A texture approach to $\alpha$-compactness in Ditop space

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**ABSTRACT**

The essence of this paper is to introduce the notion of pseudo $\alpha$-open sets and pseudo $\alpha$-closed sets. More results on compactness, co compactness, $\alpha$-compactness and $\alpha$-cocompactness in ditopological texture spaces are analysed. Many effective characterizations and properties of these concepts are also obtained.

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**Introduction**

Textures were based on point-set concept for the study of fuzzy sets in 1998 by L.M. Brown[2]. Also textures offers a convenient setting for the investigation of complement-free concepts in general, so much of the recent work has proceeded independently of the fuzzy setting.

This concept is further extended by many researchers and generalized the sets and maps in texture setting. In this paper we present some results based on compactness, $\alpha$-compactness, pseudo $\alpha$ open and closed sets. Many characterizations and properties of pseudo $\alpha$-open sets and pseudo $\alpha$-closed sets are discussed.

Definition 1.1 Let $S$ be a set, a texturing $T$ of $S$ is a subset $T \subseteq P(S)$.

1. $T$ contains $\varnothing$ and $S$.
2. $T$ is completely distributive.
3. $T$ separates the points of $S$. That is, for any $A, B \subseteq S$, if $A \cap B = \varnothing$, then $A \in T$ or $B \in T$.
4. $T$ satisfies $A \subseteq B \Rightarrow A \in T \Rightarrow B \in T$.

Let $S = \{a, b, c\}$.

If $S$ is used for using $P(S)$, then $P(S) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, S\}$ is a simple texturing of $S$.}

... and other similar content
We denote by \( O(S; T; \tau, \kappa) \), or when there can be no confusion by \( O(S) \), the set of open sets in \( S \). Likewise, \( C(S; T; \tau, \kappa) \) or \( C(S) \) will denote the set of closed sets.

**Definition 1.5.** A subset \( T \times T \) is called a difamily on \( (S, T; \tau, \kappa) \) if every open set \( F \in \{ \cup_{i \in I} F_i \mid I \subseteq V \} \) is bounded. Consider a difamily \( C \) on \( (S, T; \tau, \kappa) \). It is called a \( \tau \)-cocompact if every cover of \( S \) by \( \{ F_j \mid j \in J \} \) has a finite subcover.

**Definition 1.6.** A ditopological texture \( T \times T \) is called \( \tau \)-open if \( \{ F \cap \sigma \mid F \in \tau \} \) is \( \tau \)-closed.

**Definition 1.7.** Let \((\tau, \kappa)\) be a ditopological texture on \((S, T; \tau, \kappa)\). Then a difamily \( C \) on \((S, T; \tau, \kappa)\) is called \( \kappa \)-compact if every open set \( F \in \{ \cup_{i \in I} F_i \mid I \subseteq V \} \) is \( \kappa \)-cocompact in \( S \). That is, whenever \( F_j \in \{ \cup_{i \in I} F_i \mid I \subseteq V \} \), there exists a finite subset \( J' \) of \( J \) for which \( \cap \{ F_j \mid j \in J' \} \subseteq F \).

**Pseudo \( \alpha \)-open and \( \alpha \)-cocompact.**

**Theorem 2.1.** For a ditopological texture \((S, T; \tau, \kappa)\), \( \alpha \) is \( \tau \)-compact in \( S \). That is, whenever \( F_j \in \{ \cup_{i \in I} F_i \mid I \subseteq V \} \), there exists a finite subset \( J' \) of \( J \) for which \( \cap \{ F_j \mid j \in J' \} \subseteq F \).

**Proof.** Suppose that \( \alpha \) is \( \tau \)-compact in \( S \). Then \( \{ F \cap \sigma \mid F \in \tau \} \) is \( \tau \)-closed, \( \kappa \)-cocompact and \( \tau \)-costable and \( \kappa \)-stable.

**Corollary 3.3.** Let \((S, T; \tau, \kappa)\) be a ditopological texture space. \( (1) \) Every subset \( A \in O(S) \) is pseudo \( \alpha \)-open.

**Proof.** Suppose that \( A \in O(S) \) is pseudo \( \alpha \)-open. Then \( A = (\cap \{ \{ Q_s \mid P_s \subseteq A \} \cap \{ \{ Q_s \mid P_s \subseteq A \} \} = (\cap \{ \{ Q_s \mid P_s \subseteq A \} \cap \{ \{ Q_s \mid P_s \subseteq A \} \) = A. Hence \( (1) \) is proved.

**Theorem 3.4.** Let \((S, T; \tau, \kappa)\). Then \( (1) \) Every subset \( A \in O(S) \) is pseudo \( \alpha \)-open.
Gi. Assume if \( \alpha P(A) \subsetneq VG_i \). By the definition of \( \alpha P(A) \) there exists \( s \in S \) with \( A \subsetneq Q_s \) and \( \alpha cl(Ps) \subsetneq Gi \), for each \( i \). Now \( A \subsetneq Q_s \rightarrow VG_i \subsetneq Q_s \) (i.e), \( Ps \subsetneq VG_i \) which implies \( \alpha cl(Ps) \subsetneq Gi \) since \((\tau, \kappa)\) is \( \alpha R0 \), which is a contradiction, hence \( \alpha P(A) \subsetneq VG_i \). Now given that \( A \) is pseudo close, therefore we have \( \alpha P(A) = \alpha cl(A) \) and so \( \alpha cl(A) \subsetneq VG_i \).

Then two cases arise

**case(1)**: If \( \alpha cl(A) = S \). As \( S \) is \( \alpha \) compact, we have \( \alpha cl(A) \) is \( \alpha \) compact. Then by definition there exists \( i_1, i_2, \ldots \) in such that \( \alpha cl(A) \subsetneq Gi_1 \cup Gi_2 \cup \ldots \cup Gi_n \). Since \( A \subsetneq \alpha cl(A) \) we have \( A \subsetneq \cup Gi \), \( i = 1, 2, \ldots, n \) which shows that \( A \) is \( \alpha \) compact.

**case(2)**: \( \alpha cl(A) \neq S \). As \( S \) is \( \alpha \) stable, we have \( \alpha cl(A) \) is \( \alpha \) compact. (2) is dual of (1).

**Reference**


