On Tutoring Quality Improvement of a Mathematical Topic Using Neural Networks (With a Case Study)

Zedan M. Abdulhamid¹, Hassan. M. Mustafa² and Ayoub Al-Hamadi³
¹Faculty of Education, Educational Technology Department, Albaaha University, K.S.A.
²Educational Technology Department-Faculty of Specified Education-Banha University Egypt Currently With Computer Eng. Department Al-Baha University (K.S.A.).
³Institute for Electronics, Signal Processing and Communications (IESK) Otto-von-Guericke-University Magdeburg.

ABSTRACT

This paper motivated by an interdisciplinary research work approach integrating multi-sensory cognitive learning theory with interesting issue of tutoring quality improvement. It is worthy to note that adopted approach has been inspired by analysis and evaluation of phonics methodology applied in teaching children "how to read?". Herein, quantitative evaluation of this issue performed by considering two computer aided learning (CAL) packages concerned with a specific mathematical topic namely long division process. Via realistic modeling of packages using Artificial Neural Networks (ANN) based upon associative memory learning paradigm. In more details, at educational field practice ; both CAL packages have been applied for teaching children algorithmic steps performing long division processes. Moreover, learning performance evaluation of presented packages considers children outcomes' achievement after tutoring for suggested Mathematical Topic either with or without associated tutor's voice. Interestingly, statistical analysis of obtained educational case study results at children classrooms (for both applied packages) versus classical tutoring proved to be in well agreement with obtained after ANN computer simulation results.

1. Introduction

The field of the learning sciences is currently represented by a growing community internationally. Many educational experts now recognize that conventional ways of conceiving knowledge associated with educational systems performance as well as assessment of technology-mediated learning processes are facing increasingly challenges [1]. That is due to rapid technological and social changes arise in this time considering modified educational field applications [2]. It is announced (in U.S.A.) that last decade (1990-2000) named as Decade of the brain [3]. Accordingly, neural network theorists as well as neurobiologists and educationalists have focused their attention on making interdisciplinary contributions to investigate essential brain functions (learning and memory), and cognitive multisensory learning [1][2][4][5]. Recently, Artificial Neural Networks (ANN) combined with neuroscience, and cognitive science considered as an interdisciplinary research direction for optimal teaching children methodology how to read?. This direction motivated by a great debate given at[6]as researches at fields of psychology and linguistic were continuously cooperating in searching for optimal methodology which supported by educational field results. Nevertheless, during last decade phonics methodology is replaced –at many schools in U.S.A.- by other guided reading methods that performed by literature based activities [7]. Some promising field results have been obtained at [8] ; that support optimality of phonics methodology for solving learning/teaching issue “how to read?” [8][9]. More recently, mathematical formulation for phonics methodology has been modeled and presented in details at [10].

Accordingly, building up models of human brain functions considered as recent evaluation trend by educationalists in learning science that incorporates Neuro-physiology, psychology, and cognitive science. Interestingly, this novel trend is supported by what has been announced by National Institutes of Health (NIH) in U.S.A. that children in elementary school, may be qualified to learn "basic building blocks" of cognition and that after about 11 years of age, children take these building blocks and use them [12]. Referring to the above findings introduced at [5] and [12], as well as new research which shows the prefrontal cortex handles the work of associating numerals with matching quantities [13] motivated our approach for modelling learning / teaching mathematics using ANNs. Moreover, this paper is motivated by results of collaboration work between (Papert and Piaget) that led to consider using mathematics in the service of understanding how children learn and think [14].

Herein, this optimal approach adopted for improving teaching/ learning performance of a mathematical topic to children of about 11 years age. The suggested mathematical topic is teaching children algorithmic process for performing long division. Specifically for two arbitrary integer numbers chosen in a random manner (each composed of some number of digits). By detail, adopted principal algorithm for applied Computer Aided Learning (CAL) package consisted of five steps follows. Divide, Multiply, Subtract, Bring Down, and repeat (if necessary). [15]. For further details related to recent view and associated with the effect of application of information technology computer (ITC) on mathematical education, it is referred to [16]. The rest of this paper is organized as follows.
At next section, a basic interactive educational model is presented with a generalized block diagram. Obtained results after application of suggested CAL package at the case study at the third section, in addition obtained simulation results. At the last forth section some interesting conclusions in addition to suggestions for future work are presented. Finally, an Appendix is given for simplified flow chart of adopted CAL package.

2. General Interactive Educational Model

2.1 Basic Learning/Teaching Model

Generally, practical performing of learning process - from neurophysiologic P.O.V. - utilises two basic and essential cognitive functions and brain based learning[17]. Both functions are required to perform efficiently learning / teaching interactive process in accordance with behaviourism [18][19], as follows. Firstly, pattern classification/recognition function based on visual/audible interactive signals stimulated by CAL packages. Secondly, associative memory function is used which is originally based on classical conditioning motivated by Hebbian learning rule.

Referring to Fig.1 shown in below, the illustrated teaching model is well qualified to perform simulation of above mentioned brain functions. Inputs to the neural network learning model at that Figure, are provided by environmental stimuli (unsupervised learning).

The correction signal for the case of learning with a teacher is given by responses outputs of the model will be evaluated by either the environmental conditions (unsupervised learning) or by the teacher. Finally, the tutor plays a role in improving the input data (stimulating learning pattern), by reducing noise and redundancy of model pattern input. That is according to tutor’s experience, he provides the model with clear data by maximizing its signal to noise ratio. However, that is not our case which is based upon unsupervised Hebbian self-organized (autonomous) learning. [20][21].

Figure 1 Illustrates a general view for interactive educational process, adapted from [9].

2.2 Basic ANN Model

The presented model given at Figure 2 in above; simulates generally two diverse learning paradigms. It presents realistically both paradigms: by interactive learning/ teaching process, as well as other self organized (autonomous) learning. By some details, firstly is concerned with classical (supervised by a tutor) learning observed in our classrooms (face to face tutoring).

Accordingly, this paradigm proceeds interactively via bidirectional communication process between a teacher and his learner(s) [22][23]. However, secondly other learning paradigm performs self-organized (autonomously unsupervised) tutoring process [20].

![Generalized ANN block diagram simulating two diverse learning paradigms, adapted from [9]](image)

Referring to above Figure 2 : the error vector \( \mathbf{e}(n) \) at any time instant \( n \) observed during learning processes is given by:

\[
\mathbf{e}(n) = \mathbf{y}(n) - \mathbf{d}(n)
\]

Where \( \mathbf{y}(n) \) is the output signal of the model. \( \mathbf{d}(n) \) is the desired numeric value(s). Moreover, the following four equations are deduced:

\[
V_k(n) = X_j(n)W^T_{kj}(n)
\]

\[
Y_k(n) = \varphi(V_k(n)) = (1 - e^{-\lambda V_k(n)}) / (1 + e^{-\lambda V_k(n)})
\]

\[
e_k(n) = |d_k(n) - y_k(n)|
\]

\[
W_{kj}(n+1) = W_{kj}(n) + \Delta W_{kj}(n)
\]

Whereby \( X \) is input vector and \( W \) is the weight vector. \( \varphi \) is the activation function. \( Y \) is the output. \( e_k \) is the error value and \( d_k \) is the desired output. Noting that \( \Delta W_{kj}(n) \) is the dynamical change of weight vector value. Above four equations are commonly applied for both learning paradigms: supervised (interactive learning with a tutor), and unsupervised (learning through students’ self-study). The dynamical changes of weight vector value specifically for supervised phase is given by:

\[
\Delta W_{kj}(n) = \eta e_k(n)X_j(n)
\]

Where \( \eta \) is the learning rate value during the learning process for both learning paradigms. However, for unsupervised paradigm, dynamical change of weight vector value is given by:

\[
\Delta W_{kj}(n) = \eta Y_k(n)X_j(n)
\]

Noting that \( e_k(n) \) in (6) is substituted by \( y_k(n) \) at any arbitrary time instant \( n \) during the learning process.

3. Results

Obviously, children learning styles are relatively stable and consistent set of strategies that individual preference adopted while engaging in learning [22][23]. Herein, our practical application (case study) adopts one of these strategies namely acquiring learning information through two sensory organs (student eyes and ears). In other words, seen and heard (visual and audible) interactive signals are acquired by student's sensory organs either through his teacher or considering CAL packages (with or without teacher’s voice). Practically, children are classified in three groups in according to their diverse learning styles (preferences).

The results obtained after performing practical experimental work in classroom (case study) is shown at next subsection 3.1. Additionally, at subsection 3.2 realistic simulation results are introduced. Interestingly, it is clear that both obtained results (practical and simulation) are well in agreement and supporting each other.
3.1 Case Study Results
The obtained case study results are presented at this subsection by two graphical figures (Fig.2&Fig.3). Each figure illustrates two bi-comparative tutoring performance. Noting that both tutoring CAL packages (without audio, and with audio tools), are having superiority with respect to the classical teaching approach.

Furthermore, the two tables (Table.1 & Table.2) illustrate the obtained practical results after performing three different learning experiments. At table.1, illustrated results are classified in accordance with different students’ learning styles following three teaching methodologies. Firstly, the classical learning style is carried out by students-teacher interactive in the classroom. Secondly, learning is taken place using a suggested software learning package without teacher’s voice association. The last experiment is carried out using CAL package that is associated with teacher's voice. This table gives children's achievements (obtained marks) considering that maximum mark is 100. The statistical analysis of all three experimental marking results is given in details at Table.2 shown in below.

3.2 Simulation Results
The following program list designed for simulation of ANN supervised learning paradigm. It is written using MATLAB Version VI software. This program corresponds specifically to dynamical changes of three weight vectors for supervised learning paradigm given by equation (6) [shown at the previous subsection 2.2]. Furthermore, obtained results (after running that computer program) are depicted considering some learning rate value ($\eta = 0.4$). The comprehensive calculations of statistical averages (for various learning rate values) are presented graphically in Figure 4, however, other statistical parameters are given in Table 3. Including standard deviation ($\sqrt{\sigma}$) and coefficient of variation ($\rho = \frac{\sqrt{\sigma}}{M}$).

**Table.1: Illustrates children outcomes after performing three educational experiments.**

<table>
<thead>
<tr>
<th>Classical Learning (Marks)</th>
<th>CAL without tutor’s voice (Marks)</th>
<th>CAL with tutor’s voice (Marks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>65</td>
</tr>
<tr>
<td>2</td>
<td>43</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>29</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>75</td>
</tr>
<tr>
<td>5</td>
<td>37</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>62</td>
</tr>
<tr>
<td>8</td>
<td>60</td>
<td>90</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>85</td>
</tr>
<tr>
<td>10</td>
<td>48</td>
<td>50</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
<td>80</td>
</tr>
<tr>
<td>12</td>
<td>55</td>
<td>90</td>
</tr>
<tr>
<td>13</td>
<td>40</td>
<td>58</td>
</tr>
<tr>
<td>14</td>
<td>8</td>
<td>55</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td>60</td>
</tr>
</tbody>
</table>

**Table 2: Illustrates statistical analysis of above obtained children’s marks.**

<table>
<thead>
<tr>
<th>Teaching Methodology</th>
<th>Students’ average Achievement score (M)</th>
<th>Variance $\sigma$</th>
<th>Standard deviation $\sqrt{\sigma}$</th>
<th>Coefficient of variation $\rho = \frac{\sqrt{\sigma}}{M}$</th>
<th>Improvement of teaching Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical</td>
<td>32.46</td>
<td>265.32</td>
<td>16.28</td>
<td>0.50</td>
<td>-</td>
</tr>
<tr>
<td>CAL (without tutor’s voice)</td>
<td>46.80</td>
<td>297.49</td>
<td>17.24</td>
<td>0.36</td>
<td>44.1%</td>
</tr>
<tr>
<td>CAL (with tutor’s voice)</td>
<td>64.33</td>
<td>283.42</td>
<td>16.83</td>
<td>0.26</td>
<td>98.2%</td>
</tr>
</tbody>
</table>

**Figure 4. The effect of various learning rate values $\eta$ on average students’ learning outcomes (achievements)**
Supervising Learning Algorithm for various Learning Rate Values $\eta$

\begin{verbatim}

w=rand(1000,1000); x1=0.8; x2=0.7; x3=0.6; l=1; eta=0.4;

for g=1:100
  nog(g)=0;
end

for i=1:1000
  w1=w(1,i); w2=w(2,i); w3=w(3,i);
  net=w1*x1+w2*x2;
  y=1/(1+exp(-l*net));
  e=0.9-y;
  no(i)=0;
  while e>0.05
    no(i)=no(i)+1;
    net=w1*eta*e*x1;
    w2=w2+eta*e*x2;
    w3=w3+eta*e*x3;
  end
end

for i=1:100
  nog(i)=0;
  for x=1:1000
    if no(x)==i
      nog(i)=nog(i)+1;
    end
  end
end

i=0:99;
plot((i+1),nog(i+1),'linewidth',1.0,'color','black')
xlabel('Itr. number')
ylabel('No of occurrences for each cycle')
title('error correction algorithm')
grid on
hold on

\end{verbatim}

The suggested ANN model adapted from realistic learning simulation model given at [10] with considering various learning rate values.

It is worthy to note that learning rate value associated to CAL with teacher’s voice proved to be higher than CAL without voice. Simulation curves at Fig.4 illustrate statistical comparison for two learning processes with two different learning rates.

The lower learning rate ($\eta = 0.1$) may be relevant for simulating classical learning process. However, higher learning rate ($\eta = 0.5$) could be analogously considered to indicate (approximately) the case of CAL process applied without teacher’s voice.

4. Conclusion
This paper comes to two interesting conclusive remarks given as follows:
- Evaluation of any CAL package quality is measured after statistical analysis of educational field results. So, above suggested strategy provides specialists in educational field with fair unbiased judgment for any CAL package. That is by comparing statistical analysis of simulation results with natural analysis of individual differences obtained in by practice.
- After practical application of our suggested multimedia CAL package (case study), interesting results obtained considering diverse individuals’ learning styles. Obtained results are depending only upon two cognitive sensory systems (visual and/or audible) while performing learning process.

The shown figure in the above illustrates a simplified macro-flowchart of the two CAL packages that describing briefly algorithmic steps of long division process: Divide, Multiply, Subtract, Bring Down, and repeat (if necessary)[15].
Fig. 4 Illustrates Simulation results presented by statistical distribution for children's (students) achievements versus the frequency of occurrence for various achievements values, at different learning rate values ($\eta = 0.1$ & $\eta = 0.5$).

Table 3: Illustration of simulation results for different learning rate values $\eta$.

<table>
<thead>
<tr>
<th>Learning Rate value</th>
<th>Children's average Achievement score (M)</th>
<th>Variance $\sigma$</th>
<th>Standard deviation $\sigma$</th>
<th>Coefficient of variation $\rho = \sqrt{\sigma}/M$</th>
<th>Improve of teaching Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta = 0.1$</td>
<td>42</td>
<td>428.5</td>
<td>20.7</td>
<td>0.61</td>
<td>-</td>
</tr>
<tr>
<td>$\eta = 0.5$</td>
<td>64</td>
<td>918.1</td>
<td>30.3</td>
<td>0.47</td>
<td>66%</td>
</tr>
</tbody>
</table>

- Consequently, by future application of virtual reality technique in learning process will add one more sensory system (tactile) contributing in learning process. So, adding of the third sensory (tactile system) means being more promising for giving more additive value for learning/teaching effectiveness. Finally, for future modification of suggested CAL package measurement of time learning parameter will be promising for more elaborate measurement of learning performance in practical educational field (classroom) application. This parameter is recommended for educational field practice, [24] as well as for recently suggested measuring of e-learning systems performance [25][26].

References


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