Introduction

India had all potential to produce any sort of products and services to meet out the requirement of an individual citizen. But due to the larger consumption of fossil fuels, India is in a crunch and scarcity in generating all sort of energy to fulfill the demand of the individuals. Government is taking innumerable campaign to yield energy through various renewable sources. Still the high consumption of the individual, the growing population and the shortage of fossil fuels urged India in frantic pressure to accomplish the task.

About 55 million tonnes of Municipal Solid Waste (MSW) and 38 billion liters of sewage are generated in the urban areas of India. It is estimated that the amount of waste generated in India will increase at a per capita rate of approximately 1-1.33% annually.

The Ministry of New and Renewable Energy has predicted that there exists a power generation potential of about 1500 MW from the MSW generated in India and the Indian Government is actively promoting the waste to energy technologies, by providing various incentives and subsidies for waste to energy projects. But according to the Indian Renewable Energy Development Agency (IREDA), only 2% of the potential has been tapped in India so far.

Nagapattinam district is an eastern coastal region of Tamilnadu, India. Major population depends on fishing and its by-products. Everyday tones and tones of sea food are collected from the sea, not only for local consumption but also for export. In every value addition stage of improving the quality of sea foods, leads to lots of wastage. In the boat house and fish markets tones of fish waste due to improper size or poor quality are collected. 44% of the waste is collected from residential areas. 19% from hospitals, 17% from fish market, 11% from commercial areas and 9% waste from institutional areas. At present, waste is disposed off through dumping in a disposal yard outside the town.

The disposal yard is situated at a distance of 5 km from the town and it is spread over 19 acres. The disposal yard is sufficient for another 15 years. The disposal is done only through dumping. Nagapattinam municipality is in the process of implementing measures to develop the dumping yard and implement composting.

Necessity for Solid Waste Management:

Waste management is necessary for every country because it directly affects the health of the people and environment. In Nagapattinam town, dengue breaks out in rainy season due to the mosquitoes for which the garbage is the dwelling place. It spreads many diseases like malaria, cholera, typhoid, chikunkunia etc., The municipal solid waste mixes up with the river water which affects the quality of water. It also affects the living organisms like fish. The nutrients from the waste make the river and lakes a better place to grow for water hyacinth and other unwanted weeds, which leads to loss of water sources. From the rotten garbage, toxic gases emerge out which is deadly to human beings, animals and plants. As population grows year by year, there will be a scarcity in land which is used for disposal.

Therefore there is a urgent need for solving all those problems. Solid waste management by mathematical models will be definitely useful for decision makers for reducing waste, for minimizing travelling cost and also to maximize the usage of dumping yard.
Model 1 is what the present situation. In this model all the wastes are directly taken to the dumping yard and only a minimum quantity is taken for compost.

Model 2 is the proposed model in which the wastages are taken to treatment before dumping it. This will increase the life span of the dumping yard. The effective use of fish wastage and recoverable waste will definitely improve the revenue of the town.

Waste is a threat to land, water and air which in turn danger to the planet. Mathematical modeling is used to decision makers in day to day planning and execution of managing solid waste.

Let a represent the location point of waste source a=1,2,3,…A
Let s represent the location point separator plant s= 1, 2, 3, …S
Let i represent the location point of incinerator plant i= 1, 2, 3, … I
Let j represent the location point of bio-fuel plant j= 1, 2, 3,…J
Let k represent the location point of compost plant k= 1, 2, 3,…K
Let r represent the location point of recycling plant r= 1, 2, 3,…R
Let m represent the location point of market m= 1, 2, 3,…M
Let z represent the location point of dumping yard z= 1, 2, 3,…Z
Let w represent the total quantity of waste source at ‘a’, that is, the total waste collected from waste source ‘a’. So,  
where \(q_a\) is the total quantity of waste source at ‘a’, that is, the total waste collected from waste source ‘a’ should be at least be equal to the total quantity of waste found at that point.

The separator plant, processing plants, govt. co-operative market and dumping yard should actually exist.

The second objective function is to minimize the transportation cost from the sources to the separator plant. Let \(C_{vs}\) be the transportation cost per unit mass of waste carried by a vehicle of type ‘v’ from source ‘a’ to separator plant ‘s’.

Let \(v_s\) denote the total number of trips made by vehicle ‘v’ from source ‘a’, carrying waste type ‘w’ to the separator plant ‘s’.

To minimize the transportation cost from the sources to the separator plant, objective function is,

\[
\min C^2 = \sum_{v=1}^{V} \left( V_{v} X_{vaws} \right) - - - - - - - - -(3)
\]

Subject to the constraints,

\[
\sum_{v=1}^{V} \left( V_{v} X_{vaws} \right) \geq q_a \quad a = 1, 2,..., A \quad - - - - - - - - -(4)
\]

where \(q_a\) is the total quantity of waste source at ‘a’, that is, the total waste collected from waste source ‘a’ should be at least be equal to the total quantity of waste found at that point.

\[
w_{ts} \leq \text{cap}_s \quad p_s \quad - - - - - - - - -(5)
\]

\[
V_{v} X_{vaws} \leq \text{cap}_s \quad p_s \quad - - - - - - - - -(6)
\]

\[
X_{vaws} \geq 0 \quad - - - - - - - - -(7)
\]

(5) shows that the amount of waste transported to the separator plant should not exceed the separator plant’s capacity.(6) shows that the flow to separator plant must be positive, the plant should actually exist. (7) shows that the total number of trips made by vehicles of type v must be positive.

The third objective function is to minimize the transportation cost from the separator plant to the processing plants.

\[
\min \sum_{s=1}^{S} \left( \sum_{s=1}^{Z} \left( \delta_{s} p_{s} + \gamma_s w_{ts} \right) + \sum_{i=1}^{I} \left( \delta_{i} p_{i} + \gamma_i w_{ts} \right) + \sum_{j=1}^{J} \left( \delta_{j} p_{j} + \gamma_j w_{ts} \right) + \sum_{k=1}^{K} \left( \delta_{k} p_{k} + \gamma_k w_{ts} \right) + \sum_{m=1}^{M} \left( \delta_{m} p_{m} + \gamma_m w_{ts} \right) - - - - - - -(1) \right)
\]
Let j represent the location point of bio fuel plant
j= 1, 2, 3,...J
Let k represent the location point of compost plant
k= 1, 2, 3,...K
Let r represent the location point of recycling plant
r= 1, 2, 3,...R
Let us denote the vehicle type by v = 1, 2, ..., V  and the volume of vehicle type v be denoted as V_v.
Let C_vsi, C_vij, C_vik, C_vir  be the transportation cost per unit mass of waste carried by a vehicle of type ‘v’ from the separator plant ‘s’ to the processing plants (ie) incinerator plant , bio fuel plant, fertilizer plant and recycling plant.
Let X_vswi , X_vswj, X_vswk, X_vswr denote the total number of trips made by vehicle ‘v’ from separator plant ‘s’ carrying waste type ‘w’ to the processing plants (ie) incinerator plant, bio fuel plant, fertilizer plant and recycling plant.
To minimize the transportation cost from the separator plant to the processing plants,

\[ \text{min } C^3 = \sum_{v, s, w, i} (C_{vsi} V_v X_{vswi}) + \sum_{v, s, w, j} (C_{vij} V_v X_{vswj}) + \sum_{v, s, w, k} (C_{vik} V_v X_{vswk}) \]

\[ + \sum_{v, s, w, r} (C_{vir} V_v X_{vswr}) \]

Subject to the constraints,

\[ \sum_{v, s, w, i} (V_v X_{vswi}) + \sum_{v, s, w, j} (V_v X_{vswj}) + \sum_{v, s, w, k} (V_v X_{vswk}) \]

\[ + \sum_{v, s, w, r} (V_v X_{vswr}) \geq q_s, s \]

where q_s is the total quantity of waste at the separator plant, that is, the total waste collected from separator plant ‘s’ should be at least equal to the total quantity of waste found at that point.

\[ \text{wt}_i \leq cap_i p_i \]

\[ \text{wt}_j \leq cap_j p_j \]

\[ \text{wt}_k \leq cap_k p_k \]

\[ \text{wt}_r \leq cap_r p_r \]

\[ V_v X_{vswi} \leq cap_i p_i \]

\[ V_v X_{vswj} \leq cap_j p_j \]

\[ V_v X_{vswk} \leq cap_k p_k \]

\[ V_v X_{vswr} \leq cap_r p_r \]

\[ X_{vswi} \geq 0 \]

\[ X_{vswj} \geq 0 \]

\[ X_{vswk} \geq 0 \]

\[ X_{vswr} \geq 0 \]

(10-13) shows that the amount of waste transported to the incinerator plant, bio fuel plant, fertilizer plant and recycling plant should not exceed the incinerator plant , bio fuel plant, fertilizer plant and recycling plant’s capacity.(14-17 ) shows that the flow to incinerator plant, bio fuel plant, fertilizer plant and recycling plant must be positive, the plant should actually exist.

(18-21) shows that the total number of trips made by vehicles of type v must be positive.

The fourth objective function is to minimize the transportation cost from the processing plant to the government co-operative market carrying the recoverable products.

Let i represent the location point of incinerator plant
i= 1, 2, 3,...I
Let j represent the location point of bio fuel plant
j= 1, 2, 3,...J
Let k represent the location point of compost plant
k= 1, 2, 3,...K
Let r represent the location point of recycling plant
r= 1, 2, 3,...R
Let m represent the location point of market
m= 1, 2, 3,...M
Let us denote the vehicle type by v = 1, 2, ..., V  and the volume of vehicle type v be denoted as V_v.
Let C_vim, C_vjm, C_vkm, C_vrm  be the transportation cost per unit mass of recoverable material carried by a vehicle of type ‘v’ from the processing plants (ie) incinerator plant, bio fuel plant, fertilizer plant and recycling plant to the govt. co-operative market.
Let Y_vijm, Y_vjim, Y_vkim, Y_vrim denote the total number of trips made by vehicle ‘v’ from the processing plants (ie) incinerator plant, bio fuel plant, fertilizer plant and recycling plant carrying recovered material to the govt. co-operative market.
To minimize the transportation cost from the processing plants to the govt. co-operative market,

\[ \text{min } C^4 = \sum_{v, i, j, m} (C_{vijm} V_v Y_{vijm}) + \sum_{v, j, i, m} (C_{vijm} V_v Y_{vijm}) + \sum_{v, k, r, m} (C_{vkm} V_v Y_{vkm}) \]

Subject to the constraints,

\[ \sum_{v, i, j, m} (V_v Y_{vijm}) \geq q_i, i = 1, 2, I \]

\[ \sum_{v, j, i, m} (V_v Y_{vijm}) \geq q_j, j = 1, 2, J \]

\[ \sum_{v, k, r, m} (V_v Y_{vkm}) \geq q_k, k = 1, 2, K \]

\[ \sum_{v, r, t, m} (V_v Y_{vkm}) \geq q_r, r = 1, 2, R \]
where \( q_i \), \( q_j \), \( q_k \), \( q_r \) is the total quantity of recovered products at the processing plant, that is, the total recovered material collected from processing plants ‘i’, ‘j’, ‘k’ and ‘r’ should be at least be equal to the total quantity of recovered material found at that point. Let \( W_{tm} \) represent the amount of recovered waste transported to the market.

\[
\begin{align*}
\text{Min } C &= \sum_{i=vz} (C_{vix} V_i Y_{vix0}) + \sum_{v=zw} (C_{vix} Y_{viz0}) + \\
&+ \sum_{v=kwz} (C_{vix} Y_{vizkwz}) + \sum_{v=kwz} (C_{viz} V_i Y_{vzw}) - - - - (27)
\end{align*}
\]

\( C \) shows that the amount of recovered material transported from the incinerator plant, bio fuel plant, fertilizer plant and recycling plant to the market should not exceed the market’s capacity. (28-31) shows that the flow from incinerator plant, bio fuel plant, fertilizer plant and recycling plant to the market must be positive, the plant should actually exist. (32-35) shows that the total number of trips made by vehicles of type \( v \) must be positive.

The fifth objective function is to minimize the transportation cost from the processing plant to the dumping yard carrying the absolute waste which cannot be recovered.

\[
\begin{align*}
\text{Min } C &= \sum_{v=0z} (C_{vix} V_i Y_{vix0}) + \sum_{v=zw} (C_{vix} Y_{viz0}) + \\
&+ \sum_{v=kwz} (C_{vix} Y_{vizkwz}) + \sum_{v=kwz} (C_{viz} V_i Y_{vzw}) - - - - (36)
\end{align*}
\]

Subject to the constraints,

\[
\begin{align*}
\sum_{v=0z} (V_i Y_{vix0}) &\ge q_i, i = 1, 2, ... I - - - - (37) \\
\sum_{v=zw} (V_i Y_{viz0}) &\ge q_j, j = 1, 2, ... J - - - - (38) \\
\sum_{v=kwz} (V_i Y_{vizkwz}) &\ge q_k, k = 1, 2, ... K - - - - (39) \\
\sum_{v=kwz} (V_i Y_{vzw}) &\ge q_r, r = 1, 2, ... R - - - - (40)
\end{align*}
\]

where \( q_i \), \( q_j \), \( q_k \), \( q_r \) is the total quantity of unrecovered waste at the processing plant, that is, the total unrecovered material collected from processing plants ‘i’, ‘j’, ‘k’ and ‘r’ should be at least be equal to the total quantity of absolute waste material found at that point. Let \( W_t \) represent the amount of absolute waste transported to the dumping yard.

\[
\begin{align*}
\text{Min } C &= \sum_{i=1} (C_{ix0} V_i Y_{ix0}) + \sum_{j=1} (C_{jx0} V_j Y_{jx0}) + \\
&+ \sum_{k=1} (C_{kx0} V_k Y_{kx0}) + \sum_{r=1} (C_{rx0} V_r Y_{rx0}) - - - - (50)
\end{align*}
\]

\( C \) shows that the amount of absolute waste transported from the incinerator plant, bio fuel plant, fertilizer plant and recycling plant to the dumping yard should not exceed the dumping yard’s capacity. (42-45) shows that the flow from incinerator plant, bio fuel plant, fertilizer plant and recycling plant to the dumping yard must be positive, the plant should actually exist. (46-49) shows that the total number of trips made by vehicles of type \( v \) must be positive.

Therefore, the objective function is to minimize the overall transportation cost C.

**CONCLUSION**

Waste management includes collection, transport, processing, recycling, disposal and monitoring of waste materials. In India, more than 60% of waste is recoverable or reused. But in many places, all the waste collected are
completely taken to the dumping yard which would certainly reduce the life span of dumping yard. Therefore a careful planning is needed. Several important decisions have to be made. Solid waste management by mathematical models will be definitely useful for decision makers for reducing waste, for minimizing travelling cost and also to maximize the usage of dumping yard.

References: