A study on Q-fuzzy normal subgroups and cosets

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ABSTRACT

In this paper, we study some of the properties of Q-fuzzy normal subgroups, cosets and prove some results on these.

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Introduction


Definition: Let A be a Q-fuzzy subgroup of a group ( G, · ) and H = { x ∈ G / A(x, q) = A(e, q) }, then O(A) ,order of A is defined as O(A) = O(H).

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Some properties of Q-fuzzy normal subgroupS:

Proposition: Let ( G, · ) be a group and Q be a non-empty set. If A and B are two Q-fuzzy normal subgroups of G, then their intersection A∩B is a Q-fuzzy normal subgroup of G.

Proof: Let x and y in G and q in Q and A = { ( x, q ), A(x, q) / x in G and q in Q } and B = { ( x, q ), B(x, q) / x in G and q in Q } be a Q-fuzzy normal subgroups of G. Let C = A ∩ B = A(x, q) = A(y, q) , for all x and y in G and q in Q. Hence A∩B is a Q-fuzzy normal subgroup of G.

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group G is a Q-fuzzy subgroup of a group G. Now, \( A(xy, q) = \inf_{x, y} A(x, y, q) = A(xy, q) \).

Therefore, \( A(xy, q) = A(yx, q) \), for all x and y in G and q in Q. Hence the intersection of a family of Q-fuzzy normal subgroups of a group G is a Q-fuzzy normal subgroup of G.

**Proposition:** If \( A \) is a Q-fuzzy characteristic subgroup of a group G, then \( A \) is a Q-fuzzy normal subgroup of a group G.

**Proof:** Let \( A \) be a Q-fuzzy characteristic subgroup of a group G, x and y in G and q in Q. Consider the map \( f : G \times Q \rightarrow G \times Q \) defined by \( f(x, q) = (yx^{-1}, q) \).

Clearly, \( f \) in Q-AutG. Now, \( A(xy, q) = A(f(xy, q)) = A(yx^{-1}, q) = A(xy, q) \). Therefore, \( A(xy, q) = A(yx, q) \), for all x and y in G and q in Q. Hence \( A \) is a Q-fuzzy normal subgroup of a group G.

**Proposition:** A Q-fuzzy subgroup A of a group G is a Q-fuzzy normal subgroup of G if and only if \( A \) is constant on the conjugate classes of G.

**Proof:** Suppose that \( A \) is a Q-fuzzy normal subgroup of a group G. Let x and y in G and q in Q. Now, \( A(xy, q) = A(xyxy^{-1}, q) = A(xy, q) \). Therefore, \( A(xy, q) = A(yx, q) \), for all x and y in G and q in Q. Hence \( A \) is a Q-fuzzy normal subgroup of a group G.

**Proposition:** Let \( A \) be a Q-fuzzy normal subgroup of a group G. Then for any x in G and q in Q, we have \( A(xyx^{-1}, q) = A(yx^{-1}, q) \), for every x in G.

**Proof:** Let \( A \) be a Q-fuzzy normal subgroup of a group G. For any x in G and q in Q, we have \( A(xyx^{-1}, q) = A(yx^{-1}, q) \), for all x and y in G and q in Q. Hence \( A \) is a Q-fuzzy normal subgroup of a group G.

**Proposition:** A Q-fuzzy subgroup A of a group G is normalized if and only if \( A(e, q) = 1 \), where e is the identity element of the group G and q in Q.

**Proof:** If \( A \) is normalized, then there exists x in G such that \( A(x, q) = 1 \), but by properties of a Q-fuzzy subgroup A of G, \( A(x, q) \leq A(e, q) \), for every x in G and q in Q. Since \( A(x, q) = 1 \) and \( A(e, q) = 1 \), we have \( A(x, q) \leq A(e, q) \). Hence \( A(e, q) = 1 \). Conversely, if \( A(e, q) = 1 \), then by the definition of normalized Q-fuzzy subgroup, A is normalized.

**Proposition:** Let A and B be Q-fuzzy normal subgroups of the groups G and H, respectively. If A and B are Q-fuzzy normal subgroups, then \( A \times B \) is a Q-fuzzy normal subgroup of \( G \times H \).

**Proof:** Let A and B be Q-fuzzy normal subgroups of the groups G and H respectively. Clearly \( A \times B \) is a Q-fuzzy normal subgroup of \( G \times H \), since A and B are Q-fuzzy normal subgroups of G and H. Let \( x_1 \) and \( x_2 \) be in G, \( y_1 \) and \( y_2 \) be in H and q in Q. Then \( A(x_1, y_1)(x_2, y_2) \). Therefore, \( A \) is a Q-fuzzy normal subgroup of \( G \times H \). Hence \( A \times B \) is a Q-fuzzy normal subgroup of \( G \times H \).

**Proposition:** Let A be a Q-fuzzy normal subgroup of a group G be conjugate to a Q-fuzzy normal subgroup M of G and a Q-fuzzy normal subgroup B of a group H be conjugate to a Q-fuzzy normal subgroup N of H. Then a Q-fuzzy normal subgroup \( A \times B \) of a group \( G \times H \) is conjugate to a Q-fuzzy normal subgroup \( M \times N \) of \( G \times H \).

**Proof:** It is trivial.
subsets of $G$. To prove $B$ is a Q-fuzzy subgroup of $G$. Let $x$ and $y$ in $G$ and $q$ in $Q$. Then $xy^{-1}$ in $G$. Now, $B(xy^{-1}, q) = A(g^{-1}xg, q) = A((g^{-1}xg) (g^{-1}yg)^{-1}, q) \geq \min \{ A(g^{-1}xg, q), A((g^{-1}yg)^{-1}, q) \} \geq \min \{ A(g^{-1}xg, q), A(g^{-1}yg, q) \}$, since $A$ is a QFSG of $G = \min \{ A(x, q), B(y, q) \}$. Therefore, $B(xy^{-1}, q) \geq \min \{ B(x, q), B(y, q) \}$, for $x$ and $y$ in $G$ and $q$ in $Q$. Hence $B$ is a Q-fuzzy subgroup of the group $G$.

**Proposition:** Let $A$ be a Q-fuzzy subgroup of a group $G$. Then $(x, q)A = (y, q)A$, for $x, y$ in $G$ if and only if $A(x^{-1}y, q) = A(y^{-1}x, q) = A(e, q)$.

**Proof:** Let $A$ be a Q-fuzzy subgroup of a group $G$. Let $(x, q)A = (y, q)A$, for $x$ and $y$ in $G$ and $q$ in $Q$. Then, $(x, q)A(x, q) = (y, q)A(y, q) = \cup A(x^{-1}x, q) = A(y^{-1}y, q)$ and $A(x^{-1}y, q) = A(y^{-1}x, q)$ and $A(x^{-1}y, q) = A(e, q)$. Therefore, $A(x^{-1}y, q) = A(y^{-1}x, q) = A(e, q)$, for $x$ and $y$ in $G$ and $q$ in $Q$. Conversely, let $A(x^{-1}y, q) = A(y^{-1}x, q) = A(e, q)$, for $x$ and $y$ in $G$ and $q$ in $Q$. For every $g$ in $G$ and we have, $(x, q)A(g, q) = A(x^{-1}g, q) = A(x^{-1}y^{-1}y, q) \geq \min \{ A(x^{-1}y, q), A(y^{-1}y, g, q) \} = \min \{ A(e, q), A(g^{-1}g, q) \} = A(y^{-1}y, g, q)$, and $(y, q)A(g, q) = A(y^{-1}g, q)$ $\geq \min \{ A(y^{-1}x, q), A(x^{-1}g, q) \} = \min \{ A(e, q), A(g^{-1}g, q) \} = A(x^{-1}y, g, q) = (x, q)A(g, q)$. Therefore, $(y, q)A(g, q) \geq (x, q)A(g, q)$ $(2)$. From (1) and (2) we get, $(x, q)A(g, q) = (y, q)A(g, q)$ $(3)$. We get, $(x, q)A = (y, q)A$, for all $x$ and $y$ in $G$ and $q$ in $Q$.

**Reference**