Adaptive Equalizer for Compensation of T$^x$ And R$^x$ IQ Imbalance in the
Presence of CFO

J Tarun Kumar$^{1,*}$ and K Anitha Sheela$^2$

$^1$Department of Electronics and Communication Engineering, Sumathi reddy Institute of Technology for Women, JNT University, Warangal, India
$^2$Department of Electronics and Communication Engineering, JNT University, Hyderabad, India.

**ABSTRACT**

The OFDM based architectures severely suffers with the Impairments like IQ imbalance and Carrier Frequency Offset (CFO). These impairments lead to Inter Carrier Interference (ICI). It causes reduction in the performance of the systems in terms of Bit Error Rate (BER). In this paper we propose an adaptive equalizer to compensate for these impairments caused at T$^x$ and R$^x$ while considering Inter Block Interference (IBI) due to mismatch between Cyclic Prefix (CP) and Channel Impulse Response. If the cyclic prefix is sufficient an LMS based adaption with 2 tap is proposed which converges easily. In case of lesser length of CP a Frequency domain Per Tone Equalizer (PTEQ) is proposed which uses separate T-taps for each tone unlike Time domain Equalizer (TEQ) which equalizes all tones jointly and limits the system performance. This proposed equalizer thus increases the delay of channel response and hence reduces the IBI. The frequency domain equalizer after FFT will perform well and the simulation results shows that both the equalizers BER values are very near to the curves for ideal case.

**Keywords**

Inter Carrier Interference, Cyclic prefix, PTEQ equalizer, Lms.

**Introduction**

Orthogonal Frequency Division Multiplexing (OFDM) is a most popular and widely used multi carrier scheme to achieve higher data rates in modern wireless systems. In this the available Bandwidth is split into a number of narrow band channels called sub carriers to improve the data rates. Due to these advantages the OFDM is preferred by so many wireless networks such as 802.11, IEEE 802.16 d/e and Long Term Evolution (LTE) etc.. These features of OFDM can be achieved if and only if the parameters at the receiver and transmitter are perfectly matched. But in the real time so many constraints which cannot make the system perfectly matched. In OFDM the Zero-IF direct conversion receiver RF signal is directly converted as base band or vice-versa without using any Intermediate Frequencies (IF). In this kind of receiver the modulation and demodulation of In-phase and Quadrature phase is performed in analog domain. The imperfection in these components leads to IQ Imbalance distortion. In addition to IQ Imbalance OFDM severely suffers from the Carrier Frequency Offset (CFO) caused due to mismatch of carrier frequency at T$^x$ and R$^x$ and also Doppler shift caused due to mobility.

The I/Q imbalance and CFO leads to Inter Carrier Interference (ICI) which degrades the system performance. Various methods have been proposed to compensate these effects [2-4 &6]. A Time domain compensation technique was proposed in [3]. Joint compensation of IQ imbalance and CFO was proposed in [4]. This joint compensation technique was a low complexity and low cost but considered the effect of IQ imbalance only at receiver. An adaptive equalizer to correct the IQ imbalance in OFDM systems was proposed in [6], where the system is more complex and more operations are required to correct.

In this paper we propose an Adaptive Equalizer to compensate for Transmitter and Receiver IQ imbalance in the presence of CFO and also analysed the effect of Inter Block Interference. If the cyclic prefix (CP) is sufficient an LMS based adaption with 2 tap is proposed which converges easily. In case of lesser length of CP a Frequency domain Per Tone Equalizer (PTEQ) is proposed which uses separate T-taps for each tone unlike Time domain Equalizer (TEQ) which equalizes all tones jointly and limits the system performance. The proposed equalizer thus increases the delay of channel response and hence reduces the IBI. PTEQ uses a Recursive Least Square (RLS) algorithm for adaption.

In section 2 we describe T$^x$ and R$^x$ IQ imbalance along with CFO model in the system. A mathematical model for compensation of T$^x$ and R$^x$ IQ imbalance in the presence of CFO is discussed in section 3. The equalizer coefficients are computed in section 4. In section 5 the simulations and results were discussed. Finally, concluded the paper in section 6.

**Iq Imbalance And Cfo Model**

The IQ imbalance is caused due to mismatch in InPhase, Quadrature phase components. The IQ Imbalance is characterized by 2 parameters that is the amplitude mismatch and phase mismatch.

Let the time domain complex base band signal ‘X’ with an effect of I/Q imbalance can be given as

$$X_{iq} = \mu r + v \cdot r^*$$  

(1)

The following notations are use for mathematical operations ‘*’, ‘Θ’ and ‘@’ represent component wise vector multiplication, convolution and kroneker product. And the super scripts *,T,H represents Conjugate, Transpose and Hermitian respectively.
Transmitter IQ parameters \( \mu \) and \( v \) are Inphase and Quadrature components at transmitter. \( \epsilon \) and \( \phi \) are amplitude and phase mismatch at transmitter. The size of the OFDM symbol is \( N \times 1 \). These symbols are transformed into time domain by applying Inverse Fourier transform (IFFT).

Where \( r = (I \otimes P)F^{-1}[r_{k+1}^T] \) represents the inverse fourier transform matrix

![Fig.1. Transmitter IQ imbalance with CFO.](image)

The IQ imbalance at transmitter is represented by two padded with \( N-L \) zero elements. In (2) and (3) if the \( \epsilon_t \) and \( \Phi_t \) is the \( k^{th} \) complex OFDM symbol, which is of size \( N \times 1 \). In this section we also analyse the Inter Block Interference (IBI). The interference is caused due to previous block transmitted K-1 on K. If a cyclic prefix is added of length \( \nu \), then the resulting signal \( r \) can be given as

\[
r = (I \otimes P)(I \otimes F^{-1})[r_{k+1}^T]
\]

(4)

Substituting (5) in (6)

\[
Z = \mu S + v S^*
\]

Similarly, the IQ Imbalance is also causes at the receiver as specified in (1) and it also can be considered. Let us assume \( Z \) as the complex base band signal with IQ Imbalance at receiver and transmitter and it can be given as

\[
Z = \mu S + v S^*
\]

(6)

Assuming IBI then the \( \nu \) of (8) can be replaced by (4)

\[
y = h_i \Theta r \cdot e^{j/2(\Delta f)} + h_\nu r^* \cdot e^{-j/2(\Delta f)}
\]

(8)

Where \( \nu \) is of dimensions \( N \times 1 \) and \( \nu = 0 \) then the I-phase and Q-phase components becomes equal and leads to \( H_{0} = H_{\nu} \).

The OFDM symbol is then transmitted through a frequency selective (time dispersive) channel of length L. Let us assume \( C \) is the impulse response of the multipath channel. The received base band vector \( \nu \) can be given as

\[
S = C \Theta X + w
\]

(5)

![Fig.2. Receiver IQ Imbalance with CFO.](image)

When \( C_1 \) & \( C_2 \) is a convolution of IQ imbalance and channel response of length \( N+L-1 \), and \( W \) is a Additive white Gaussian Noise (AWGN).
Then the size of the received symbol ‘y’ is to be adjusted after multiplication which cause an ICI. The signals y are involved with S and Q and these can be compensated in Frequency domain. Let us write (13) in Frequency domain.

\[ y_1 = y_e^{-2j\Pi x_0} \]

(11)

Correspondingly, the mirror image (complex conjugate) of the signal received is also be compensated by same as

\[ y_2 = y_e^{-2j\Pi x_0} \]

\[ = (\mu, \Theta S, e^{-j2\Pi x_0}) e^{-j2\Pi x_0} \]

\[ = (\mu, \Theta S, e^{-j2\Pi x_0}) + \nu, \Theta S, e^{-j2\Pi x_0} \]

(12)

Let us assume \( S^*, e^{-2j2\Pi x_0} \) to be ‘q’ the (12) can be rewritten as

\[ y_1 = \mu, S + v, q \]

\[ y_2 = \mu, S^* q + \nu, S \]

(13)

The ‘S’ is the desired signal and ‘q’ is an undesired signal after multiplication which cause an ICI. The signals \( y_1 \) and \( y_2 \) are involved with S and Q and these can be compensated in Frequency domain. Let us write (13) in Frequency domain.

\[ Y_1 = M_r S + V_r Q \]

\[ Y_2 = M_r S^* Q + V_r S \]

(14)

The above (14) can be more elaborated for each OFDM symbol

\[ [Y_1 \ Y_2] = [S \ \hat{Q}] \begin{bmatrix} M_r & M_r^* \\ V_r & V_r^* \end{bmatrix} \]

(15)

\( S(k) \) is the desired signal.

In addition to ICI caused due undesired signal ‘q’. If the cyclic prefix is less than the Channel Impulse Response, and another interference caused by previous OFDM symbol is known to be IBI. It needs to be compensated by a time domain equalizer.

The \( y_1 \) is applied to a TEQ with a weight of \( W_1 \), and \( y_2 \) is conjugate of a real signal applied to another TEQ with weight of \( W_2 \). The no. of taps used in each TEQ are \( L' \).

Then the size of the received symbol ‘y’ is to be adjusted for the size \( 2(N + L' - 2 \times L) \).

Hence, \( O_1 = 0 \) \( (2N + L' - 1) \times (2N + L' - 2) \), \( T_{zk} \) is a Toeplitz matrix of size \( (2N + L' - 2 \times N + 2v - L - L' + 2) \) with first column of dimension, with first column and first row \( \left[ h_{k}(z L-1), \ldots \ h_{k}(0) \right] \). Then ‘y_1’ and ‘y_2’ are converted to frequency domain by applying FFT. Finally a FEQ with 2 taps is applied to recover the original signal. The original estimated signal is given as

\[ \hat{r}(k) = V_1^*(k) [F(k) W_1^H Y_1(k)] + V_2^*(k) [F(k) W_2^H Y_2(k)] \]

(16)

Where \( V_1(k) \) and \( V_2(k) \) are the FEQ taps acting on a \( k \)th sub carrier \( W_k \) which are the weights from TEQ. It is a Toeplitz matrix of size \( (N + L' - 1 \times N) \) with first column elements are \( [w_{L' - 1}, \ldots w_{k - 1}, 0, 1, \ldots, -1] \) and first row \( [W_{k N - 1}, 0, 1, \ldots, -1] \) for \( k=1,2 \). \( F(k) \) is the \( k \)th row of FFT matrix F. Then the (16) can be modified to Per Tone and employing an FEQ

\[ \hat{r}(k) = V_1^H(k) F(k) Y_1(k) + V_2^H(k) F(k) Y_2(k) \]

(17)

The F(K) is defined as

\[ F(k) = \begin{bmatrix} I_{L'-1} & 0 \\ O_{1 \times L'-1} & F(k) \end{bmatrix} \]

(18)

And \( V_1(k) \) is the FEQ co efficient.

These coefficients can be estimated for a \( k \)th sub carrier. Such that, they minimize the Mean Square Error (MSE)

\[ \min_{V_1(k), V_2(k)} \sum_{k} \left[ r(k) - V_1^H(k) V_2^H(k) \right] \]

(19)

If no IBI is present, the length of cyclic prefix is sufficient and then the equalizer of order 1 is also sufficient.

The \( r(k) \) are estimated by the linear combination of \( V_1 \) and \( V_2 \)

\[ r(k) = [V_1^*(k) V_2^*(k)] \begin{bmatrix} Y_1(k) \\ Y_2(k) \end{bmatrix} \]

(20)

The coefficients \( V_1 \) and \( V_2 \) are selected to minimise the MSE.

\[ \min_{V_1(k), V_2(k)} \sum_{k} \left[ r(k) - [V_1^*(k) V_2^*(k)] \begin{bmatrix} Y_1(k) \\ Y_2(k) \end{bmatrix} \right] \]

(21)

The \( V_1 \) and \( V_2 \) are estimated from \( \mu_1, \mu_2, v_1, v_2 \), and channel response C.

Co-Efficients Computing Using Adaptive Algorithm

In this paper we propose two cases to compute the equalizer coefficients

**Case (i) : CP ≥ Channel Impulse Response.**

In this case, the LMS or NLMS algorithm is sufficient to compute the equalizer coefficients. This algorithm is an extended solution proposed in [6]. It uses only two taps to compensate the distortion produced by \( \delta^* \), \( R^* \) IQ imbalance and CFO. As the distortion is very severe it requires more number of symbols to converge. \( y_1(k) \) and \( y_2(k) \) are the two inputs to the equalizer \( V_1(k) \) and \( V_2(k) \) are the taps.

Let us assume ‘\( j \)’ is the no. of training symbols required to minimise the MSE as per (21). Then the estimated symbol \( r(k) \) at the output of the equalizer is given as

\[ \hat{r}(k) = V_1^{(j)}(k) Z_1^{(j)}(k) + V_2^{(j)}(k) Z_2^{(j)}(k) \]

where \( j = 1, \ldots, k \)
As per the LMS algorithm, the coefficients are updated based on the error signal

\[ e^{(j)}(k) = r^{(j)}(k) - \hat{r}^{(j)}(k) \]  

(23)

Based on the above, the future coefficients are given as per the LMS rule.

\[
\begin{align*}
V_1^{(j+1)}(k) &= V_1^{(j)}(k) + \mu e^{(j)}(k)Y_1^{(j)}(k) \\
V_2^{(j+1)}(k) &= V_2^{(j)}(k) + \mu e^{(j)}(k)Y_2^{(j)}(k)
\end{align*}
\]

(24)

The equalizer coefficients are estimated by the suitable number of training symbols and the equalization can be achieved on the received signal.

**Case (i):** Based on the error signal

In this case assuming the coefficients are computed initially by setting

\[
\begin{align*}
V_1^{(j=0)} &= O_{L\times N} \\
V_2^{(j=0)} &= O_{L\times N}
\end{align*}
\]

(25)

For all OFDM systems k=1 to N compute

\[
B^{(j=0)} = \delta^{-1}I_{2L \times 2L}
\]

\[u^{(j)}(k) = [F_j(k)Z_1^{(j)} - F_j(k)Z_2^{(j)}]^T\]

Where D^{(j)}(k) is the training symbol transmitted at time instant 'j' then

\[
\begin{align*}
\xi^{(j)} &= D^{(j)}(k) - [V_1^{H}(j-1)(k) - V_2^{H}(j-1)(k)]u^{(j)}(k) \\
B^{(j)} &= \frac{B^{(j-1)}u^{(j)}(k)}{1 + u^{H}(j)(k)B^{(j-1)}u^{(j)}(k)} \\
V_1^{(j)}(k) &= V_1^{(j-1)}(k) + [B^{(j)}u^{(j)}(k)]^T \xi^{(j)} \\
V_2^{(j)}(k) &= V_2^{(j-1)}(k) + [B^{(j)}u^{(j)}(k)]^T \xi^{(j)} \\
B^{(j)} &= B^{(j-1)} - B^{(j-1)}u^{H}(j)(k)B^{(j-1)}
\end{align*}
\]

In the similar manner, the equalizer coefficients are substituted in (22) to compute the transmitted OFDM symbol.

As the coefficients need to be updated for each OFDM symbol. In case I, the proposed LMS algorithm which acquire convergence slower. In case II of WIMAX the IEEE 802.16 uses 256 OFDM symbols out of which 192 are carrier data. So, it requires 192 equalizers. The load on the equalizers is compared with [6].

The number of multiplications and additions required are less for 192 carrier data. The coefficients should be updated after each symbol. Every time it requires 3 multiplications and 3 additions. So, it requires 576 multiplications and 576 additions for the algorithm specified in [6]. But in this scheme the no. of taps required is very less. So, the convergence becomes faster. And also the convergence is achieved for lesser no. of symbols.

**Simulation Results**

In this paper the simulations are carried out on the parameters of IEEE802.16 an OFDM based systems.

The performance of the system is compared with an ideal system with no distortions present and also the systems with no compensation employed. The IEEE 802.16 uses 256 subs carriers with a maximum cyclic prefix of 64. The channel models considered are AWGN and Rayleigh. The simulations are carried out for both the cases.

**Case (i):**

In this case assuming the CP is greater than or equal to the Channel Impulse Response (0≥L) means the IBI is zero. The performance of the LMS adaptive equalizer is analysed for the parameters of FFT size N=256, v=64, L=64,amplitude imbalance at transmitter and receiver εr, εr=5% of actual amplitude, phase imbalance is Φr , Φr=50° and the CFO (Δf)=240KHz when local oscillator is operating at a frequency of 5 to 6GHz. In this the step size ‘Δ’ is assumed to be 0.35.

In fig (7) shows the performance curves of LMS 2 tap equalizer in AWGN channel. The performance is compared with the ideal system with no IQ imbalance at transmitter and receiver and CFO.

A LMS equalizer is sufficient if CP≥L the weight of LMS equalizer converges easily. The simulations are carried out for SNR value up to 40 db. The BER is approximately 10^4 however it is very near to ideal case.

In this case the CP is lesser than the Channel Impulse Response (υ≤L) means the IBI exists in this case, assuming υ=48 and rest of parameters similar to case1. The LMS equalizer in fig (6) is replaced with a adaptive RLS based PTEQ equalize with more taps.

The fig (8) shows the performance of Rayleigh fading channel. And the fading is realized using jakes model. When CP≤L the LMS linear equalizer will perform well. In case of CP≤L the LMS equalizer converges easily but the IBI cannot be eliminated. In such case a PTEQ equalizer is replaced with LMS shown in fig(6).

In case of CP≥L the fig(9) shows the performance curves of PTEQ equalizer with variable tap lengths L=1,10 and 12 and assuming a channel length of 64and CP of length 48. It shows that the above equalizer compensate the Transmitter and Receiver IQ imbalance along with CFO. At the same time it is observed that as the no of taps increasing the BER curve is approaching towards to curve of ideal case. In the assumed case the length difference between the channel and CP is 12. So, 12 taps can compensate the BER loss. As per the simulations the BER loss varies for the value of L = 1,10and 12 are 11.42, 2.3 and 1.13 in dB. The performance also depends on the weights of adaptive equalizer how fast and accurate they converge.

In fig (10) it shows the comparison between LMS and RLS based equalization in case of CP ≥ L. It seems to be the LMS based equalization is very near to the PTEQ equalizer with L’=1. So, the LMS based adaption is very suitable for case i and the performance is close to ideal curve. The PTEQ equalizer with RLS adaption with suitable no. of taps will perform well in case of CP ≥ L.

**Conclusions**

In this paper the compensation of both Transmitter and Receiver IQ imbalance along with CFO was discussed using adaptive equalizer. The proposed equalizer with LMS algorithm has performed well in case of CP ≥ L. Similarly the PTEQ equalizer has performed well in case of CP ≤ L and the BER is very close to the ideal system. The PTEQ equalizer with more no. of taps can compensate better and the BER approaches to ideal case. But increasing the number of taps is a costly effort. However this method is very accurate since BER has approached to ideal case. This proposed equalizer is suitable for
the systems which operate for lesser difference in the length of cyclic prefix and Channel Impulse Response, which results in less no. of taps.

Fig.7. AWGN channel with IQ imbalance and CFO in case i.

Similarly for CP ≤ L, PTEQ equalizer has performed well compared to LMS. However, CP > L the PTEQ equalizer with more number of taps could compensate better for CFO and IQ imbalance in presence of IBI. And the BER has approached to ideal case.

Fig.8. Rayleigh fading channel using L1 = 12 in case ii

Fig.9. Rayleigh fading channel using L1 = 1, 10, 12 in case ii

Fig.10. Comparison for Rayleigh fading using LMS and RLS in case ii

References