Fuzzy Generalized Super Closed Sets
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**Abstract**
In this paper we introduced the concept of fuzzy g- super closed and explore various properties fuzzy topological space.

**Introduction**
Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family \( \{A_\alpha; \alpha \in \Lambda\} \) of fuzzy sets of X is defined by to be the mapping \( \sup A_\alpha \) (resp. \( \inf A_\alpha \)). A fuzzy set A of X is contained in a fuzzy set B of X if \( A(x) \leq B(x) \) for each \( x \in X \). A fuzzy point \( x_\beta \) in X is a fuzzy set defined by \( x_\beta(y) = \beta \) for \( y = x \), \( \beta \in [0,1] \) and \( y \in X \). A fuzzy point \( x_\beta \) is said to be quasi-coincident with the fuzzy set A denoted by \( x_\beta \preceq A \) if and only if \( \beta + A(x) > 1 \). A fuzzy set A is quasi-coincident with a fuzzy set B denoted by \( A \preceq B \) if and only if there exists a point \( x \in X \) such that \( A(x) + B(x) > 1 \).

A family \( \tau \) of fuzzy sets of X is called a fuzzy topology [2] on X if 0, 1 belongs to \( \tau \) and \( \tau \) is super closed with respect to arbitrary union and finite intersection. The members of \( \tau \) are called fuzzy super open sets and their complement are fuzzy super closed sets.

For any fuzzy set A of X the closure of A (denoted by \( \text{cl}(A) \)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by \( \text{int}(A) \)) is the union of all fuzzy super open subsets of A.

**Definition 1.1**[5]:- Let \( (X,\tau) \) fuzzy topological space and \( A \subseteq X \) then
1. Fuzzy Super closure \( \text{scl}(A) = \{x \in X : \text{cl}(U) \cap A \neq \emptyset\} \)
2. Fuzzy Super interior \( \text{sint}(A) = \{x \in X : \text{cl}(U) \leq A \neq \emptyset\} \)

**Definition 1.2**[5]:- A fuzzy set A of a fuzzy topological space \( (X,\tau) \) is called:
(a) Fuzzy super closed if \( \text{scl}(A) \subseteq A \).
(b) Fuzzy super open if \( 1-A \) is fuzzy super closed \( \text{sint}(A) = A \).

**Remark 1.1**[5]:- Every fuzzy closed set is fuzzy super closed but the converses may not be true.

**Remark 1.2**[5]:- Let A and B are two fuzzy super closed sets in a fuzzy topological space \( (X,\tau) \), then \( A \cup B \) is fuzzy super closed.

**Remark 1.3**[5]:- The intersection of two fuzzy super closed sets in a fuzzy topological space \( (X,\tau) \) may not be fuzzy super closed.

**Definition 1.5**[3,8,9,10, 11]:- A fuzzy set A of a fuzzy topological space \( (X,\tau) \) is called:
1. fuzzy g- super closed if \( \text{cl}(A) \leq G \) whenever \( A \leq G \) and G is super open.
2. fuzzy g-super open if its complement $1-A$ is fuzzy g-super closed.

**Definition 1.8.** [3,8,9,10, 11]: A fuzzy point $x_{p} \in A$ is said to be quasi-coincident with the fuzzy set $A$ denoted by $x_{p} q A$ iff $p + A(x) > 1$. A fuzzy set $A$ is quasi-coincident with a fuzzy set $B$ denoted by $A q B$ iff there exists $x \in X$ such that $A(x) + B(x) > 1$. If $A$ and $B$ are not quasi-coincident then we write $A q B$. Note that $A \leq B , A q (1-B)$.

**Fuzzy g-super Closed Sets.**

**Definition 2.1.**: A fuzzy set $A$ of a fuzzy topological space $(X, \tau)$ is called fuzzy generalized super closed (fuzzy g-super closed) if $Cl(A) \leq O$ whenever $A \leq O$ and $O$ is fuzzy super open.

**Remark 2.1.**: Every fuzzy closed set is fuzzy g-super closed but its converse may not be true. For,

**Example 2.1.**: Let $X \{a,b\}$ and $A$ and $U$ be defined as follows: $A(a) = 0.3, A(b) = 0.2; UQ) = 0.5, U(b) = 0.7$; Let $\tau = \{\emptyset, U, X\}$ be a fuzzy topology on $X$. Then $A$ is fuzzy g-super closed but not fuzzy super closed.

**Theorem 2.1.**: If $A$ and $B$ are fuzzy g-super closed in a fuzzy topological space $(X, \tau)$ then $A \cup B$ is fuzzy g-super closed.

**Proof.**: Let $O$ be a fuzzy open set in $X$, such that $A \cup B \leq O$ then $A \leq O$ and $B \leq O$ so $Cl(A) \leq O$ and $Cl(B) \leq O$. Therefore $Cl(A) \cup Cl(B) = Cl(A \cup B) \leq O$. Hence $A \cup B$ is fuzzy g-super closed.

**Remark 2.2.**: The intersection of two fuzzy g-super closed sets in a fuzzy topological space $(X, \tau)$ may not be fuzzy g-super closed.

**Example 2.2.**: Let $X = \{a, b\}$ and $U, A$ and $B$ be defined as follows $U(a) = 0.7, U(b) = 0.6; A(a) = 0.6, A(b) = 0.7; B(a) = 0.8, B(b) = 0.5$; Let $\tau = \{\emptyset, U, X\}$, then $A$ and $B$ are fuzzy g-super closed in $(X, \tau)$ but $A \cap B$ is not fuzzy g-super closed.

**Theorem 2.2.**: Let $A \leq B \leq Cl(B)$ and $A$ is fuzzy g-super closed in a fuzzy topological space $(X, \tau)$. Then $B$ is fuzzy g-super closed.

**Proof.**: Let $O$ be a fuzzy super open set such that $B \leq O$ then $A \leq O$ and since $A$ is fuzzy g-super closed $Cl(A) \leq O$. Now $B \leq Cl(A) \Rightarrow Cl(B) \leq Cl(A) \leq O$. Consequently $B$ is fuzzy g-super closed.

**Definition 2.2.**: A fuzzy set $A$ of a fuzzy topological space $(X; -\tau)$ is called fuzzy g-super open iff $A^c$ is fuzzy g-super closed.

**Remark 2.3.**: Every fuzzy open set is fuzzy g-super open. The converse may not be true.

**Theorem 2.3.**: A fuzzy set $A$ of fuzzy topological space $(X, \tau)$ is fuzzy g-super open iff $F \leq \text{int}(A)$ whenever $F$ is fuzzy closed and $F \subset A$.

**Theorem 2.4.**: Let $A$ and $B$ be Q-separated fuzzy g-super open subsets of a fuzzy topological space $(X, \tau)$ then $A \cup B$ is fuzzy g-super open.

**Proof.**: Let $F$ be a fuzzy super closed subset of $A \cup B$. Then $F \cap (A \cup B) \leq (A \cup B) \cap Cl(A) = (A \cap Cl(A)) \cup (B \cap Cl(A)) \leq \text{int}(A)$. Similarly $F \cap (B \cap Cl(B)) \leq \text{int}(B)$. Now $F \cap (A \cup B) \leq (F \cap Cl(A)) \cup (F \cap Cl(B)) \leq \text{int}(A) \cup \text{int}(B) \leq \text{int}(A \cup B)$. Hence $F \leq \text{int}(A \cup B)$ and by theorem (2.2) $A \cup B$ is fuzzy g-super open.

**Theorem 2.5.**: Let $A$ and $B$ be two fuzzy g-super closed sets of a fuzzy topological space $(X, \tau)$ and suppose that $A^c$ and $B^c$ are Q-separated, then $A \cap B$ is fuzzy g-super closed.

**Theorem 2.6.**: Let $A$ be a fuzzy g-super open subset of a fuzzy topological space $(X, \tau)$ and $\text{int}(A) \leq B \leq A$ then $B$ is fuzzy g-super open.

**Proof.**: Since $A^c \leq B^c \leq Cl(A^c)$ and $A^c$ is fuzzy g-super closed it follows that $B^c$ is fuzzy g-super closed by theorem (2.2), thus $B$ is fuzzy g-super open.

**Theorem 2.7.**: Let $(Y, \tau_Y)$ be a subspace of a fuzzy topological space $(X, \tau)$ and $A$ be a fuzzy set in $Y$. If $A$ is fuzzy g-super closed in $X$ then $A$ is fuzzy g-super closed in $Y$. 

Proof. : Let \( A \subseteq O_Y \), where \( O_Y \) is fuzzy super open in \( Y \). Then there exists a fuzzy super open set \( O \) in \( X \) such that \( O_Y = O \cap Y \). Therefore \( A \subseteq O \) and since \( A \) is fuzzy \( g \)-super closed in \( X \), \( Cl(A) \subseteq O \). It follows that \( Cl_Y(A) = Cl(A) \cap Y \subseteq O \cap Y = O_Y \). Hence \( A \) is fuzzy \( g \)-super closed in \( Y \).

**Theorem 2.8.** Let \((X, \tau)\) be a fuzzy topological space and \( \mathcal{F} \) be the family of all fuzzy super closed sets of \( X \). Then \( \tau = \mathcal{F} \) iff every fuzzy subset of \( X \) is fuzzy \( g \)-super closed.

**Proof:** Necessity.: Suppose that \( \tau = \mathcal{F} \) and that \( A \subseteq O \in \tau \) then \( Cl(A) \subseteq Cl(O) = O \) and \( A \) is fuzzy \( g \)-super closed.

Sufficiency. Suppose that every fuzzy subset of \( X \) is fuzzy \( g \)-super closed. Let \( O \in \tau \) then since \( O \subseteq O \) and \( O \) is fuzzy \( g \)-super closed where \( Cl(O) \subseteq O \) and \( O \in \mathcal{F} \). Thus \( \tau \subseteq \mathcal{F} \) and \( \mathcal{F} \subseteq \tau \) consequently \( \mathcal{F} = \tau \) and \( \tau = \mathcal{F} \).

**Theorem 2.12.** Let \( A \) be a fuzzy \( g \)-super closed set in a fuzzy topological space \((X, \tau)\) and \( f: (X, \tau) \rightarrow (Y, \tau^*) \) is fuzzy super continuous and fuzzy super closed then \( f(A) \) is fuzzy \( g \)-super closed in \( Y \).

**Proof:** If \( f(A) \subseteq G \) where \( G \) is fuzzy super open in \( Y \) then \( A \subseteq f^{-1}(G) \) and hence \( Cl(A) \subseteq f^{-1}(G) \). Thus \( f(Cl(A)) \subseteq G \) and \( f(Cl(A)) \) is a fuzzy super closed set. It follows that \( Cl(f(A)) \subseteq Cl(f(Cl(A))) = f(Cl(A)) \subseteq G \). Then \( Cl(f(A)) \subseteq G \) and \( f(A) \) is fuzzy \( g \)-super closed.

**References**


