td>
<td>-22.11</td>
<td>-70.05</td>
</tr>
<tr>
<td>5</td>
<td>$\sigma_{xy}$</td>
<td>1.285</td>
<td>1.3</td>
<td>1.08</td>
<td>0.107</td>
<td>-3.135</td>
<td>-13.24</td>
<td>-44</td>
</tr>
</tbody>
</table>
Conclusion and discussion:

In this paper, we have investigated the velocity by the graphs of table -1 of equation (5) between velocity and distance in porous medium. Velocity increases in the interval $0 \leq y \leq 1$ at $h = .3$, velocity decreases in the interval $1 \leq y \leq 3$ at $h = .3$, and velocity increases with negative sign at $h = .3$ in the interval $4 \leq y \leq .6$. Again velocity increases in the interval $0 \leq y \leq 1$, velocity decreases in the interval $2 \leq y \leq 4$, and increases with negative sign in the interval $5 \leq y \leq .6$ at $h = .4$. Velocity increases in the interval $0 \leq y \leq 2$, decreases in the interval $3 \leq y \leq 5$, and velocity is negative at $y = .6$ at the height $h = .5$. Again velocity increases at $h = .6$ in the interval $0 \leq y \leq 1$, and decreases in the interval $3 \leq y \leq .6$ at $h = .6$. Again the velocity increases in the interval $0 \leq y \leq .3$, and decreases in the interval $4 \leq y \leq .6$ at $h = .7$. The points with zero velocity are stagnation point. The value of velocity increases correspondingly in the interval $0 \leq y \leq .6$ when $h$ increases.

Again from the table -3 the velocity decreases correspondingly in the interval $0 \leq y \leq .6$ when $A$ increases from 1 to 5. Since the velocity is zero at $y = .5$ for all values of $A$, $y = .5$ is a stagnation point.

Again from the table -2 the value of skin friction increases correspondingly in the interval $0 \leq y \leq .2$ and the values of skin friction decreases with negative sign in the interval $3 \leq y \leq .6$ when $h$ increases from .3 to .7.
Again from the table -4 it is clear that the skin friction decreases with positive sign in the interval \(0 \leq y \leq 3\) when \(A\) increases from 1 to 5 and decreases with negative sign correspondingly in the interval \(4 \leq y \leq 6\) when \(A\) increases from 1 to 5. Also we have investigated the shearing stress, the volumetric flow, drag coefficients and stream lines by equations (7), (9), (11), (12), (13), (14) and (15). The fluid is rotational.

References: