New algorithm for solving mixed intuitionistic fuzzy assignment problem

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**Abstract**

In conventional assignment problem, cost is always certain. In this paper, Assignment problem with crisp, fuzzy and intuitionistic fuzzy numbers as cost coefficients is investigated. There is no systematic approach for finding an optimal solution for mixed intuitionistic fuzzy assignment problem. This paper develops an approach to solve a mixed intuitionistic fuzzy assignment problem where cost is not in deterministic numbers but imprecise ones. The solution procedure of mixed intuitionistic fuzzy assignment problem is proposed to find the optimal assignment and also obtain an optimal value in terms of triangular intuitionistic fuzzy numbers. Numerical examples show that an intuitionistic fuzzy ranking method offers an effective tool for handling an intuitionistic fuzzy assignment problem.

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**Introduction**

Assignment Problem (AP) is used worldwide in solving real world problems. An assignment problem plays an important role in an assigning of persons to jobs, or classes to rooms, operators to machines, drivers to trucks, trucks to routes, or problems to research teams, etc. The assignment problem is a special type of linear programming problem (LPP) in which our objective is to assign n number of jobs to n number of machines (persons) at a minimum cost. To find solution to assignment problems, various algorithm such as linear programming [8,9,13,17], Hungarian algorithm [15], neural network [12], genetic algorithm [6] have been developed.

However, in real life situations, the parameters of assignment problem are imprecise numbers instead of fixed real numbers because time/cost for doing a job by a facility (machine/person) might vary due to different reasons. The theory of fuzzy set introduced by Zadeh [25] in 1965 has achieved successful applications in various fields. In 1970, Belmann and Zadeh introduce the concepts of fuzzy set theory into the decision-making problems involving uncertainty and imprecision [7]. Amit Kumar et al investigated Assignment and Travelling salesman Problems with cost coefficients as LR fuzzy parameters [1]. Fuzzy linear programming approach for solving transportation problems with transshipment [2], Method for solving fully fuzzy assignment problems using triangular fuzzy numbers [3]. In [18], Sathi Mukherjee et al presented an Application of fuzzy ranking method for solving assignment problems with fuzzy costs. Lin and Wen [16] proposed an efficient algorithm based on labeling method for solving the linear fractional programming case. Y.L.P. Thorani and N. Ravi Sankar did Fuzzy assignment problem with generalized fuzzy numbers [23]. Different kinds of fuzzy assignment problems are solved in the papers [1,3,10,11,12,24].

The concept of Intuitionistic Fuzzy Sets (IFSs) proposed by Atanassov [5] in 1986 is found to be highly useful to deal with vagueness. In [14], Jahir Hussain et al presented An Optimal More-for-Less Solution of Mixed Constrains Intuitionistic Fuzzy Transportation Problems. P. Senthil Kumar et al did a systematic approach for solving mixed intuitionistic fuzzy transportation problems, A method for solving balanced intuitionistic fuzzy assignment problem [20,21]. Here we investigate a more realistic problem, namely mixed intuitionistic fuzzy assignment problem. Let $z_{ij}^d$ be the intuitionistic fuzzy cost of assigning the $i^{th}$ job to the $i^{th}$ machine. We assume that one machine can be assigned exactly one job; also each machine can do at most one job. The problem is to find an optimal assignment so that the total intuitionistic fuzzy cost of performing all jobs is minimum or the total intuitionistic fuzzy profit is maximum.

Assignment problem with crisp, fuzzy and intuitionistic fuzzy numbers as cost coefficients is called mixed intuitionistic fuzzy assignment problem. Here the objective function is considered with crisp, fuzzy and intuitionistic fuzzy numbers. Then there is no
systematic approach for finding an optimal solution for mixed intuitionistic fuzzy assignment problem but when we search our literature it has demonstrated intuitionistic fuzzy assignment problems\cite{19,21,22} only. In this paper, ranking procedure of Annie Varghese and Sunny Kuriakose \cite{4} is used to rank the intuitionistic fuzzy numbers and also compare the minimum and maximum of it. The proposed method is used to transform the mixed intuitionistic fuzzy assignment problem into balanced intuitionistic fuzzy assignment problem so that an intuitionistic fuzzy Hungarian method may be applied to solve the AP.

This paper is organized as follows: Section 2 deals with some basic terminology, In section 3, provides the definition of intuitionistic fuzzy assignment problem and its mathematical formulation, Section 4, consists of solution procedure for mixed intuitionistic fuzzy assignment problem. In section 5, to illustrate the proposed method a numerical example with results and discussion is discussed and followed by the conclusions are given in Section 6.

Preliminaries

**Definition 2.1** Let A be a classical set, \( \mu_A(x) \) be a function from A to \([0,1]\). A fuzzy set \( A^- \) with the membership function \( \mu_A(x) \) is defined by

\[
A^- = \{ (x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0, 1] \}
\]

**Definition 2.2** Let X be denote a universe of discourse, then an intuitionistic fuzzy set A in X is given by a set of ordered triples, \( \tilde{A} = \{ x, \mu_A(x), \theta_A(x) \} \in X \)

Where \( \mu_A, \theta_A : X \rightarrow [0, 1] \) are functions such that \( 0 \leq \mu_A(x) + \theta_A(x) \leq 1 \), \( \forall x \in X \). For each x the membership \( \mu_A(x) \) and \( \theta_A(x) \) represent the degree of membership and the degree of non-membership of the element \( x \in X \) to \( A \subset X \) respectively.

**Definition 2.3** A fuzzy number A is defined to be a triangular fuzzy number if its membership functions \( \mu_A: \mathbb{R} \rightarrow [0, 1] \) is equal to

\[
\mu_A(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1} & \text{if } x \in [a_1, a_2] \\
\frac{a_3 - x}{a_3 - a_2} & \text{if } x \in [a_2, a_3] \\
0 & \text{otherwise}
\end{cases}
\]

where \( a_1 \leq a_2 \leq a_3 \). This fuzzy number is denoted by \( (a_1, a_2, a_3) \).

**Definition 2.4** A Triangular Intuitionistic Fuzzy Number \( \tilde{A} \) is an intuitionistic fuzzy set in \( \mathbb{R} \) with the following membership function \( \mu_A(x) \) and non membership function \( \theta_A(x) \):

\[
\mu_A(x) = \begin{cases} 
0 & \text{for } x < a_1 \\
\frac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2 \\
1 & \text{for } x = a_2 \\
\frac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\
0 & \text{for } x > a_3
\end{cases} \quad \theta_A(x) = \begin{cases} 
1 & \text{for } x < a_1' \\
\frac{a_2 - x}{a_2 - a_1'} & \text{for } a_1' \leq x \leq a_2 \\
\frac{a_2 - a_1'}{a_2 - a_1} & \text{for } a_1' \leq x \leq a_2 \\
0 & \text{for } x = a_2 \\
\frac{x - a_2}{a_3' - a_2} & \text{for } a_2 \leq x \leq a_3' \\
\frac{a_3' - a_2}{a_3' - a_2} & \text{for } a_2 \leq x \leq a_3' \\
1 & \text{for } x > a_3'
\end{cases}
\]

Where \( a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3' \) and \( \mu_A(x), \theta_A(x) \leq 0.5 \) for \( \mu_A(x) = \theta_A(x) \ \forall x \in R \). This TrIFN is denoted by \( \tilde{A} = (a_1, a_2, a_3)(a_1', a_2, a_3') \)

Particular Cases

Let \( \tilde{A} = (a_1, a_2, a_3)(a_1', a_2, a_3') \) be a TrIFN. Then the following cases arise

Case 1: \( a_1' = a_1, a_3' = a_3 \), then \( \tilde{A} \) represent Tringular Fuzzy Number (TFN). It is denoted by \( \tilde{A} = (a_1, a_2, a_3) \).

Case 2: \( a_1' = a_1 = a_2 = a_3 = a_3' = m \), then \( \tilde{A} \) represent a real number m.

**Definition 2.5** Let \( \tilde{A} \) and \( \tilde{B} \) be two TrIFNs. The ranking of \( \tilde{A} \) and \( \tilde{B} \) by the \( \mathbb{R} (\cdot) \) on \( E \), the set of TrIFNs is defined as follows:

i. \( \mathbb{R} (\tilde{A}) > \mathbb{R} (\tilde{B}) \) iff \( \tilde{A} \succ \tilde{B} \)

ii. \( \mathbb{R} (\tilde{A}) < \mathbb{R} (\tilde{B}) \) iff \( \tilde{A} \prec \tilde{B} \)
iii. \( \mathfrak{R}(\overline{A}^t) = \mathfrak{R}(\overline{B}^t) \) iff \( \overline{A}^t = \overline{B}^t \)

iv. \( \mathfrak{R}(\overline{A}^t + \overline{B}^t) = \mathfrak{R}(\overline{A}^t) + \mathfrak{R}(\overline{B}^t) \)

v. \( \mathfrak{R}(\overline{A}^t - \overline{B}^t) = \mathfrak{R}(\overline{A}^t) - \mathfrak{R}(\overline{B}^t) \)

**Arithmetic Operations**

Let \( \overline{A}^t = (a_{1,1}, a_{2,1}, a_{3,1})(a_{1,2}, a_{2,2}, a_{3,2}) \) and \( \overline{B}^t = (b_{1,1}, b_{2,1}, b_{3,1})(b_{1,2}, b_{2,2}, b_{3,2}) \) be any two TrIFNs then the arithmetic operations as follows:

**Addition:** \( \overline{A}^t \oplus \overline{B}^t = (a_{1,1} + b_{1,1}, a_{2,1} + b_{2,1}, a_{3,1} + b_{3,1})(a_{1,2} + b_{1,2}, a_{2,2} + b_{2,2}, a_{3,2} + b_{3,2}) \)

**Subtraction:** \( \overline{A}^t \ominus \overline{B}^t = (a_{1,1} - b_{1,1}, a_{2,1} - b_{2,1}, a_{3,1} - b_{3,1})(a_{1,2} - b_{1,2}, a_{2,2} - b_{2,2}, a_{3,2} - b_{3,2}) \)

**Ranking of triangular intuitionistic fuzzy numbers**

The Ranking of a triangular intuitionistic fuzzy number \( \overline{A}^t = (a_{1,1}, a_{2,1}, a_{3,1})(a_{1,2}, a_{2,2}, a_{3,2}) \) is defined by Annie Varghese and Sunny Kuriakose [4].

\[
R(\overline{A}^t) = \frac{1}{3} \left[ (a_{3,1} - a_{1,1})(a_{2,1} - 2a_{1,1} - 2a_{1,2}) + (a_{3,1} - a_{1,1})(a_{2,1} + a_{1,2} + a_{3,1}) + 3(a_{3,1}^2 - a_{1,1}^2) \right] / a_{3,1}^2 - a_{1,1}^2 + a_{3,1} - a_{1,1}
\]

The ranking technique [4] is:

If \( \mathfrak{R}(\overline{A}^t) \leq \mathfrak{R}(\overline{B}^t) \), then \( \overline{A}^t \leq \overline{B}^t \) i.e., \( \min \{ \overline{A}^t, \overline{B}^t \} = \overline{A}^t \)

**Example:** Let \( \overline{A}^t = (8,10,12)(6,10,14) \) and \( \overline{B}^t = (3,5,8)(1,5,10) \) be any two TrIFN, then its rank is defined by \( \mathfrak{R}(\overline{A}^t) = 10, \mathfrak{R}(\overline{B}^t) = 5.33 \) this implies \( \overline{A}^t \succ \overline{B}^t \)

**Intuitionistic Fuzzy Assignment Problem and its Mathematical Formulation**

Consider the situation of assigning \( n \) machines to \( n \) jobs and each machine is capable of doing any job at different costs. Let \( c_{ij}^t \) be an intuitionistic fuzzy cost of assigning the \( j \)-th job to the \( i \)-th machine. Let \( x_{ij} \) be the decision variable denoting the assignment of the machine \( i \) to the job \( j \). The objective is to minimize the total intuitionistic fuzzy cost of assigning all the jobs to the available machines (one machine per job) at the least total cost. This situation is known as balanced intuitionistic fuzzy assignment problem.

(IFAP) Minimize \( Z^t = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^t x_{ij} \)

Subject to,

\[
\sum_{j=1}^{n} x_{ij} = 1, \text{ for } i = 1,2,\ldots,n
\]

\[
\sum_{i=1}^{n} x_{ij} = 1, \text{ for } j = 1,2,\ldots,n
\]

\( x_{ij} \in \{0,1\} \)

Where \( x_{ij} = \begin{cases} 1, & \text{if the } i\text{-th machine is assigned to } j\text{-th job} \\ 0, & \text{if } i\text{-th machine is not assigned to } j\text{-th job} \end{cases} \)

\( c_{ij}^t = (c_{ij}^t, c_{ij}^t, c_{ij}^t)(c_{ij}^t, c_{ij}^t, c_{ij}^t) \)

The above IFAP can be stated in the below tabular form as follows:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>\cdots</th>
<th>j</th>
<th>\cdots</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>( c_{i1}^t )</td>
<td>( c_{i2}^t )</td>
<td>\cdots</td>
<td>( c_{ij}^t )</td>
<td>\cdots</td>
<td>( c_{in}^t )</td>
</tr>
<tr>
<td>j</td>
<td>( c_{1j}^t )</td>
<td>( c_{2j}^t )</td>
<td>\cdots</td>
<td>( c_{nj}^t )</td>
<td>\cdots</td>
<td>( c_{nj}^t )</td>
</tr>
<tr>
<td></td>
<td>( c_{1n}^t )</td>
<td>( c_{2n}^t )</td>
<td>\cdots</td>
<td>( c_{nj}^t )</td>
<td>\cdots</td>
<td>( c_{nn}^t )</td>
</tr>
</tbody>
</table>
The Computational Procedure for Mixed Intuitionistic Fuzzy Assignment Problem

Step 1. Construct an assignment problem with crisp, fuzzy and an intuitionistic fuzzy numbers as cost coefficients.

Step 2. Check whether the given mixed intuitionistic fuzzy cost matrix of a mixed intuitionistic fuzzy assignment problem is a balanced one or not. If it is a balanced one (i.e., number of jobs is equal to the number of machines) then go to step 4. If not, it is an unbalanced one (i.e., number of jobs is not equal to the number of machines) then go to step 3.

Step 3. Introduce the required number of dummy rows and/or columns with zero intuitionistic fuzzy costs.

Step 4. Convert BMIFAP into balanced intuitionistic fuzzy assignment problem (BIFAP) using the following steps

i. If any one or more in the costs (profits) of an assignment problem having a crisp number say \( a_1 \) that can be expanded as a TrIFN \( (a_1', a_1, a_1) \).

ii. If any one or more in the costs (profits) of an assignment problem having a triangular fuzzy number say \( (a_1, a_2, a_3) \) that can be expanded as a TrIFN \( (a_1', a_2', a_3') \).

iii. If any one or more in the costs (profits) of an assignment problem having a TrIFN say \( (a_1', a_2', a_3') \) that can be kept as it is.

Step 5. In the given intuitionistic fuzzy cost matrix, subtract the smallest element in each row from every element of that row by using ranking procedure as mentioned in section II.

Step 6. In the reduced intuitionistic fuzzy cost matrix, subtract the smallest element in each column from every element of that column by using ranking procedure as mentioned in section II.

Step 7. Make the assignment for the reduced intuitionistic fuzzy cost matrix obtained from Step 6 in the following way:

a. Examine the rows successively until a row with exactly one unmarked intuitionistic fuzzy zero is found. Enclose this intuitionistic fuzzy zero in a box (□) as an assignment will be made there and cross (×) all other intuitionistic fuzzy zeros appearing in the corresponding column as they will not be considered for further assignment. Proceed in this way until all the rows have been examined.

b. After examining all the rows completely, examine the columns successively until a column with exactly one unmarked intuitionistic fuzzy zero is found. Make an assignment to this single intuitionistic fuzzy zero by putting a box (□) and cross out (×) all other intuitionistic fuzzy zeros in the corresponding row. Proceed in this way until all the columns have been examined.

c. Repeat the operation (a) and (b) until all the intuitionistic fuzzy zeros are either marked (□) or crossed (×).

Step 8. If there is exactly one assignment in each row and in each column then the optimum assignment policy for the given problem is obtained. Otherwise go to Step-9.

Step 9. Draw minimum number of vertical and horizontal lines necessary to cover all the intuitionistic fuzzy zeros in the reduced intuitionistic fuzzy cost matrix obtained from Step-7 by inspection or by adopting the following procedure

i. Mark (✓) all rows that do not have assignment

ii. Mark (✓) all columns (not already marked) which have intuitionistic fuzzy zeros in the marked rows

iii. Mark (✓) all rows (not already marked) that have assignments in marked columns,

iv. Repeat steps 9(ii) and 9(iii) until no more rows or columns can be marked.

v. Draw straight lines through all unmarked rows and marked columns.

Step 10. Select the smallest element among all the uncovered elements. Subtract this least element from all the uncovered elements and add it to the element which lies at the intersection of any two lines. Thus, we obtain the modified matrix. Go to Step 7 and repeat the procedure.

Numerical Example:
Example: Let us consider an mixed intuitionistic fuzzy assignment problem with rows representing 3 machines $M_1, M_2, M_3$ and columns representing the 3 jobs $J_1, J_2, J_3$. The cost matrix $[c]^{T}$ is given whose elements are different types of real, fuzzy and intuitionistic fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>(3,5,8)(1,5,10)</td>
<td>10</td>
<td>(10,15,20)</td>
</tr>
<tr>
<td>$M_2$</td>
<td>3</td>
<td>(1,3,8)</td>
<td>(4,5,7)(0,5,11)</td>
</tr>
<tr>
<td>$M_3$</td>
<td>(1,2,3)</td>
<td>(3,5,8)(2,5,9)</td>
<td>6</td>
</tr>
</tbody>
</table>

Solution:

The corresponding balanced intuitionistic fuzzy assignment problem is

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>(3,5,8)(1,5,10)</td>
<td>(10,10,10)(10,10,10)</td>
<td>(10,15,20)(10,15,20)</td>
</tr>
<tr>
<td>$M_2$</td>
<td>(3,3,3)(3,3,3)</td>
<td>(1,3,8)(1,3,8)</td>
<td>(4,5,7)(0,5,11)</td>
</tr>
<tr>
<td>$M_3$</td>
<td>(1,2,3)(1,2,3)</td>
<td>(3,5,8)(2,5,9)</td>
<td>(6,6,6)(6,6,6)</td>
</tr>
</tbody>
</table>

An intuitionistic fuzzy assignment problem can be formulated in the following mathematical programming form

$$\text{Min}[(3,5,8)(1,5,10)x_{11}+(10,10,10)(10,10,10)x_{12}+(10,15,20)(10,15,20)x_{13}+(3,3,3)(3,3,3)x_{21}+(1,3,8)(1,3,8)x_{22}+(4,5,7)(0,5,11)x_{23}+(2,3,3)(2,3,3)x_{31}+\mathcal{R}(3,5,8)(2,5,9)x_{32}+(6,6,6)(6,6,6)x_{33}$$

Subject to $x_{11} + x_{12} + x_{13} = 1$, $x_{11} + x_{21} + x_{31} = 1$,

$x_{21} + x_{22} + x_{23} = 1$, $x_{12} + x_{22} + x_{32} = 1$,

$x_{31} + x_{32} + x_{33} = 1$, $x_{13} + x_{23} + x_{33} = 1$,

$x_{ij} \in \{0, 1\}$.

Now, using the step 5 of the proposed method, we have the following reduced intuitionistic fuzzy assignment table

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>(-2,0,2)(-2,0,2)</td>
<td>(0,3,7)(-1,3,8)</td>
<td>(3,4,5)(3,4,5)</td>
</tr>
<tr>
<td>$M_2$</td>
<td>(0,0,0)(0,0,0)</td>
<td>(-2,0,5)(-2,0,5)</td>
<td>(1,2,4)(-3,2,8)</td>
</tr>
<tr>
<td>$M_3$</td>
<td>(-2,0,2)(-2,0,2)</td>
<td>(-3,0,3)(-3,0,3)</td>
<td>(3,5,9)(-5,5,9)</td>
</tr>
</tbody>
</table>

Now, using the step 6 of the proposed method, we have the following reduced intuitionistic fuzzy assignment table

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>(-2,0,2)(-2,0,2)</td>
<td>(-3,0,3)(-3,0,3)</td>
<td>(3,5,9)(-5,5,9)</td>
</tr>
<tr>
<td>$M_2$</td>
<td>(0,0,0)(0,0,0)</td>
<td>(-2,0,5)(-2,0,5)</td>
<td>(1,2,4)(-3,2,8)</td>
</tr>
<tr>
<td>$M_3$</td>
<td>(-2,0,2)(-2,0,2)</td>
<td>(-3,0,3)(-3,0,3)</td>
<td>(3,5,9)(-5,5,9)</td>
</tr>
</tbody>
</table>

Now, using step 7 to step 10, we have the following optimum assignment table

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>(-2,0,2)(-2,0,2)</td>
<td>(-9,1,10)(-14,1,15)</td>
<td>(-5,0,5)(-13,0,13)</td>
</tr>
<tr>
<td>$M_2$</td>
<td>(0,0,0)(0,0,0)</td>
<td>(-2,0,5)(-2,0,5)</td>
<td>(1,2,4)(-3,2,8)</td>
</tr>
<tr>
<td>$M_3$</td>
<td>(-2,0,2)(-2,0,2)</td>
<td>(-3,0,3)(-3,0,3)</td>
<td>(3,5,9)(-5,5,9)</td>
</tr>
</tbody>
</table>

The optimal solution is

$$x_{11}^* = x_{22}^* = x_{33}^* = 1, \quad x_{12}^* = x_{13}^* = x_{23}^* = x_{31}^* = x_{32}^* = 0,$$

With the optimal objective value $\mathcal{R}(\hat{Z}) = 15.33$ which represents the optimal total cost. In other words the optimal assignment is $M_1 \rightarrow J_1$, $M_2 \rightarrow J_2$, $M_3 \rightarrow J_3$.

The intuitionistic fuzzy minimum total cost is calculated as

$$\hat{c}_{11} + \hat{c}_{22} + \hat{c}_{33} = (3,5,8)(1,5,10) + (1,3,8)(1,3,8) +$$
Also we find that
\[ \Re(\tilde{Z}) = \Re([10, 14, 22](8, 14, 24)) = \text{Rs. 15.33} \]

In the above example it has been shown that the total optimal cost obtained by our method remains same as that obtained by converting the total intuitionistic fuzzy cost by applying the ranking method Annie Varghese and Sunny Kuriakose [4].

**Results and discussion**

The minimum total intuitionistic fuzzy assignment cost is
\[ \tilde{Z}^* = (10, 14, 22)(8, 14, 24) \]

Figure 1 Graphical Representation of IFAC

The result in (1) can be explained (Refer to figure 1) as follows:

(i) Assignment cost lies in [10,22].

(ii) 100% expect are in favour that an assignment cost is 14 as \( \mu_{Z^*}(x) = 1 \) \( x = 14 \).

(iii) Assuming that \( \mu \) is a membership value and \( \theta \) is a non-membership value at \( c \). Then 100\% experts are in favour and 100\% experts are opposing but 100\%(1-\mu - \theta)\% are in confusion that an assignment cost is \( c \).

Values of \( \mu_{Z^*}(c) \) and \( \theta_{Z^*}(c) \) at different values of \( c \) can be determined using equations given below.

\[
\mu_{Z^*}(c) = \begin{cases} 
0 & \text{for } x < 10 \\
\frac{x - 10}{4} & \text{for } 10 \leq x \leq 14 \\
1 & \text{for } x = 14 \\
\frac{22 - x}{8} & \text{for } 14 \leq x \leq 22 \\
0 & \text{for } x > 22 
\end{cases}
\]

\[
\theta_{Z^*}(c) = \begin{cases} 
1 & \text{for } x < 8 \\
\frac{14 - x}{6} & \text{for } 8 \leq x \leq 14 \\
0 & \text{for } x = 14 \\
\frac{x - 14}{10} & \text{for } 14 \leq x \leq 24 \\
1 & \text{for } x > 24 
\end{cases}
\]

**Conclusion**

In this paper, Assignment problem with crisp, fuzzy and intuitionistic fuzzy numbers as cost coefficients is discussed. The proposed method is a systematic approach for solving an assignment problem under mixed intuitionistic fuzzy environment. The total optimal cost obtained by our method remains same as that obtained by converting the total intuitionistic fuzzy cost by applying the ranking method of Annie Varghese and Sunny Kuriakose [4]. Also the membership and non-membership values of an intuitionistic fuzzy costs are derived. This technique can also be used in solving other types of problems like, project schedules, transportation problems and network flow problems.

**References**

