Introduction

The theory of micropolar fluids was originally formulated by Eringen [1]. In essence, the theory introduces new material parameters, an additional independent vector field, the microrotation and new constitutive equations which must be solved simultaneously with the usual equations for Newtonian flow. The desire to model the non-Newtonian flow of fluid containing rotating micro-constituents prporous media, turbulent shear flows, and flowing capillaries and microchannels is driven by Lukaszewiez [2].

We analyze the effect of the variable viscosity and the variable thermal conductivity on self-similar boundary layer flow of a micropolar fluid in a porous channel, where the flow is driven by uniform mass transfer through the channel walls. The corresponding Newtonian fluid model was first studied by Berman [3], who described an exact solution of the Navier-Stokes equations by assuming a self-similar solution and reducing the governing partial differential equations to a nonlinear ordinary differential equation of fourth order. The solution is of potential value in understanding more realistic flow in channels and pipes, and study of Berman’s exact solution and generalizations of it have attracted numerous studies subsequently, for example Yuan [4], Robinson [5], Zaturska et. al. [6], Desseaux [7]. Through the viscosity and thermal conductivity are assumed as constant properties but in actual these are temperature dependent (Schlichiting [8], Eckert[9]). Therefore, in this paper we consider the effect of variable viscosity and variable thermal conductivity on stagnation flow of a micropolar fluid towards a vertical permeable surface.

Formulation of the problem:

Consider a laminar two-dimensional stagnation flow of an incompressible micropolar fluid impinges normal to a vertical plate. It is assumed that the free stream velocity $U(x)$ and the temperature of the plate $T_w(x)$ vary linearly with the distance $x$ from the stagnation point, i.e. $U(x) = ax$ and $T_w(x) = T_w + bx$, where $a, b$ are positive constants. The steady laminar boundary layer equations governing the flow are

Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0$$

Equation of momentum
\[
\frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = U \frac{dU}{dx} + \left( \frac{\mu + k}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho} \frac{\partial N}{\partial y} \pm g \beta (T - T_\infty) \tag{2}
\]

The angular momentum equation
\[
\rho j \left( \frac{\partial u}{\partial x} + v \frac{\partial N}{\partial y} \right) = \gamma \frac{\partial^2 N}{\partial y^2} - k \left( 2N + \frac{\partial u}{\partial y} \right) \tag{3}
\]

The energy equation
\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{4}
\]

subject to the boundary conditions
\[
u = V_w, \quad N = -\frac{1}{2} \frac{\partial u}{\partial y} \quad T = T_w(x) \quad \text{at} \quad y = 0
\]
\[
u \rightarrow U(x) \cdot N \rightarrow 0 \cdot T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty
\]

Where \( u \) and \( v \) are the velocity components along the x-axis respectively, \( T \) is the fluid temperature, \( N \) is the component of the microrotation vector normal to the x-y plane, \( \rho \) is the density, \( j \) is the microinertia density, \( \mu \) is the dynamic viscosity, \( k \) is the gyro-viscosity, \( \gamma \) is the spin gradient viscosity and \( V_w \) is the uniform surface mass flux. The last term on the right hand side of equation (2) represents the influence of the thermal buoyancy force on the flow field with “+” and “-” signs pertaining respectively to the buoyancy assisting and the buoyancy opposing flow regions. We assume that
\[
\gamma = \left( \mu + \frac{k}{2} \right) j = \mu \left( 1 + \frac{K}{2} \right) j, \quad \text{where} \quad K = \frac{k}{\mu}
\]

To seek similarity solutions for equations (1)-(4) subject to the boundary conditions (5), introduce the following dimensionless similarity variables:
\[
\eta = y \left( \frac{U}{v_x} \right)^{\frac{1}{2}}, \quad f(\eta) = \frac{\psi}{(v_x U)^{\frac{1}{2}}}, \quad g(\eta) = \left( \frac{v_x}{U} \right)^{\frac{1}{2}} N, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}
\]

where \( \eta \) is the independent similarity variable, \( f(\eta) \) is dimensionless stream function, \( g(\eta) \) is dimensionless microrotation, \( \theta(\eta) \) is dimensionless temperature and \( V \) is the kinematic viscosity of the fluid. Further, \( \psi \) is the stream function which is defined in the usual way as \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \) so as to identically satisfy equation (1). Using (6), we get
\[
u = U f'(\eta), \quad v = -\left( v_x \right)^{\frac{1}{2}} f(\eta) \tag{7}
\]

The fluid viscosity is assumed to be inverse linear function of temperature as
\[
\frac{1}{\mu} = \frac{1}{\mu_\infty} \left[ 1 + \alpha (T - T_\infty) \right], \quad \frac{1}{\mu} = a (T - T_r), \quad a = \frac{\alpha}{\mu_\infty} \quad \text{and} \quad T_r = T_\infty - \frac{1}{\alpha}
\]

where \( a \) and \( T_r \) are constants and their values depends on the reference state and the thermal property of the fluid. In general \( a > 0 \) for liquids and \( a < 0 \) for gases. \( T_r \) is transformed reference temperature related to viscosity parameter. \( \alpha \) is constant based on thermal property and \( \mu_\infty \) is the viscosity at \( T = T_\infty \) similarly, consider the variation of thermal conductivity as,
\[
\frac{1}{\lambda} = \frac{1}{\lambda_\infty} \left[ 1 + \xi (T - T_\infty) \right], \quad \frac{1}{\lambda} = b(T - T_k) \quad b = \frac{\xi}{\lambda_\infty} \quad \text{and} \quad T_k = T_\infty - \frac{1}{\xi} \tag{9}
\]

where \( b \) and \( T_k \) are constants and their values depend on the reference state and thermal property of the fluid. \( \xi \) is constant based on thermal property and \( \lambda_\infty \) is the viscosity at \( T=\infty \).

Using equation (6), it can be easily verified that the continuity equation is satisfied automatically and using equation (6) - (9) in the equations (2),(3) and (4) become,

\[
f'' = \frac{\theta_c - \theta}{K(\theta_c - \theta) + \theta_c} f'^2 - \frac{\theta_c \theta'}{(\theta_c - \theta) \left[ K(\theta_c - \theta) + \theta_c \right]} f'' - \frac{\theta_c - \theta}{K(\theta_c - \theta) + \theta_c} \{ ff'' + Kg' + B\theta + 1 \} \tag{10}
\]

\[
g'' = \frac{2}{2 + K} \left\{ \frac{1}{G} (2g + f'^2) + (f'g' - fg') \right\} \tag{11}
\]

\[
\theta'' = -\frac{\theta_c^2}{\theta_c - \theta} - \frac{\theta_c - \theta}{\theta_c} P_r (f \theta' - f' \theta) \tag{12}
\]

The corresponding boundary conditions are

\[
f(0) = f_0, \quad f'(0) = 0, \quad g(0) = -\frac{1}{2} f'', \quad \theta(0) = 1 \tag{13}
\]

\[
f'(\eta) \to 1, \quad g(\eta) \to 0 \quad \text{as} \quad \eta \to \infty
\]

**Results and discussion:**

The equations (10)-(12) together with the boundary conditions (13) are solved for various values of the parameters involved in the equations using algorithms based on the shooting method. Results are presented for velocity distribution, microrotation distribution and temperature distribution with the variation of different parameters.

Initially solution was taken for constant values of taking \( Pr=0.70, G=0.51, K=2.00, \theta v=-10.00, \theta c=-10.00 \) with the viscosity parameter \( V_r \) ranging from -10 to -1 at the certain values of \( \theta c=-10.00 \). Similarly the solutions have been found with varying the thermal conductivity parameter \( \theta c \) ranging from -10 to -1 at the certain values of \( \theta v=-10 \) keeping other values remaining same. We have considered in some detail the influence of physical parameters on velocity distribution, microrotation distribution and temperature distribution which is shown in figures 1-5. The figures 1 and 2 show the variations in velocity and microrotation distribution with the variation of viscosity parameter \( \theta v \). From the figures it is clear that the velocity decreases as \( \theta v \) increases. Figures 3 and 4 show the variations in temperature and velocity distribution with the variation of thermal conductivity parameter \( \theta c \). From the figures it is seen that both temperature and velocity decreases as \( \theta c \) increases. From figure 5, it is clear that the velocity increases with the increasing value of \( B \).

![Fig.1. Velocity distribution profile (f') with the variation of \( \theta v \)](image)
Fig. 2. Microrotation distribution profile ($g$) with the variation of $\Theta_v$

Fig. 3. Temperature distribution profile ($\Theta$) with the variation of $\Theta_c$

Fig. 4. Velocity distribution profile ($f'$) with the variation of $\Theta_c$
Conclusion

In this study, the stagnation flow of a micropolar fluid towards a vertical permeable surface is investigated when the viscosity and thermal conductivity are assumed to vary with temperature. The results presented demonstrate clearly that the viscosity parameter has a substantial effect on velocity and microrotation distribution, while the effect on temperature distribution is very pronounced due to the variation of thermal conductivity parameter. In addition, buoyancy parameter B takes a major role in the variation of velocity distribution. Thus the assumption of constant properties may lead severe errors in the design of fluid machinery and in various flow problems.

References