Reliability and sensitivity analysis of a two unit warm standby system with low efficiency unit

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Abstract

In the present paper the system considered consists of two subsystems A and B. Subsystem A consists of identical operating and warm standby units. While subsystem B has two dissimilar units: main unit and a unit in cold standby. Main unit of subsystem B is assumed to be more efficient than the standby unit so when the main unit fails the system goes to the state of low efficiency. Main unit is connected to cold standby unit with a switching over device. Further, whenever there is a failure in one of the units of A and in the main unit of subsystem B, the system goes to critical state where system has to stop functioning to avoid the further failures. Also we have considered that the company providing repair facility has appointed a repairman. The repairman repairs the system in case of minor failures but when the system fails completely he has to take it to the nearest service station of the company for repair. By applying Supplementary variable technique, Laplace transformations and copula methodology transition state probabilities, asymptotic behaviour, reliability, availability, M.T.T.F., cost effectiveness and sensitivity of the system have been determined. Particular cases corresponding to the situations when the standby unit of subsystem A is in cold standby and in hot standby have also been considered. At last some numerical examples have been taken to illustrate the model.
the main unit is more efficient than the standby unit. Whenever the main unit fails, load transfers to the standby unit with the help of a
switching over device and the system goes to the state of low efficiency. Further, whenever one unit of Subsystem A and the main unit
of B fail, system goes to critical state and is stopped deliberately to avoid the further risks of failure. The company providing the
repair facility has appointed a person (repairman) to look after the system. The repairman repairs the system whenever there is a minor
failure in the system. But if the system fails completely due to major failures, policy of the company is to take it to the nearest service
station of the company for repair. Therefore, in this situation repairing of the system cannot be started immediately, hence it has to
wait for the repair. At the service station better facilities and expert repairmen are available where the system can be repaired
effectively in a short period of time. Failure rates of the system are assumed to be constant whereas repairs follow general
distributions. It is also assumed that from state $S_0$ to $S_3$ and from $S_1$ to $S_3$ there are two different types of failures. This is a realistic
assumption since the warm standby unit can also fail in state of standby with a failure rate less than that of operating unit. The joint
probability distribution of failure rates has been analysed by using copula methodology [5, 7]. Transition state probabilities,
asymptotic behaviour, various reliability measures such as reliability, availability, M.T.T.F., cost analysis and sensitivity analysis of
the system have been obtained by using Supplementary variable technique, Laplace transformation and copula. Transition state
probabilities when the second unit of subsystem A is a cold standby or a hot standby with the operating unit have also been examined
and a comparison on the basis of the reliability obtained in these three cases has also been made. Numerical examples have been
provided to illustrate the model at last.

**Assumptions**

1. Initially the system is in perfectly operating state.
2. In subsystem A one unit is in warm standby with the operating unit. Both the units are similar.
3. In subsystem B one unit is in cold standby with the main unit. The standby unit is not as efficient as the main unit so when the
   main unit of subsystem B fails the system goes to the state of low efficiency.
4. The system $S_5$ in which one unit of subsystem A and the main unit of subsystem B have failed is a critical state. In this state the
   functioning of system stopped deliberately with the emergency failure rate $\lambda_E$.
5. The switchover device is perfect and switchover is instantaneous.
6. The main unit of subsystem B has given priority over other units for repair.
7. Subsystem A can repair only when both of its units have failed.
8. When the system is in complete failure state the repairman provided by company carries it to the nearest service station of
   company due to which the system has to wait for some time.
9. After repair the system is as good as new.
10. The joint probability distribution of failure rates is given by Gumbel-Hougaard family of copula.

**State Specification**

<table>
<thead>
<tr>
<th>States</th>
<th>Subsystem A: Number of good units</th>
<th>Subsystem B: Number of good units</th>
<th>System state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0$</td>
<td>2</td>
<td>2</td>
<td>G</td>
</tr>
<tr>
<td>$S_1$</td>
<td>2</td>
<td>1</td>
<td>L</td>
</tr>
<tr>
<td>$S_2$</td>
<td>2</td>
<td>0</td>
<td>$F_w$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>1</td>
<td>2</td>
<td>G</td>
</tr>
<tr>
<td>$S_4$</td>
<td>0</td>
<td>2</td>
<td>$F_w$</td>
</tr>
<tr>
<td>$S_5$</td>
<td>1</td>
<td>1</td>
<td>C</td>
</tr>
<tr>
<td>$S_6$</td>
<td>1</td>
<td>1</td>
<td>$F_w$</td>
</tr>
<tr>
<td>$S_7$</td>
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<td>0</td>
<td>$F_R$</td>
</tr>
<tr>
<td>$S_8$</td>
<td>0</td>
<td>2</td>
<td>$F_R$</td>
</tr>
<tr>
<td>$S_9$</td>
<td>1</td>
<td>1</td>
<td>$F_R$</td>
</tr>
</tbody>
</table>

**Table 1: State specification**

G: Good state, L: Low efficiency state, C: Critical state, $F_w$: Failed under waiting, $F_R$: Failed under repair
Notations

\( \lambda_1 \): Failure rate of operating unit of A.

\( \lambda_2 \): Failure rate of standby unit of A.

\( \mu_1 \): Failure rate of main unit of B.

\( \mu_2 \): Failure rate of standby unit of B.

\( \lambda_E \): Rate of emergency failure in the system.

\( \phi_i (r) \): Repair rate of main unit and standby unit of subsystem B. If \( i = 1/2 \) then \( r = y/z \).

\( \phi_A (x) \): Repair rate of subsystem A.

\( x, y, z \): Elapsed repair time for both the units of subsystem A, the main unit of subsystem B and the standby unit of B.

\( P_i (t) \): Probability that the system is in \( S_i \) state at instant \( t \) for \( i = 1 \) to \( i = 9 \).

\( \tilde{P}_i (s) \): Laplace transform of \( P_i (t) \).

\( P_d(x, t) \): Probability density function that at time \( t \) the system is in failed state \( S_4 \) and the system is under repair, elapsed repair time is \( x \).

\( E_p (t) \): Expected profit during the interval \((0, t]\).

\( K_1, K_2 \): Revenue per unit time and service cost per unit time respectively.
\[ S_\eta(x) : \eta(x) \exp(-\int_0^x \eta(x) \, dx) \]

\[ \mathcal{S}_\eta(x) : \text{Laplace transform of } S_\eta(x) = \int_0^\infty \eta(x) \exp(-s x - \int_0^x \eta(x) \, dx) \]

If \( u_1 = \phi_p(y) \), \( u_2 = \psi_p(y) \) then the expression for the joint probability according to Gumbel-Hougaard family of copula is given as

\[ C_\theta(u_1, u_2) = \exp\left[-\left((\log \phi_p(y))^\theta + (\log \psi_p(y))^\theta\right)^{1/\theta}\right] \]

5. Formulation of Mathematical Model

By elementary probability and continuity arguments, one can obtain the following set of integro-differential equations.

\[ \frac{d}{dt} \mu_1 + \mu_1 \exp\left[-\left((\log \lambda_1)^\theta + (\log \lambda_2)^\theta\right)^{1/\theta}\right] P_1(t) = \int_0^\infty \phi_1(y) P_1(y, t) \, dy \]

\[ + \int_0^\infty \phi_1(y) P_1(y, t) \, dy + \int_0^\infty \phi_1(y) P_1(y, t) \, dy + \int_0^\infty \phi_1(y) P_1(y, t) \, dy \]

\[ \frac{d}{dy} P_2(y,w,t) + \mu_2 + \phi_2(y) + \exp\left[-\left((\log \lambda_1)^\theta + (\log \lambda_2)^\theta\right)^{1/\theta}\right] \]

\[ = P_2(y,w,t) = 0 \]

\[ \frac{d}{dt} P_3(t) = \exp\left[-\left((\log \lambda_1)^\theta + (\log \lambda_2)^\theta\right)^{1/\theta}\right] P_3(t) \]

\[ \frac{d}{dy} P_4(t) + \mu_1 + \lambda_2 = P_4(t) = 0 \]

\[ \frac{d}{dt} P_5(t) + \mu_1 + \lambda_2 + \phi_1(y) = P_5(t) = 0 \]

\[ \frac{d}{dy} P_6(t) + \mu_1 + \lambda_2 + \phi_1(y) = P_6(t) = 0 \]

\[ \frac{d}{dt} P_7(t) + \mu_1 + \lambda_2 + \phi_1(y) = P_7(t) = 0 \]

\[ \frac{d}{dy} P_8(t) + \mu_1 + \lambda_2 + \phi_1(y) = P_8(t) = 0 \]

\[ \frac{d}{dt} P_9(t) + \mu_1 + \lambda_2 + \phi_1(y) = P_9(t) = 0 \]

Boundary conditions:

\[ P_1(0,t) = \mu_1 P_0(t) \]

\[ P_2(0,w,t) = \mu_2 P_1(y,t) \]

\[ P_3(0,t) = 0 \]

\[ P_4(0,w,t) = \lambda_1 P_3(t) \]

\[ P_5(0,t) = \mu_1 P_3(t) + \exp\left[-\left((\log \lambda_1)^\theta + (\log \lambda_2)^\theta\right)^{1/\theta}\right] P_1(y,t) \]

\[ P_6(0,w,t) = \lambda_2 P_3(y,t) \]
\[ P_3(0,t) = wP_2(y,w,t) \quad \ldots(17) \]
\[ P_8(0,t) = wP_4(x,w,t) \quad \ldots(18) \]
\[ P_9(0,t) = wP_6(y,w,t) \quad \ldots(19) \]

Initial condition:
\[ P_0(t) = 1 \text{ at } t = 0 \text{ and all other probabilities are zero initially.} \quad \ldots(20) \]

Solution of the model

Taking Laplace transformation of equations (1-19) and using (20), we get

\[ \left[s + \mu_1 + \exp\left[-\left(\log \lambda_1\right)^{\theta} + \left(-\log \lambda_2\right)^{\theta}\right]\right] P_0(s) = 1 + \int_0^\infty \phi_1(y)P_1(y,s)dy \]
\[ + \int_0^\infty \phi_7(y)P_7(y,s)dy + \int_0^\infty \phi_8(x)P_8(x,s)dy + \int_0^\infty \phi_9(y)P_9(y,s)dy \quad \ldots(21) \]

\[ \left[s + \frac{\partial}{\partial y} + \mu_2 + \phi_1(y) + \exp\left[-\left(\log \lambda_1\right)^{\theta} + \left(-\log \lambda_2\right)^{\theta}\right]\right] P_1(y,s) = 0 \quad \ldots(22) \]

\[ \left[s + \frac{\partial}{\partial y} + w\right] P_2(y,w,s) = 0 \quad \ldots(23) \]

\[ \left[s + \mu_1 + \lambda_1\right] P_3(s) = \exp\left[-\left(\log \lambda_1\right)^{\theta} + \left(-\log \lambda_2\right)^{\theta}\right] P_0(s) \quad \ldots(24) \]

\[ \left[s + \frac{\partial}{\partial x} + w\right] P_4(x,w,s) = 0 \quad \ldots(25) \]

\[ \left[s + \frac{\partial}{\partial y} + \lambda_E\right] P_5(y,s) = 0 \quad \ldots(26) \]

\[ \left[s + \frac{\partial}{\partial y} + w\right] P_6(y,w,s) = 0 \quad \ldots(27) \]

\[ \left[s + \frac{\partial}{\partial y} + \phi_1(y)\right] P_7(y,s) = 0 \quad \ldots(28) \]

\[ \left[s + \frac{\partial}{\partial x} + \phi_A(x)\right] P_8(x,s) = 0 \quad \ldots(29) \]

\[ \left[s + \frac{\partial}{\partial y} + \phi_1(y)\right] P_9(y,s) = 0 \quad \ldots(30) \]

Boundary conditions:
\[ P_1(0,s) = \mu_1 P_0(s) \quad \ldots(31) \]
\[ P_2(0,w,s) = \mu_2 P_1(y,s) \quad \ldots(32) \]
\[ P_3(s) = 0 \quad \ldots(33) \]
\[ P_4(0,w,s) = \lambda_1 P_3(s) \quad \ldots(34) \]
\[ P_5(0,s) = \mu_1 P_3(s) + \exp\left[-\left(\log \lambda_1\right)^{\theta} + \left(-\log \lambda_2\right)^{\theta}\right] P_1(y,s) \quad \ldots(35) \]
\[ P_6(0,w,s) = \lambda_E P_5(y,s) \quad \ldots(36) \]
\[ P_7(0,s) = wP_2(y,w,s) \quad \ldots(37) \]
Solving equations (21-30) and using equations (31-39), we get the following transition state probabilities:

\[ \overline{P}_0(s) = \frac{1}{D(s)} \]  

\[ \overline{P}_1(s) = \mu_1 \overline{P}_0(s) \left[ \frac{1 - \overline{S}_0(s + \lambda + \mu_2)}{s + \lambda + \mu_2} \right] \]  

\[ \overline{P}_2(s) = \mu_1 \mu_2 \overline{P}_0(s) \left[ \frac{1 - \overline{S}_0(2s + \lambda + w + \mu_2)}{2s + \lambda + w + \mu_2} \right] \]  

\[ \overline{P}_3(s) = \overline{\lambda} \overline{P}_0(s) \left[ \frac{1}{s + \lambda_1 + \mu_1} \right] \]  

\[ \overline{P}_4(s) = \lambda_1 \overline{P}_3(s) \left[ \frac{1}{s + w} \right] \]  

\[ \overline{P}_5(s) = \mu_1 \overline{P}_3(s) \left[ \frac{1}{s + \lambda_E} \right] + \mu_1 \overline{\lambda} \overline{P}_0(s) \left[ \frac{1 - \overline{S}_0(2s + \lambda + \lambda_E + \mu_2)}{2s + \lambda + \lambda_E + \mu_2} \right] \]  

\[ \overline{P}_6(s) = \mu_1 \overline{\lambda}_E \overline{P}_3(s) \left[ \frac{1}{2s + \lambda_E + w} \right] + \mu_1 \overline{\lambda} \overline{P}_0(s) \left[ \frac{1 - \overline{S}_0(3s + \lambda + w + \lambda_E + \mu_2)}{3s + \lambda + w + \lambda_E + \mu_2} \right] \]  

\[ \overline{P}_7(s) = \mu_1 \mu_2 \overline{P}_0(s) \left[ \frac{1 - \overline{S}_2(2s + w + \mu_2)}{2(3s + \lambda + w + \mu_2)} \right] \]  

\[ \overline{P}_8(s) = \lambda_1 \overline{P}_3(s) \left[ \frac{1 - \overline{S}_0(2s + w)}{2s + w} \right] \]  

\[ \overline{P}_9(s) = \mu_1 \overline{\lambda}_E \overline{P}_3(s) \left[ \frac{1 - \overline{S}_0(3s + \lambda + \lambda_E)}{3s + w + \lambda_E} \right] + w \mu_1 \overline{\lambda}_E \overline{P}_0(s) \times \]  

\[ \left[ \frac{1 - \overline{S}_2(4s + \lambda + w + \lambda_E + \mu_2)}{2(4s + \lambda + w + \lambda_E + \mu_2)} \right] \]  

where

\[ D(s) = s + \mu_1 + \lambda - \mu_1 \overline{S}_0(s + \mu_2 + \lambda) - \frac{w}{2} \mu_1 \mu_2 \overline{S}_2(3s + \lambda + \mu_2) - \frac{w \overline{\lambda} \lambda_1}{s + \lambda_1 + \mu_1} \times \]  

\[ \overline{S}_2(2s + w) - \frac{w \overline{\lambda}_E \mu_1 \overline{\lambda}_E}{s + \lambda_1 + \mu_1} \overline{S}_0(3s + \lambda + \lambda_E) - \frac{w \overline{\lambda} \mu_1 \overline{\lambda}_E}{2 \overline{\lambda}_E \mu_1} \overline{S}_2(4s + \lambda + \mu_2 + \lambda + \lambda_E) \]  

\[ \lambda = \exp\left[\left(-\log \lambda_1\right)^\theta + (-\log \lambda_2)^\theta \right]^{1/\theta} \]  

Also up and down state probabilities of the system are given by

\[ \overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_3(s) + \overline{P}_5(s) \]  

\[ = \left[ 1 + \mu_1 \left( \frac{1 - \overline{S}_0(s + \lambda + \mu_2)}{s + \lambda + \mu_2} \right) + \frac{\lambda}{s + \lambda_1 + \mu_1} + \frac{\mu_1 \overline{\lambda}}{(s + \lambda_1)(s + \lambda_1 + \lambda)} \right] \overline{P}_0(s) \]  

\[ + \mu_1 \overline{\lambda} \left( \frac{1 - \overline{S}_0(2s + \lambda + \lambda_E + \mu_2)}{2s + \lambda + \lambda_E + \mu_2} \right) \overline{P}_0(s) \]  

\[ \overline{P}_{down}(s) = \overline{P}_2(s) + \overline{P}_4(s) + \overline{P}_6(s) + \overline{P}_7(s) + \overline{P}_8(s) + \overline{P}_9(s) \]
\[
\begin{align*}
&= \mu_1 \mu_2 \left[ 1 - \frac{s}{\lambda + w + \mu_2} \right] + \frac{\lambda_1 \lambda}{(s + w)(s + \lambda_1 + \lambda_1)} \\
&\quad + \frac{\mu_1 \lambda_1 \lambda}{(s + \lambda + w)(s + \lambda_1 + \mu_1)} + \mu_1 \lambda_1 \lambda_1 \left[ 1 - \frac{s}{3s + \lambda + w + \lambda_E + \mu_2} \right] \\
&\quad + \mu_1 w \mu_2 \left[ \frac{1 - \frac{s}{3s + w + \lambda_2 + \mu_2}}{2(3s + \lambda + w + \mu_2)} \right] + \mu_1 \lambda_1 \lambda_1 \left[ 1 - \frac{s}{(2s + w)(s + \lambda_1 + \mu_1)} \right] \\
&\quad \times \left[ 1 - \frac{s}{2(4s + \lambda + w + \lambda_E + \mu_2)} \right] P_0(s) \\
&= \frac{1}{D(0)} \quad \text{(53)}
\end{align*}
\]

Also it is noticeable that
\[
\overline{P}_{up}(s) + \overline{P}_{down}(s) = 1/s \quad \text{(54)}
\]

**Asymptotic behaviour**

Using Abel’s lemma
\[
\lim_{s \to 0} s F(s) = \lim_{t \to \infty} F(t) = F(\text{say})
\]

in equations (52) and (53), one can obtain the following time independent probabilities
\[
\begin{align*}
P_{up} &= 1 + \mu_1 \left[ 1 - \frac{s}{\lambda + \mu_2} \right] + \frac{\lambda}{\lambda_1 + \mu_1} + \frac{\mu_1 \lambda}{\lambda_1 + \mu_1} \\
&\quad + \mu_1 \lambda_1 \lambda_1 \left[ 1 - \frac{s}{3s + \lambda + \lambda_2 + \mu_2} \right] \frac{1}{D(0)} \\
P_{down} &= \mu_1 \mu_2 \left[ 1 - \frac{s}{\lambda + w + \mu_2} \right] + \frac{\lambda_1 \lambda}{w(\lambda_1 + \mu_1)} + \frac{\mu_1 \lambda_1 \lambda_1 \lambda_1}{(\lambda_1 + w)(\lambda_1 + \mu_1)} \\
&\quad + \mu_1 \lambda_1 \lambda_1 \left[ 1 - \frac{s}{3s + \lambda + \lambda_2 + \mu_2} \right] + \mu_1 w \mu_2 \left[ 1 - \frac{s}{2(3s + \lambda + \lambda_2 + \mu_2)} \right] \\
&\quad + \mu_1 \lambda_1 \lambda_1 \left[ 1 - \frac{s}{w(\lambda_1 + \mu_1)} \right] + \mu_1 \lambda_1 \lambda_1 \left[ 1 - \frac{s}{(w + \lambda_2)(\mu_1 + \lambda_1)} \right] + \frac{w \mu_1 \lambda_1 \lambda_1}{D(0)} \frac{1}{D(0)} \\
&= \frac{1}{D(0)} \quad \text{(56)}
\end{align*}
\]

where
\[
D(0) = \lim_{s \to 0} D(s)
\]

\[
\lambda = \exp\left[ -(\log \lambda_1)^\theta + (-\log \lambda_2)^\theta \right]^{1/\theta}
\]

**Particular cases**

(i) When repair follows exponential distribution.

In this case the results can be derived by putting
\[
\overline{S}_\phi(s) = \frac{\phi_\lambda(y)}{s + \phi_\lambda(y)} \overline{S}_\phi(s) = \frac{\phi_\lambda(x)}{s + \phi_\lambda(x)} \quad \text{(57)}
\]
in equations (52) and (53), we get

\[
\bar{P}_{\text{up}}(s) = \left[1 + \frac{\mu_1}{s + \lambda + \mu_2 + \phi_1(y)} + \frac{\lambda}{s + \lambda_1 + \mu_1} + \frac{\mu_1 \lambda}{(s + \lambda_E)(s + \mu_1 + \lambda_1)} \right] \frac{1}{D_1(s)} 
\]

\[
\bar{P}_{\text{down}}(s) = \left[\frac{\mu_1 \mu_2}{2s + \lambda + w + \mu_2 + \phi_1(y)} + \frac{\lambda \lambda_1}{(s + w)(s + \mu_1 + \lambda_1)} \right] \frac{1}{D_1(s)} 
\]

\[
\frac{\mu_1 \lambda_1}{(2s + \lambda_E + \mu_1)(s + \mu_1 + \lambda_1)} + \frac{\mu_1 \lambda \lambda}{3s + \lambda + w + \lambda_E + \mu_2 + \phi_1(y)} 
\]

\[
+ \frac{\mu_1 w \mu_2}{2(3s + \lambda + w + \mu_2 + 2\phi_1(y))} + \frac{w \lambda_1 \lambda}{(2s + w + \phi_A(x))(s + \lambda_1 + \mu_1)} \]

\[
+ \frac{\mu_1 \lambda_1 \lambda\lambda}{(3s + w + \lambda_E + \phi_1(y))(s + \mu_1 + \lambda_1)} + \frac{w \mu_1 \lambda \lambda\lambda}{2(4s + \lambda + w + \lambda_E + \mu_2 + 2\phi_1(y))} \frac{1}{D_1(s)} 
\]  

where

\[
D_1(s) = s + \mu_1 + \lambda - \frac{\mu_1 \phi_1(y)}{s + \mu_2 + \lambda + \phi_1(y)} - \frac{w \mu_1 \mu_2 \phi_1(y)}{3s + \lambda + \mu_2 + 2\phi_1(y)} + \frac{w \lambda \lambda_1}{s + \mu_1 + \lambda_1} \times 
\]

\[
\times \frac{\phi_A(x)}{2s + w + \phi_A(x)} - \frac{w \lambda \lambda_1 \lambda \lambda}{s + \mu_1 + \lambda_1} - \frac{\phi_1(y)}{3s + w + \lambda_E + \phi_1(y)} \frac{w \lambda \lambda_1 \lambda \lambda}{4s + \lambda + \mu_2 + w + \lambda_E + 2\phi_1(y)} 
\]

\[
\lambda = \exp[-\{(\log \lambda_1)^\theta + (\log \lambda_2)^\theta \}^{1/\theta}] 
\]

(ii) When waiting time is zero.

In this case up and down state probabilities of the system can be obtain by putting w = 0 in equations, we get

\[
\bar{P}_{\text{up}}(s) = \left[1 + \mu_1 \left\{\frac{1 - s_\lambda (s + \lambda + \mu_2)}{s + \lambda + \mu_2}\right\} + \frac{\lambda}{s + \lambda_1 + \mu_1} + \frac{\mu_1 \lambda}{(s + \lambda_E)(s + \mu_1 + \lambda_1)} \right] \frac{1}{D_2(s)} 
\]

\[
\bar{P}_{\text{down}}(s) = \left[\frac{\mu_1 \mu_2}{2s + \lambda + \lambda_2 + \mu_2}\right] + \frac{\lambda \lambda_1}{2s + \lambda + \lambda_2 + \mu_2} \frac{1}{D_2(s)} 
\]

where

\[
D_2(s) = s + \mu_1 + \lambda - \mu_1 s_\lambda (s + \mu_2 + \lambda) 
\]

\[
\lambda = \exp[-\{(\log \lambda_1)^\theta + (\log \lambda_2)^\theta \}^{1/\theta}] 
\]

(iii) When the standby unit in subsystem A is in cold standby.

This can be derived by putting \(\lambda_2 = 0\) or \(\lambda = \lambda_1 = \lambda_A\) in equations (40-49). The Laplace transformations of various transition state probabilities are as follows:

\[
\bar{P}_0(s) = 1/ D_3(s) 
\]
\[ P_1(s) = \mu_1 P_0(s) \left[ \frac{1 - \overline{S}_d(s + \lambda_A)}{s + \lambda_A + \mu_2} \right] \]  
\[ P_2(s) = \mu_1 \mu_2 P_0(s) \left[ \frac{1 - \overline{S}_d(2s + \lambda_A + w + \mu_2)}{2s + \lambda_A + w + \mu_2} \right] \]  
\[ P_3(s) = \lambda_A P_0(s) \left[ \frac{1}{s + \lambda_A + \mu_1} \right] \]  
\[ P_4(s) = \lambda_A P_3(s) \left[ \frac{1}{s + w} \right] \]  
\[ P_5(s) = \mu_1 P_3(s) \left[ \frac{1}{s + \lambda_E} \right] + \mu_1 \lambda_A \lambda_E P_0(s) \left[ \frac{1 - \overline{S}_d(2s + \lambda_A + \lambda_E + \mu_2)}{2s + \lambda_A + \lambda_E + \mu_2} \right] \]  
\[ P_6(s) = \mu_1 \lambda_E P_3(s) \left[ \frac{1}{2s + \lambda_E + w} \right] + \mu_1 \lambda_A \lambda_E P_0(s) \left[ \frac{1 - \overline{S}_d(3s + \lambda_A + \lambda_E + \mu_2)}{3s + \lambda_A + \lambda_E + \mu_2} \right] \]  
\[ P_7(s) = \mu_1 \mu_2 P_0(s) \left[ \frac{1 - \overline{S}_d(3s + \lambda_A + w + \mu_2)}{2(3s + \lambda_A + w + \mu_2)} \right] \]  
\[ P_8(s) = w \lambda_A P_3(s) \left[ \frac{1 - \overline{S}_d(2s + w)}{2s + w} \right] \]  
\[ P_9(s) = \mu_1 \lambda_E w P_3(s) \left[ \frac{1 - \overline{S}_d(3s + w + \lambda_E)}{3s + w + \lambda_E} \right] + \mu_1 \lambda_A \lambda_E P_0(s) \times \left[ \frac{1 - \overline{S}_d(4s + \lambda_A + \lambda_E + \mu_2)}{2(4s + \lambda_A + \lambda_E + \mu_2)} \right] \]  

where

\[ D_3(s) = s + \mu_1 + \lambda_A - \mu_1 \overline{S}_d(s + \mu_2 + \lambda_A) - \frac{w}{2} \mu_1 \mu_2 \overline{S}_d(3s + w + \lambda_A + \mu_2) \]

\[ - \frac{w\lambda_A^2}{s + \mu_1 + \lambda_A} \times \overline{S}_d(2s + w) - \frac{w\lambda_E \lambda_A}{s + \mu_1 + \lambda_A} \overline{S}_d(3s + w + \lambda_E) \]

\[ - \frac{w}{2} \lambda_A \lambda_E \mu_1 \overline{S}_d(4s + \lambda_A + \mu_2 + w + \lambda_E) \]  

Also up and down state probabilities of the system are given by

\[ P_{\text{up}}(s) = \left[ 1 + \mu_1 \left( \frac{1 - \overline{S}_d(s + \lambda_A + \mu_2)}{s + \lambda_A + \mu_2} \right) + \frac{\lambda_A}{s + \lambda_A + \mu_1} + \frac{\mu_1 \lambda_A}{(s + \lambda_E)(s + \mu_1 + \lambda_A)} \right] P_0(s) \]  
\[ P_{\text{down}}(s) = \left[ \mu_1 \mu_2 \left( \frac{1 - \overline{S}_d(2s + \lambda_A + w + \mu_2)}{2s + \lambda_A + w + \mu_2} \right) + \frac{\lambda_A^2}{(s + w)(s + \mu_1 + \lambda_A)} \right] \]  
\[ + \frac{\mu_1 \lambda_A \lambda_E}{(2s + \lambda_E + w)(s + \lambda_A + \mu_1)} + \mu_1 \lambda_A \lambda_E \left[ \frac{1 - \overline{S}_d(3s + \lambda_A + w + \lambda_E + \mu_2)}{3s + \lambda_A + w + \lambda_E + \mu_2} \right] \]
\[ + \mu_1 \lambda_A \lambda_E \left[ \frac{1 - \overline{S}_d(3s + \lambda_A + w + \lambda_E + \mu_2)}{3s + \lambda_A + w + \lambda_E + \mu_2} \right] + \frac{1 - \overline{S}_d(2s + w)}{(2s + w)(s + \lambda_A + \mu_1)} \]
\( + \mu_1 \lambda_E w \lambda_A \left[ \frac{1 - \bar{S}_A (3s + w + \lambda_E)}{(3s + w + \lambda_E)(s + \mu_1 + \lambda_A)} \right] + w \mu_1 \lambda_A \lambda_E \times \)
\( \times \left[ \frac{1 - \bar{S}_{2\lambda} (4s + \lambda_A + w + \lambda_E + \mu_2)}{2(4s + \lambda_A + w + \lambda_E + \mu_2)} \right] P_0(s) \) …(78)

(iv) When standby unit in subsystem A is in hot standby with the operating unit.

In this case by putting \( \lambda_2 = \lambda_1 = \lambda_A \) and \( \lambda = 2\lambda_A \) in equations (40-49), one can obtain following transition state probabilities.

\( \bar{P}_0(s) = 1/D_4(s) \) …(79)

\( \bar{P}_1(s) = \mu_1 \bar{P}_0(s) \left[ \frac{1 - \bar{S}_A (s + 2\lambda_A + \mu_2)}{s + 2\lambda_A + \mu_2} \right] \) …(80)

\( \bar{P}_2(s) = \mu_1 \mu_2 \bar{P}_0(s) \left[ \frac{1 - \bar{S}_A (2s + 2\lambda_A + w + \mu_2)}{2s + 2\lambda_A + w + \mu_2} \right] \) …(81)

\( \bar{P}_3(s) = 2\lambda_A \bar{P}_0(s) \left[ \frac{1}{s + \lambda_A + \mu_1} \right] \) …(82)

\( \bar{P}_4(s) = \lambda_A \bar{P}_3(s) \left[ \frac{1}{s + w} \right] \) …(83)

\( \bar{P}_5(s) = \mu_1 \lambda_A \bar{P}_3(s) \left[ \frac{1}{s + \lambda_A} \right] + 2\mu_1 \lambda_A \bar{P}_0(s) \left[ \frac{1 - \bar{S}_A (2s + 2\lambda_A + \lambda_E + \mu_2)}{2s + 2\lambda_A + \lambda_E + \mu_2} \right] \) …(84)

\( \bar{P}_6(s) = \mu_1 \lambda_E \bar{P}_3(s) \left[ \frac{1}{2s + \lambda_E + w} \right] + \mu_2 \lambda_A \lambda_E \bar{P}_0(s) \left[ \frac{1 - \bar{S}_A (3s + 2\lambda_A + w + \lambda_E + \mu_2)}{3s + 2\lambda_A + w + \lambda_E + \mu_2} \right] \) …(85)

\( \bar{P}_7(s) = \mu_1 \mu_2 \bar{P}_0(s) \left[ \frac{1 - \bar{S}_{2\lambda} (3s + 2\lambda_A + w + \mu_2)}{2(3s + 2\lambda_A + w + \mu_2)} \right] \) …(86)

\( \bar{P}_8(s) = w \lambda_A \bar{P}_3(s) \left[ \frac{1 - \bar{S}_{\phi_1} (2s + w)}{2s + w} \right] \) …(87)

\( \bar{P}_9(s) = \mu_1 \lambda_E w \bar{P}_3(s) \left[ \frac{1 - \bar{S}_A (3s + w + \lambda_E)}{3s + w + \lambda_E} \right] + w \mu_1 \lambda_A \lambda_E \bar{P}_0(s) \times \left[ \frac{1 - \bar{S}_{2\lambda} (4s + 2\lambda_A + w + \lambda_E + \mu_2)}{2(4s + 2\lambda_A + w + \lambda_E + \mu_2)} \right] \) …(88)

where

\( D_4(s) = s + \mu_1 + 2\lambda_A - \mu_1 S_{\phi_1} (s + \mu_2 + 2\lambda_A) - \frac{w}{2} \mu_1 \mu_2 \bar{S}_{2\lambda} (3s + 2\lambda_A + \mu_2) \)

\( \quad - \frac{2w\lambda_A^2}{s + \mu_1 + \lambda_A} \times \bar{S}_{\phi_1} (2s + w) - \frac{2w\lambda_E \mu_1 \lambda_A}{s + \mu_1 + \lambda_A} \bar{S}_A (3s + w + \lambda_E) \)

\( \quad - w \lambda_A \lambda_E \mu_1 \bar{S}_{2\lambda} (4s + 2\lambda_A + \mu_2 + w + \lambda_E) \) …(89)

Also the up and down state probabilities of the system are given by

\( \bar{P}_{up}(s) = 1 + \mu_1 \left[ \frac{1 - \bar{S}_A (s + 2\lambda_A + \mu_2)}{s + 2\lambda_A + \mu_2} \right] + \frac{2\lambda_A}{s + \lambda_A + \mu_1} + \frac{2\mu_1 \lambda_A}{(s + \lambda_E)(s + \mu_1 + \lambda_A)} \)

\( + 2\mu_1 \lambda_A \left[ \frac{1 - \bar{S}_A (2s + 2\lambda_A + \lambda_E + \mu_2)}{2s + 2\lambda_A + \lambda_E + \mu_2} \right] P_0(s) \) …(90)
\[ \overline{P}_{\text{down}}(s) = \left[ \mu_1 \mu_2 \left( 1 - \sum_{n=0}^{\infty} \left( 2s + 2\lambda_A + w + \mu_2 + \mu_1 \right) \right) \right] + \frac{2\lambda_A^2}{(s+w)(s+\mu_1 + \lambda_A)} + \frac{2\mu_1 \lambda_E \lambda_A}{(2s + \lambda_E + w)(s + \lambda_A + \mu_1)} + 2\mu_1 \lambda_E \lambda_A \left( \frac{1 - \sum_{n=0}^{\infty} \left( 3s + 2\lambda_A + w + \lambda_E + \mu_2 \right)}{3s + 2\lambda_A + w + \lambda_E + \mu_2} \right) + \lambda_1 \mu_2 \left( \frac{1 - \sum_{n=0}^{\infty} \left( 3s + w + \lambda_E \right)}{(3s + w + \lambda_E)(s + \mu_1 + \lambda_A)} \right) + 2\mu_1 \lambda_E w \lambda_A \left( \frac{1 - \sum_{n=0}^{\infty} \left( 4s + 2\lambda_A + w + \lambda_E + \mu_2 \right)}{2(4s + 2\lambda_A + w + \lambda_E + \mu_2)} \right) \] 

Numerical computation

(1) Availability analysis

Let us take \( \lambda_1 = 0.6, \lambda_2 = 0.2, \mu_1 = 0.3, \mu_2 = 0.2, \lambda_E = 0.5, w = 0.3, \Phi_1 = \Phi_2 = \Phi_A = 1, \theta = 1 \) and \( x = y = z = 1 \). Also let the repair follows exponential distribution i.e. equation (57) holds, then putting all these values in equation (52), taking inverse Laplace transformation, we get

\[ P_{wp}(t) = e^{-0.5000000000t} - 0.0031131041003 e^{-0.9100000001t} - 0.058466089696 \]

\[ e^{1.577110005t} - 0.1942010441 e^{1.8601482323t} \cos(0.007394121216t) - 0.2512567850 e^{1.577110005t} \sin(0.007394121216t) + 0.4613082557 e^{-0.7862665523t} + 0.1663598755 e^{0.7258577930t} + 0.002024434936 e^{0.6042513821t} + 1.17130413 e^{0.126841365t} \]

Now setting \( t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \), in above equation (92), one can obtain Table 2 and correspondingly Fig. 2 which represents the variation of availability with respect to time.

(2) Reliability analysis

Let the failure rates be \( \lambda_1 = 0.6, \lambda_2 = 0.2, \mu_1 = 0.3, \mu_2 = 0.2, \) emergency failure rate be \( \lambda_E = 0.5 \), waiting rate \( w = 0.3 \), repair rates be \( \Phi_1 = \Phi_2 = \Phi_A = 0, \theta = 1 \) and \( x = y = z = 1 \). Also let the repair follows exponential distribution. Now by putting all these values in equations (52), (77) and (90) taking inverse Laplace transform, using (57) and varying time from \( t = 0 \) to \( t = 10 \), one can obtain Table 3 and Fig. 3 which demonstrate the manner in which reliability varies as time passes when the unit of subsystem A is in warm standby, cold standby or in hot standby.

(3) M.T.T.F. analysis

We know that \( \text{M.T.T.F.} = \lim_{s \to 0} \overline{P}_{wp}(s) \)

Also suppose that repair follows exponential distribution then using equation (57) and

(a) Setting \( \Phi_1 = \Phi_2 = \Phi_A = 0, \lambda_2 = 0.2, \mu_1 = 0.3, \mu_2 = 0.2, \) \( \lambda_E = 0.5, w = 0.3, x = y = z = 1, \theta = 1 \) and varying \( \lambda_1 \) as \( 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 \), one can obtain Table 4 which demonstrates variation of M.T.T.F. with respect to \( \lambda_1 \).

(b) Let us take \( \Phi_1 = \Phi_2 = \Phi_A = 0, \lambda_1 = 0.6, \mu_1 = 0.3, \mu_2 = 0.2, \lambda_E = 0.5, w = 0.3, x = y = z = 1, \theta = 1 \) then by varying \( \lambda_2 \) as \( 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 \) Table 5 can be obtained which shows how M.T.T.F. varies as the value of \( \lambda_2 \) increases.

(c) Fixing \( \Phi_1 = \Phi_2 = \Phi_A = 0, \lambda_1 = 0.6, \lambda_2 = 0.2, \mu_2 = 0.2, \lambda_E = 0.5, w = 0.3, x = y = z = 1, \theta = 1 \) and varying \( \mu_1 \) as \( 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 \) one can obtain Table 6 which shows variation of M.T.T.F. with respect to \( \mu_1 \).

(d) Putting \( \Phi_1 = \Phi_2 = \Phi_A = 0, \lambda_1 = 0.6, \lambda_2 = 0.2, \mu_1 = 0.3, \lambda_E = 0.5, w = 0.3, x = y = z = 1, \theta = 1 \) and varying \( \mu_2 \) from 0.10 to 0.90 one can obtain Table 7 which represents the manner in which M.T.T.F. varies with respect to \( \mu_2 \).

Variations of M.T.T.F with respect to \( \lambda_1, \lambda_2, \mu_1 \) and \( \mu_2 \) in the cases (a), (b), (c) and (d) have been shown by the Figs. 4, 5, 6 and 7 respectively.
(4) Cost Analysis
Letting $\lambda_1 = 0.6$, $\lambda_2 = 0.2$, $\mu_1 = 0.3$, $\mu_2 = 0.2$, $\lambda_E = 0.5$, $w = 0.3$, $\Phi_1 = \Phi_2 = \Phi_A = 1$, $\theta = 1$ and $x = y = z = 1$. Furthermore, if the repair follows exponential distribution then using equations (57), we can obtain equation (92). If the service facility is always available, then expected profit during the interval $(0, t]$ is given by

$$E_P(t) = K_1 \left[ P_{up}(t) dt - K_2 t \right]$$

where $K_1$ and $K_2$ are the revenue and service cost per unit time respectively, then

$$E_P(t) = K_1 [0.2602109240 e^{-0.5000000000 t} + 0.003420894509 e^{-0.9100000000 t} + 0.3707166241 e^{-1.577110005 t} + 0.00350319082 e^{-0.8601482323 t} - 0.2283382234 e^{-0.7862665523 t} - 9.234941008 e^{-0.1268411365 t} - 8.996326343 - K_2 t]$$

Keeping $K_1 = 1$ and varying $K_2$ at 0.1, 0.2, 0.3, 0.4, 0.5 in equation (93), one can obtain Table 8 which is depicted by Fig. 8.

(5) Sensitivity Analysis
We have performed sensitivity analysis of system reliability along with the change in specific values of system parameters. For this it is assumed that $\Phi_1 = \Phi_2 = \Phi_A = 0$, $\theta = 1$ and $x = y = z = 1$. Putting all these values in equation (52), we get

$$R(s) = \left[ 1 + \frac{\mu_1}{s + \lambda + \mu_2} + \frac{\lambda}{s + \lambda_1 + \mu_1} + \frac{\mu_1 \lambda}{(s + \lambda E)(s + \mu_1 + \lambda_1)} + \frac{\mu_1 \lambda}{2s + \lambda + \lambda E + \mu_2} \right] \frac{1}{s + \mu_1 + \lambda} \quad ...(94)$$

Sensitivity of any reliability characteristic with respect to some specific parameter concludes how that particular reliability characteristic of the system changes with the change in the value of that specific parameter. In the present work we have done sensitivity analysis of reliability of the system with the change in the values of $\mu_1$ and $\lambda_E$ in two different cases as given below.

(a) Let us find $\frac{\partial R(s)}{\partial \mu_1}$ i.e. differentiate equation (94) with respect to $\mu_1$, take its inverse Laplace Transform, put $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $\mu_2 = 0.2$, $\lambda_E = 0.2$ and then varying the time as $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$ for $\mu_1 = 0.1, 0.3$ and 0.5, we get Table 9 and Fig. 9.

(b) Calculating $\frac{\partial R(s)}{\partial \lambda_E}$ by using equation (94), taking its inverse Laplace Transform and putting $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $\mu_2 = 0.2$, $\mu_1 = 0.2$. Now varying the time as $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$ for $\lambda_E = 0.1, 0.3$ and 0.5, one can get Table 10 and Fig. 10.

Table 2: Time vs. Availability

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Table 3: Time vs. Reliability for warm, cold and hot standby

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Table 4: \( \lambda_1 \) vs. M.T.T.F

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Fig. 4: $\lambda_1$ vs. M.T.T.F.

Table 5: $\lambda_2$ vs. M.T.T.F

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Fig. 5: $\lambda_2$ vs. MTTF

Table 6: $\mu_1$ vs. M.T.T.F.

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</table>
Fig. 6: $\mu_1$ vs. M.T.T.F

Table 7: $\mu_2$ vs. M.T.T.F

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Fig. 7: $\mu_2$ vs. M.T.T.F

Table 8: Time vs. expected profit

<table>
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Fig. 8: Time vs. expected profit

Table 9: Sensitivity analysis of the system reliability with respect to $\mu_1$

<table>
<thead>
<tr>
<th>Time</th>
<th>Value of $\frac{\partial R(s)}{\partial \mu_1}$</th>
<th>$\mu_1 = 0.1$</th>
<th>$\mu_1 = 0.3$</th>
<th>$\mu_1 = 0.5$</th>
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</table>

Fig. 9: Sensitivity analysis of the system reliability with respect to $\mu_1$

Table 10: Sensitivity analysis of the system reliability with respect to $\lambda_E$

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<th>Time</th>
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### Conclusions

The following conclusions may be drawn on the basis of study conducted in the present paper.

1. When $\lambda_1 = 0.6$, $\lambda_2 = 0.2$, $\mu_1 = 0.3$, $\mu_2 = 0.2$, $\lambda_E = 0.5$, $w = 0.3$ the availability of the system decreases as the time increases. This variation of availability with respect to time is depicted in Fig. 2.

2. Table 3 is corresponding to the reliabilities obtained for the cases when the second unit of subsystem A is in warm standby, cold standby and in hot standby. One can easily conclude by observing Fig. 3 that reliability in each case decreases as the time increases but the system has highest reliability in case when second unit of subsystem A is in warm standby.

3. By critically examine Figs. 4, 5, 6 and 7 one can see that the M.T.T.F. of the system decreases with the increment in the values of $\lambda_1$, $\lambda_2$, $\mu_1$ and $\mu_2$. M.T.T.F. is found to be highest with respect to $\lambda_1$. Also in case of $\mu_1$ and $\mu_2$ the decrement is more rapid in comparison to the cases of $\lambda_1$ and $\lambda_2$. The value of M.T.T.F. varies from 7.662-4.355, 6.344-2.971, 6.967-4.086 and from 6.254-3.645 with respect to $\lambda_1$, $\lambda_2$, $\mu_1$ and $\mu_2$ respectively for considered parameters.

4. Keeping revenue cost per unit time at 1 and varying service cost from 0.1 to 0.5, one can obtain Fig. 8. It is very clear from Fig. 8 that increasing service cost implies decrement in profit. Here highest and lowest values of expected profit are obtained to be 5.40 and 0.4871 respectively for considered values.

5. Tables 9 and 10 are corresponding to the sensitivity analysis of the system reliability with respect to change in $\mu_1$ and $\lambda_E$ respectively. This behaviour of sensitivity has been shown in Figs. 9 and 10. One can observe that sensitivity of the system reliability decreases with the increase in the value of $\mu_1$ and $\lambda_E$. Also one can analyze that the system reliability is more sensitive in case of $\mu_1$ than $\lambda_E$.

### References


