The Riemann zeta function and its extension into continuous optimization equation

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ABSTRACT
In this paper, the Riemann Zeta function is presented as a function with real and imaginary parts. Thus we are able to evaluate
\[
\zeta(z) = \frac{\varphi(z)}{\rho(z)}
\]

By writing \( \zeta(z) \) as a bilinear function, and through the use of Sobolev space theorem, an optimization problem with a variable coefficient is derived. Some methods of solution are presented.

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Introduction
Given that
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(1)}
\]
Such that
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(2)}
\]
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(3)}
\]
Equation (4) given
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(4)}
\]
This implies that (1) can be written as
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(5)}
\]
If one substitutes the Taylor’s series expansions for
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(6)}
\]
On further simplification, it can be shown that
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(7)}
\]
The above equation (8) is also equivalent to (9) on using integrating by part;
Recall that
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(10)}
\]
Thus:
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(11)}
\]
If we replace \( \frac{d}{dz} \left[ z \right] \) by \( \zeta(z) \), the resulting function will be;
Riemann presented in [Riemann (1859)] that;
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(12)}
\]
It follows that
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(13)}
\]
Thus (13) can be written as
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(14)}
\]
If one substitutes \( \frac{d}{dz} \left[ z \right] \) into (16) and rationalizes the emerging equation, this will lead to;
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(15)}
\]
Where:
\[
A = \frac{2^{2n+1} \pi^{2n+1}}{\pi^{n+1} \Gamma(n+1)} \left( \frac{n+1}{2(n+1)} \right) \quad \text{(16)}
\]
\[
B = \frac{2^{2n+1} \pi^{2n+1}}{\pi^{n+1} \Gamma(n+1)} \left( \frac{n+1}{2(n+1)} \right) \quad \text{(17)}
\]
Using binomial theorem on equation (19), we obtain
\[
\frac{d}{dz} \left[ z \right] = e^{\frac{d}{dz} \left[ z \right]} \quad \text{(18)}
\]
If we choose \( k = n + 1 \) then B becomes

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To evaluate the value of $B^2$, we simply compute the square of (23) such that:

\[
B^2 = \left( \frac{\log z}{2} \right) + \sum_{n=1}^{L} \left( \frac{\log z}{n} \right) \left( \frac{1}{n} \right) \left( x \right) \left( y \right) \left( z \right) \left( w \right)
\]

The above equation allows us to write (17) as follows:

\[
\zeta(z) = \sum_{n=1}^{L} \left( \frac{\log z}{4} \right) (2n^2 \pi^2 x^2) \left( \frac{A}{(nL+1)^2} + \frac{E}{2n^2 \pi^2} \right)
\]

If the above series is truncated at L= even number then, (27) becomes:

\[
\zeta(z) = \sum_{n=1}^{L} \left( \frac{\log z}{4} \right) (2n^2 \pi^2 x^2) \left( \frac{A}{(nL+1)^2} + \frac{E}{2n^2 \pi^2} \right)
\]

On the other hand, if L is an odd number then the series in (27) becomes:

\[
\zeta(z) = \sum_{n=1}^{L} \left( \frac{\log z}{4} \right) (2n^2 \pi^2 x^2) \left( \frac{A}{(nL+1)^2} + \frac{E}{2n^2 \pi^2} \right)
\]

where $\delta$ and $\rho$ could be either $-1$ or $+1$.

On multiplying (17) by its conjugate, we obtain $\zeta(x \bar{z}(x))$ to be;

\[
\lim_{n \to \infty} \sum_{n=1}^{\infty} \left( \frac{\log z}{4} \right) (2n^2 \pi^2 x^2) \left( \frac{A}{(nL+1)^2} + \frac{E}{2n^2 \pi^2} \right)
\]

This can be neatly written as;

\[
\xi(z) = \sum_{n=1}^{\infty} \left( \frac{\log z}{4} \right) (2n^2 \pi^2 x^2) \left( \frac{A}{(nL+1)^2} + \frac{E}{2n^2 \pi^2} \right)
\]

where

\[
\pi(z) = \sum_{n=1}^{\infty} \left( \frac{\log z}{4} \right) (2n^2 \pi^2 x^2) \left( \frac{A}{(nL+1)^2} + \frac{E}{2n^2 \pi^2} \right)
\]

From the above, it is clear that (17) gives $\gamma(z)$ as the state variable and $\beta(t)$ as the control variable.

\[
\gamma(z) = \sum_{n=1}^{\infty} \left( \frac{\log z}{4} \right) (2n^2 \pi^2 x^2) \left( \frac{A}{(nL+1)^2} + \frac{E}{2n^2 \pi^2} \right)
\]

Conclusion

If we choose to minimize the integral of (31), we come to obtain:

\[
\min_{\gamma(z)} \frac{1}{2} \left( \frac{d\gamma(z)}{dt} + \frac{d\gamma(z)}{dt} \right)
\]

Furthermore, (35) is a quadratic function for which its bilinear transformation is given as;

\[
\min_{\gamma(z)} \frac{1}{2} \left( \frac{d\gamma(z)}{dt} + \frac{d\gamma(z)}{dt} \right)
\]

On imposing some constraints on (36), it becomes an optimization problem of the form;

\[
\min_{\gamma(z)} \frac{1}{2} \left( \frac{d\gamma(z)}{dt} + \frac{d\gamma(z)}{dt} \right)
\]

Subject to the constraints;

The constrained problem (37) can be turned into unconstrained problem via the penalty method and the multiplier method (34) as;

\[
\min_{\gamma(z)} \frac{1}{2} \left( \frac{d\gamma(z)}{dt} + \frac{d\gamma(z)}{dt} \right)
\]

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