Introduction

The fundamental concept of fuzzy sets was initiated by Zadeh [10] in 1965 and opened a new path of thinking to mathematicians, engineers, physicists, chemists and many others due to its diverse applications in various fields. The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer science, control engineering, information science, coding theory, group theory, real analysis, measure theory etc. In 1971, Rosenfeld [5] first introduced the concept of fuzzy subgroups, which was the first fuzzification of any algebraic structure. Thereafter the notion of different fuzzy algebraic structures such as fuzzy ideals in rings and semirings etc. have seriously studied by many mathematicians. In 1979, Anthony and Sherwood redefined fuzzy groups. Since then many results in group theory has been generalized to fuzzy setting. Some properties of Products, set products, unions, intersections, the homomorphic images and pre images of fuzzy groups are introduces. V.N.Dixit, et.al [4] gave characterizations of fuzzy conjugate subgroups and fuzzy characteristic subgroups by their level subgroups. The philosophy is developing a technique with which we no longer merely decide whether a fuzzy subset is a fuzzy subgroup or not but with which we actually degree to which it is a fuzzy group. A.Solairaju and R.Nagarajan [16], [7], [8], [9] introduced the concept of Q - fuzzy groups. Our aim is to introduce the notion of degree to which a Q- fuzzy subset of a group G is a Q- fuzzy subgroup by means of the implication operator of [0,1] and only if its subgroup degree \( d_g(A) = 1 \). Some properties of subgroup degree are investigated.

Preliminaries

Throughout this paper , \([0,1]^X \) (or \( I^X \)) denote the set of all Q- fuzzy subsets on X. \( A \) denotes the constant Q-fuzzy sets on X taking the value A. For A \([0,1]^X \) and a \([0,1] \), we can use the notations

\[
\begin{align*}
A[a] &= \{ x \in X ; A(x,q) \geq a \} \\
A[a] &= \{ x \in X ; A(x,q) = a \}
\end{align*}
\]

**Definition: (T-norm):** A function \( T : [0,1] \times [0,1] \rightarrow [0,1] \) is called a t-norm if it satisfies, for all \( x,y,z \in [0,1] \), the following conditions:

1. **(T1)** \( T(x,y) = T(y,x) \)
2. **(T2)** \( T(x, T(y,z)) = T(T(y,x),z) \)
(T3) if \( y \leq z \), then \( T(x, y) \leq T(x, z) \), that is \( T(x, \cdot) \) is increasing.

(T4) \( T(x, 1) = x \).

**Definition:** A function \( I_T : [0,1] \times [0,1] \rightarrow [0,1] \) is called an R-implication if there exists a t-norm \( T \) such that \( I_T(x, y) = \sup \{ t \in [0,1] : T(x, y) \leq y \} \), \( x, y \in [0,1] \). \( I_T \) is also called the residual of the t-norm \( T \).

**Definition:** A Q-fuzzy subset \( A \) of a group \( G \) is said to be Q-fuzzy group of \( G \) if for any \( x, y \in G \), \( A(xy, q) \leq \min \{ A(x, q), A(y, q) \} \). If the degree of \( a \leq b \) is defined by \( I_T(a, b) \), then we can naturally introduce the notion of subgroup degree as follows.

**Definition:** Let \( A \) be a Q-fuzzy subset in a group \( G \) and \( T \) be a t-norm on \([0,1]\). The subgroup degree \( d_g(A) \) of \( A \) is defined as

\[
d_g(A) = \bigcap ( (A(x, q) \cap A(y, q)) \cap (A(x, q) \cap A(x^{-1}, q)))
\]

Moreover by

\[
\bigcap ( (A(x, q) \cap A(y, q)) \cap (A(x, q) \cap A(x^{-1}, q))) \leq \bigcap ( (A(x, q) \cap A(y, q)) \cap (A(x, q) \cap A(y^{-1}, q))) = d_g(A).
\]

We obtain

\[
d_g(A) = \bigcap ( (A(x, q) \cap A(y, q)) \cap (A(x, q) \cap A(x^{-1}, q))) = d_g(A).
\]

The following lemma is obvious.

**Lemma:** Let \( A \) be a Q-fuzzy subset in a group \( G \). Then \( d_g(A) \geq a \) if and only if for any \( x, y \in G \), \( A(x, q) \cap A(y, q) \cap a \leq A(xy^{-1}, q) \). By lemma 3.2 we can easily obtain the following results.

**Proposition:** Let \( A \) be a Q-fuzzy subset in a group \( G \). Then

\[
d_g(A) = \bigcup \{ a \in [0,1] / A(x, q) \cap A(y, q) \cap a \leq A(xy^{-1}, q) \} \text{ for all } x, y \in G \text{ and } q \in Q.
\]

**Proposition:** Let \( A \) be a Q-fuzzy subset in a group \( G \). Then

\[
d_g(A) = \bigcup \{ a \in [0,1] / \text{ for all } b < (0, a) , A_{[b]} \text{ is a subgroup of } G \}.
\]

Proof: Suppose that \( A(x, q) \cap A(y, q) \cap a \leq A(xy^{-1}, q) \) for all \( x, y \in G \). Then for any \( b \in (0, a) \) and for any \( x, y \in A_{[b]} \).

We have

\[
A(xy^{-1}, q) \geq A(x, q) \cap A(y, q) \cap a
\]
\[ \geq A(x,q) \cap A(y,q) \cap b = A(x,q) \cap A(y,q) \cap b, \]

This shows \( xy^{-1} \in A[b] \) and \( q \in Q \). Therefore \( A[b] \) is a subgroup of \( G \). Hence

\[ d_g(A) = \bigcup \{ a \in [0,1] / A(x,q) \cap A(y,q) \cap a \leq A(xy^{-1},q) \} \text{ for all } x,y \in G \text{ and } q \in Q. \]

Conversely, assume that \( a \in (0,1] \), \( A[a] \) is a subgroup of \( G \). For any \( x,y \in G \), let

\[ b = A(x,q) \cap A(y,q) \cap a, \text{ then } b \leq a \text{ and } x,y \in A[b]. \]

Thus \( x y^{-1} \in A[a] \) that is \( A(xy^{-1},q) \geq b = A(x,q) \cap A(y,q) \cap a. \)

This means that

\[ d_g(A) = \bigcup \{ a \in [0,1] / A(x,q) \cap A(y,q) \cap a \leq A(xy^{-1},q) \} \text{ for all } x,y \in G \text{ and } q \in Q. \]

Proposition: Let \( \{ A_i \}_{i \in I} \) be a family of Q-fuzzy subsets in a group \( G \). Then

\[ \text{deg}(\bigcap A_i) \geq \bigcap \text{deg}(A_i) \text{ for all } i \in I. \]

Proposition: Let \( f \) be homomorphism from a group \( G \) to a group \( G'. \)

(i) If \( A \) is Q-fuzzy subset in \( G \), then \( d_g(f^{-1}(A)) \geq d_g(A) \), where \( f^{-1}(A) = \bigcup \{ A(x,q) \cap A(y,q) \cap a \leq A(xy^{-1},q) \} \text{ for all } x,y \in G \text{ and } q \in Q. \)

(ii) If \( \eta \) is Q-fuzzy subset in \( G' \), then \( d_g(f^{-1}(\eta)) \geq d_g(\eta) \), where \( f^{-1}(\eta) = \bigcup \{ A(x,q) \cap A(y,q) \cap a \leq A(xy^{-1},q) \} \text{ for all } x,y \in G \).

Conclusion: In this paper, the notion of degree to which a Q- fuzzy subset is a Q- fuzzy group by means of implication operator of \([0,1]\) is discussed and Q-fuzzy subset \( A \) in a group \( G \) is a Q- fuzzy group if and only if its subgroup degree \( d_g(A) = 1. \)

References
8. A.Solairaju and R.Nagarajan, Lattice valued Q- fuzzy sub modules of near rings with respect to To norms , Advances in Fuzzy mathematics, 4(2)(2009),137-145.