Method of telecommunications channel throughput distribution based on linear programming and neuro fuzzy predicting

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ABSTRACT
Decision making method of telecommunications channel throughput distribution based on linear programming and neuro fuzzy predicting tool

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Keywords
Telecommunications channel throughput,
Linear programming,
Neuro fuzzy predicting.

Introduction

Known methods channel bandwidth distribution application do not allow maximum providers income attainment and do not facilitate cutting time of end users messages receiving.

Reasonable telecommunications channel throughput distribution for traffic classes with different cost is an actual scientific and technical problem [1, 2]. This problem successful solution allows income maximizing of telecommunications services providers and improving quality of service for end users.

Main part

Let us consider telecommunications network channel where two traffic classes are transmitted during consequent cycles, which consist of \( C \) time units. One time unit can be used for any one packet transmission. Time units’ quantity within one cycle is defined by channel throughput, i.e. the maximum packet quantity which can be transmitted during this cycle.

For the reasonable throughput distribution in such channel it is proposed to use linear programming mathematical tool [3]. In this case it is necessary to use the following objective function:

\[
\gamma_1 (Y_{11} + Y_{12}) + \gamma_2 (Y_{21} + Y_{22}) \rightarrow \text{max},
\]

where \( \gamma_1 \) – income received by provider as a result of one packet of the 1st class transmission; \( \gamma_2 \) – income received by provider as a result of one packet of the 2nd class transmission; \( Y_v \) – packets of the \( v \) class quantity which will be transmitted in the channel during the cycle \( u \). It is necessary to find such values of \( Y_{11}, Y_{12}, Y_{21}, Y_{22} \), at which sum income received by provider during two nearest cycles is maximum.

For making a decision about channel throughput distribution the following constraints system is composed:

\[
\begin{align*}
Y_{11} & \leq q_{11} + Z_{11}; \\
Y_{12} & \leq q_{12} + Z_{12}; \\
Y_{11} + Y_{12} & \leq q_{11} + Z_{11} + Z_{12}; \\
Y_{21} + Y_{22} & \leq q_{21} + Z_{21} + Z_{22}; \\
Y_{31} & \leq Q_1 + Z_{31}; \\
Y_{32} & \leq Q_2 + Z_{32}; \\
Y_{31} + Y_{32} & \leq C; \\
Y_{21} + Y_{22} & \leq C; \\
Y_{11} & \geq 0; \ Y_{12} \geq 0; \ Y_{21} \geq 0; \ Y_{22} \geq 0.
\end{align*}
\]
where \( q_{uv} \) – packets quantity in the queue of the class \( v \) at \( u \) class starting; \( Z_{uv} \) – class \( v \) packets quantity, entering the system in the cycle \( u \); \( Q_i \) – maximum packet queue size of the class \( 2 \). Proposed method is based on simplex tables’ [4] usage for the solving of linear programming problem with defined inequalities system (2) and objective function (1):

\[
\begin{aligned}
Y_{11} & \leq b_1 ; \\
Y_{12} & \leq b_2 ; \\
Y_{11} + Y_{21} & \leq b_3 ; \\
Y_{12} + Y_{22} & \leq b_4 ; \\
Y_{21} & \leq b_5 ; \\
Y_{22} & \leq b_6 ; \\
Y_{11} + Y_{12} & \leq b_7 ; \\
Y_{21} + Y_{22} & \leq b_8 ,
\end{aligned}
\]

where

\[
\begin{align*}
b_1 &= q_{11} + Z_{11} , \quad (4) \\
b_2 &= q_{12} + Z_{12} , \quad (5) \\
b_3 &= q_{11} + Z_{11} + Z_{21} , \quad (6) \\
b_4 &= q_{12} + Z_{12} + Z_{22} , \quad (7) \\
b_5 &= Q_i + Z_{21} , \quad (8) \\
b_6 &= Q_i + Z_{22} , \quad (9) \\
b_7 &= b_k = C . \quad (10)
\end{align*}
\]

With help of additional variables \( X_1 \), \( X_2 \), ..., \( X_8 \) inequalities system (3) can be modified into linear equations system:

\[
\begin{align*}
Y_{11} + X_1 &= b_1 ; \\
Y_{12} + X_2 &= b_2 ; \\
Y_{11} + Y_{21} + X_3 &= b_3 ; \\
Y_{12} + Y_{22} + X_4 &= b_4 ; \\
Y_{21} + X_5 &= b_5 ; \\
Y_{22} + X_6 &= b_6 ; \\
Y_{11} + Y_{12} + X_7 &= b_7 ; \\
Y_{21} + Y_{22} + X_8 &= b_8 ;
\end{align*}
\]

The system (11) can be represented in canonic form; initial simplex table corresponding to it is shown in Table 1:

<table>
<thead>
<tr>
<th>( i )</th>
<th>( j )</th>
<th>( b_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

For initial simplex table leading columns definition it is necessary to use the following rule:

**Rule 1.** Leading column – is \( n \) number column which contains \( d_n \) element with maximum value.
It is necessary to define leading element in the found leading columns according to the following rule:

Rule 2. Leading element – is $a_{mn}$ element of a leading column with such values $m$ and $n$ that $a_{mn}b_m$ product is minimal positive number.

Afterwards initial simplex table should be modified into the simplex table of the first iteration. For this end it is necessary to apply the following formulas:

\[
a_{nj}^{k+1} = \frac{a_{nj}}{a_{mn}}, \quad j = \left[ \frac{1}{12} \right];
\]

\[
b_m^{k+1} = \frac{b_m^*}{a_{mn}};
\]

\[
a_i^{k+1} = a_i - \frac{a_i^*}{a_{mn}}, \quad i \neq m, \quad j = \left[ \frac{1}{12} \right];
\]

\[
b_i^{k+1} = b_i - \frac{b_i^*}{a_{mn}}, \quad i \neq m,
\]

\[
a_j^{k+1} = a_j - \frac{a_j^*}{a_{mn}}, \quad j = \left[ \frac{1}{12} \right],
\]

where $k$ – is iteration number. For initial simplex table elements $k = 0$. Afterwards it is necessary to check whether unknown solution is found. Received solution attribute is positive values absence in the row of modified simplex table. If the solution is not received, then procedures of founding leading column and element are provided till the solution is not received.

If the solution is received, but any element negative value in $d_j$ row is changed to 0 as a result of modification, then alternative solution exists. In this case the column containing such an element is considered as leading column. New leading element is found in it, simplex table is modified once more. Then it is necessary to check if alternative solution is found. If new solution is not found, all the above mentioned procedures are repeated again. This process is ended when all solutions are found.

The definite simplex table form corresponds to every solution and it is possible to find in it unknown values of $Y_{11}$, $Y_{12}$, $Y_{21}$ and $Y_{22}$ due to the following rule:

Rule 3. The value of $Y_{11}$ is equal to the element $b_i$ value with such $i$ that $a_i = 1$; the value of $Y_{12}$ is equal to the element $b_i$ value with such $i$ that $a_{i2} = 1$; the value of $Y_{21}$ is equal to the element $b_i$ value with such $i$ that $a_{i3} = 1$; the value of $Y_{22}$ is equal to the element $b_i$ value with such $i$ that $a_{i4} = 1$.

It is necessary to choose one solution among all received which leads to the least packet losses, which can be calculated by the following formula:

\[
R_2 = R_{12} + R_{21} + R_{22}
\]

For $R_{11}$, $R_{12}$, $R_{21}$ and $R_{22}$ values calculations the following expression can be used:

\[
R_{11} = q_{21} - Q_1, \quad q_{21} > Q_1; \quad q_{21} \leq Q_1;
\]

\[
R_{12} = q_{22} - Q_2, \quad q_{22} > Q_2; \quad q_{22} \leq Q_2;
\]

\[
R_{21} = q_{12} + Z_{21} - Y_{21} - Q_1, \quad q_{12} + Z_{21} - Y_{21} > Q_1; \quad q_{12} + Z_{21} - Y_{21} \leq Q_1;
\]

\[
R_{22} = q_{22} + Z_{22} - Y_{22} - Q_2, \quad q_{22} + Z_{22} - Y_{22} > Q_2; \quad q_{22} + Z_{22} - Y_{22} \leq Q_2;
\]

where $q_{21}$ – packets quantity in the class 1 queue in the beginning of the cycle 2, if this queue size is unlimited.
\[ q_{21}' = Z_{11} + q_{11} - Y_{11}; \]  
\[ q_{22}' = Z_{12} + q_{12} - Y_{12}. \]  
(22)  
(23)

Thus, for decision making of the telecommunication channel bandwidth distribution during two classes of data packets transmission, a new method consisting of the following steps is proposed.

Step 1. Initial simplex table formation. In this step elements of table 1 are defined. At that \( b_i^0 \) elements are calculated by the formulas (4) – (10) and values of \( a_0^0, d_0^0, d_1^0, d_2^0, d_3^0, d_4^0 \) elements are considered as: \( d_0^0 = d_1^0 = \gamma_1 \) and \( d_2^0 = d_4^0 = \gamma_2 \).

Step 2. Leading column founding. 
In this step according to rule 1 leading column is chosen.

Step 3. Leading element founding. 
In this step according to the rule 2 in simplex table leading element is found.

**Step 4. Simplex table modification.**
In this step modified simplex table elements are calculated by the formulas (12) – (16).

Step 5. Solution founding. 
In this step elements with positive values in the row \( d_j \) availability is checked. If there are such elements it is necessary to return to step 2. Otherwise the solution is found. According to it unknown values \( Y_{11}, Y_{12}, Y_{21}, Y_{22} \) are defined according to the Rule 3.

Step 6. Alternative solution checking. 
In this step in the row \( d_j \) the element which value is equal to 0 in simplex table containing solution is found. This value is also equal to the negative value in the simplex table of the previous iteration. If there is no such an element, then pass to the Step 7 is accomplished. 
Otherwise, column containing such an element is considered as leading and for alternative solution search it is necessary to return to the Step 3.

Step 7. Decision making. 
In this step packet losses are calculated for every solution found by formulas (17) – (23). Then solution which provides the least packet losses is chosen respectively to the values \( Y_{11}, Y_{12}, Y_{21}, Y_{22} \), from the set of found solutions.

The subject of further investigation is effective prediction of different classes packets arrival intensity method prediction for the transmission in the telecommunication channel. 

Computations show that proposed method application allows 15% providers’ income increase and 40% packet loss decrease in comparison with classic priority servicing.

Efficient application of this method can be achieved if reliable data about \( Z_{11}, Z_{12}, Z_{21}, Z_{22} \) are available. To receive these data it is necessary to apply fuzzy neural forecasting tool [5 – 6].

For prediction of packets quantity, entering the system during \( i \) time unit (outcome value \( \tilde{Z}_i \)) it is offered to use fuzzy neural network (Fig.1). Packets quantity, entering the system, measured during four previous time units (income values \( Z_{i-4}, Z_{i-3}, Z_{i-2}, Z_{i-1} \)) are applied to this system input.
It is reasonable to construct fuzzy neural network according to the technique [6], assuming the following steps accomplishment:

1) fuzzy inference algorithm choosing;
2) membership functions definition for the income values;
3) membership functions quantity choosing;
4) membership functions form choosing;
5) fuzzy neural network learning algorithm choosing;
6) learning selection creation for fuzzy neural network parameters setup;
7) cycles quantity choosing for fuzzy neural network choosing;
8) fuzzy neural network parameters setup (learning).

Neural fuzzy network should be maximum simple, although it should provide sufficient prediction quality.

With due regard to these criteria the synthesized neural fuzzy system, following parameters are chosen: fuzzy output algorithm – Sugeno of the first order [7], membership functions quantity for every income value – 2, membership functions form for every income value – triangle, neurons training algorithm – back propagation of error algorithm [8].

As a result equations for income values membership functions are received:

\[
\mu_i(Z_{i-4}) = \begin{cases} 
1, & Z_{i-4} < 0; \\
\frac{94.9 - Z_{i-4}}{94.9}, & 0 \leq Z_{i-4} < 94.9; \\
0, & Z_{i-4} \geq 94.9; 
\end{cases} 
\]  \hspace{1cm} \text{(24)}

\[
\mu_i(Z_{i-3}) = \begin{cases} 
0, & Z_{i-3} < 0; \\
\frac{Z_{i-3} - 94.9}{94.9}, & 0 \leq Z_{i-3} < 94.9; \\
1, & Z_{i-3} \geq 94.9; 
\end{cases} 
\]  \hspace{1cm} \text{(25)}

where \( k = 1, 2, 3, 4 \).

Except for that, fuzzy neural network setup allowed receiving individual output fuzzy rules coefficients. Received neuro fuzzy system functioning is based on usage of fuzzy rules base:

If \( (Z_{i-4} = \alpha^1) \) and \( (Z_{i-3} = \beta^1) \) and \( (Z_{i-2} = \gamma^1) \) and \( (Z_{i-1} = \delta^1) \), then \( \tilde{Z}_i = Y_1 \).

\hspace{1cm} \text{(26)}

If \( (Z_{i-4} = \alpha^2) \) and \( (Z_{i-3} = \beta^2) \) and \( (Z_{i-2} = \gamma^2) \) and \( (Z_{i-1} = \delta^2) \), then \( \tilde{Z}_i = Y_2 \).

\hspace{1cm} \text{(27)}

If \( (Z_{i-4} = \alpha^3) \) and \( (Z_{i-3} = \beta^3) \) and \( (Z_{i-2} = \gamma^3) \) and \( (Z_{i-1} = \delta^3) \), then \( \tilde{Z}_i = Y_3 \).

\hspace{1cm} \text{(28)}

If \( (Z_{i-4} = \alpha^4) \) and \( (Z_{i-3} = \beta^4) \) and \( (Z_{i-2} = \gamma^4) \) and \( (Z_{i-1} = \delta^4) \), then \( \tilde{Z}_i = Y_4 \).

\hspace{1cm} \text{(29)}
If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_5 \) \hfill (30)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_6 \) \hfill (31)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_7 \) \hfill (32)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_8 \) \hfill (33)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_9 \) \hfill (34)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_{10} \) \hfill (35)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_{11} \) \hfill (36)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_{12} \) \hfill (37)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_{13} \) \hfill (38)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_{14} \) \hfill (39)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_{15} \) \hfill (40)

If \( Z_{i-4} = \alpha^i \) and \( Z_{i-3} = \beta^i \) and \( Z_{i-2} = \gamma^i \) and \( Z_{i-1} = \delta^i \), then \( \tilde{Z}_i = Y_{16} \) \hfill (41)

where \( \alpha^1 \) – term number 1 of income value \( Z_{i-4} \); \( \alpha^2 \) – term number 2 of income value \( Z_{i-4} \); \( \beta^1 \) – term number 1 of income value \( Z_{i-3} \); \( \beta^2 \) – term number 2 of income value \( Z_{i-3} \); \( \gamma^1 \) – term number 1 of income value \( Z_{i-2} \); \( \gamma^2 \) – term number 2 of income value \( Z_{i-2} \); \( \delta^1 \) – term number 1 of income value \( Z_{i-1} \); \( \delta^2 \) – term number 2 of income value \( Z_{i-1} \); \( Y_r \) – individual fuzzy rule output number \( r \), \( r = 1, 2, \ldots, 16 \).

According to Sugeno algorithm of the 1st order the values of individual outputs are defined with help of the following expression:

\[
y_r = a_r Z_{i-4} + b_r Z_{i-3} + c_r Z_{i-2} + d_r Z_{i-1} + e_r .
\] \hfill (42)

Neuro fuzzy system contains 5 layers. The first layer accomplishes fuzzification procedure which consists in values \( \mu(Z'_{i-4}) \) and \( \mu(Z'_{i-3}) \) calculation by the formulas (24) and (25) for definite income values \( Z'_{i-4}, Z'_{i-3}, Z'_{i-2} \) and \( Z'_{i-1} \).

The second layer of neuro fuzzy system performs aggregation procedure, during it conditions truth degrees for every rule at concise income values:

\[
G_1 = \mu(Z'_{i-4}) \land \mu(Z'_{i-3}) \land \mu(Z'_{i-2}) \land \mu(Z'_{i-1}) ,
\] \hfill (43)

\[
G_2 = \mu(Z'_{i-4}) \land \mu(Z'_{i-3}) \land \mu(Z'_{i-2}) \land \mu(Z'_{i-1}) ,
\] \hfill (44)
\[ G_3 = \mu_3(Z_{i-2}) \land \mu_2(Z_{i-1}) \land \mu_2(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (45)
\[ G_4 = \mu_3(Z_{i-2}) \land \mu_4(Z_{i-3}) \land \mu_2(Z_{i-1}) \land \mu_4(Z_{i-4}), \] (46)
\[ G_5 = \mu_2(Z_{i-4}) \land \mu_4(Z_{i-3}) \land \mu_1(Z_{i-1}) \land \mu_4(Z_{i-4}), \] (47)
\[ G_6 = \mu_2(Z_{i-4}) \land \mu_2(Z_{i-3}) \land \mu_3(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (48)
\[ G_7 = \mu_2(Z_{i-4}) \land \mu_2(Z_{i-3}) \land \mu_4(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (49)
\[ G_8 = \mu_2(Z_{i-4}) \land \mu_3(Z_{i-3}) \land \mu_5(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (50)
\[ G_9 = \mu_2(Z_{i-4}) \land \mu_3(Z_{i-3}) \land \mu_4(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (51)
\[ G_{10} = \mu_2(Z_{i-4}) \land \mu_1(Z_{i-3}) \land \mu_4(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (52)
\[ G_{11} = \mu_2(Z_{i-4}) \land \mu_2(Z_{i-3}) \land \mu_4(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (53)
\[ G_{12} = \mu_2(Z_{i-4}) \land \mu_3(Z_{i-3}) \land \mu_5(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (54)
\[ G_{13} = \mu_2(Z_{i-4}) \land \mu_3(Z_{i-3}) \land \mu_4(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (55)
\[ G_{14} = \mu_2(Z_{i-4}) \land \mu_3(Z_{i-3}) \land \mu_2(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (56)
\[ G_{15} = \mu_2(Z_{i-4}) \land \mu_3(Z_{i-3}) \land \mu_5(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (57)
\[ G_{16} = \mu_2(Z_{i-4}) \land \mu_3(Z_{i-3}) \land \mu_4(Z_{i-2}) \land \mu_4(Z_{i-4}), \] (58)

With help of the third neurons layer aggregation normalization results is accomplished:
\[ \overline{G}_r = \frac{G_r}{\sum_{i=1}^{16} G_i}, \] (59)

The fourth layer provides activation by the formula (42) and calculates normalization and activation results product:
\[ y_r = \overline{G}_r \cdot Y_r. \] (60)

Defuzzification procedure (explicit value of outcome value) is provided by the fifth neurons layer. At that the results of fourth system layer functioning are added:
\[ \overline{Z}_r = \sum_{i=1}^{16} y_r. \] (61)

Synthesized fuzzy neural network can be applied for receive the values \( Z_{11}, Z_{12}, Z_{21} \) and \( Z_{22} \). Imitation experiments results show that neuro fuzzy system application allows affordable prediction quality getting.

**Conclusion**

Decision making method of telecommunications channel throughput distribution is proposed. This method is based on linear programming and neuro fuzzy predicting tool usage. Application of this method allows provider’s income maximizing and packet loss decrease for transmission of different classes’ data flows.

**References**


