Some properties of Q-Intuitionistic L-Fuzzy subnearrings of a nearring

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ABSTRACT
In this paper, we study some of the properties of Q-intuitionistic L-fuzzy subnearring of a nearring and prove some results on these.

1. Introduction
After the introduction of fuzzy sets by L.A.Zadeh[15], several researchers explored on the generalization of the notion of fuzzy set. The concept of intuitionistic L-fuzzy subset was introduced by K.T.Atanassov[4,5], as a generalization of the notion of fuzzy set. Azriel Rosenfeld[6] defined a fuzzy groups. Asok Kumer Ray[3] defined a product of fuzzy subgroups and A.Solairaju and R.Nagarajan[13,14] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. We introduce the concept of Q-intuitionistic L-fuzzy subnearring of a nearring and established some results.

1.1 Preliminaries:
1.1 Definition: Let X be a non-empty set and L = (L, ≤) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A of X is a function  A : X × Q → L. Let A and B be any two Q-intuitionistic L-fuzzy subsets of X respectively and for every x in X and y in Q. Also, µA(x, y) ≥ µB(x, y), for all x and y in X and q in Q. Therefore, µA(x, q) ≥ µB(x, q), for all x and y in X and q in Q.

1.2 Definition: Let (L, ≤) be a complete lattice with an involutive order reversing operation N : L → L. Let (R, +, .) be a nearring. A Q-intuitionistic L-fuzzy subset A of R is said to be a Q-intuitionistic L-fuzzy subnearring (QILFSNR) of R if it satisfies the following axioms:
(i) µA(x, q) · µB(x, q) ≤ µC(x, q)
(ii) µA(x, q) · µB(x, q) ≤ µC(x, q)
(iii) µA(x, q) · µB(x, q) ≤ µC(x, q)
(iv) µA(x, q) · µB(x, q) ≤ µC(x, q)

1.3 Definition: Let (R, +, ·) be a nearring. A Q-intuitionistic L-fuzzy subset A of R is said to be a Q-intuitionistic L-fuzzy subnearring (QILFSNR) of R if it satisfies the following axioms:
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(iv) µA(x, q) · µB(x, q) ≤ µC(x, q)

1.4 Definition: Let A and B be any two Q-intuitionistic L-fuzzy subnearrings of nearrings R₁ and R₂ respectively. The product of A and B denoted by AxB is defined as AxB = { ((x, y), q), µAxB((x, y), q), νAxB((x, y), q) } / for all x in R₁ and y in R₂ and q in Q, where µAxB(x, y, q) = µA(x, q) · µB(y, q) and νAxB(x, y, q) = νA(x, q) ∨ νB(y, q).

1.5 Definition: Let A be a Q-intuitionistic L-fuzzy subset in a set S, the strongest Q-intuitionistic L-fuzzy relation on S, that is a Q-intuitionistic L-fuzzy relation on A is V given by µV(x, y, q) = µA(x, q) ∧ µA(y, q) and νV(x, y, q) = νA(x, q) ∨ νA(y, q), for all x and y in S and q in Q.

2. Some properties of Q-intuitionistic L-fuzzy subnearrings of a nearring
2.1 Theorem: Intersection of any two Q-intuitionistic L-fuzzy subnearrings of a nearring is a Q-intuitionistic L-fuzzy subnearring of R.
Therefore, $v_C(xy, q) \leq v_C(x, q) \lor v_C(y, q)$, for all $x$ and $y$ in $R$. Thus, $C$ is a Q-intuitionistic L-fuzzy subnearring of a nearring $R$. Hence, intersection of any two Q-intuitionistic L-fuzzy subnearrings of a nearring $R$ is a Q-intuitionistic L-fuzzy subnearring of $R$.

2.2 Theorem: Let $(R, +, \cdot)$ be a nearring. The intersection of a family of Q-intuitionistic L-fuzzy subnearrings of $R$ is a Q-intuitionistic L-fuzzy subnearring of $R$.

Proof: Let $\{V_i : i \in I\}$ be a family of Q-intuitionistic L-fuzzy subnearrings of a nearring $R$ and let $A = \bigcap_{i \in I} V_i$. Let $x$ and $y$ in $R$ and $q$ in $Q$. Then, $\mu_A(x - y, q) = \inf_{i \in I} \mu_{V_i}(x - y, q) \geq \mu_{V_i}(x, y, q)$ for all $x$ and $y$ in $R$ and $q$ in $Q$. Thus, $\mu_A(x - x, q) = 1$ for all $x$ and $y$ in $R$ and $q$ in $Q$. Therefore, $\mu_A(x, y, q) \subseteq \mu_A(x, y, q)$, for all $x$ and $y$ in $R$ and $q$ in $Q$. Hence, $A$ is a Q-intuitionistic L-fuzzy subnearring of $R$.

2.4 Theorem: Let $A$ and $B$ be Q-intuitionistic L-fuzzy subnearrings of the nearrings $R_1$ and $R_2$ respectively. Suppose that $e$ and $e'$ are the identity element of $R_1$ and $R_2$ respectively. If $A$ and $B$ are Q-intuitionistic L-fuzzy subnearrings of $R_1$ and $R_2$, then at least one of the following two statements must hold.

(i) $\mu_A(x, q) \leq \mu_A(x, q)$ and $v_A(e, q) \leq v_A(x, q)$, for all $x$ in $R_1$ and $q$ in $Q$.

(ii) $\mu_A(e, q) \leq \mu_B(e, q)$ and $v_A(e, q) \leq v_B(e, q)$, for all $y$ in $R_2$ and $q$ in $Q$.

Proof: Let $A$ and $B$ be Q-intuitionistic L-fuzzy subnearrings of $R_1$ and $R_2$, respectively. Then, we can find a $x$ in $R_1$ and $y$ in $R_2$ such that $\mu_A(x, q) > \mu_B(y, q)$ and $v_A(x, q) < v_B(y, q)$ for all $x$ and $y$ in $R_1$ and $R_2$, respectively. Suppose that $A$ and $B$ are Q-intuitionistic L-fuzzy subnearrings of $R_1$ and $R_2$, respectively. Then, at least one of the following two statements must hold.

(i) $\mu_A(x, q) \leq \mu_A(x, q)$ and $v_A(e, q) \leq v_A(x, q)$, for all $x$ in $R_1$ and $q$ in $Q$.

(ii) $\mu_A(e, q) \leq \mu_B(e, q)$ and $v_A(e, q) \leq v_B(e, q)$, for all $y$ in $R_2$ and $q$ in $Q$.

2.5 Theorem: Let $A$, $B$, and $C$ be any two Q-intuitionistic L-fuzzy subnearrings of the nearrings $R_1$, $R_2$, and $R_3$, respectively, then $A \cap B$ is a Q-intuitionistic L-fuzzy subnearring of $R_1 \times R_2$.

Proof: Let $A$, $B$, and $C$ be any two Q-intuitionistic L-fuzzy subnearrings of the nearrings $R_1$, $R_2$, and $R_3$, respectively. Then, $A \cap B$ is a Q-intuitionistic L-fuzzy subnearring of $R_1 \times R_2$. Now, using the property that $\mu_A(x, q) \leq \mu_B(x, q)$ and $v_A(x, q) \leq v_B(x, q)$, for all $x$ in $R_1$ and $q$ in $Q$, we get, $\mu_A(x, y, q) \leq \mu_B(x, y, q)$ and $\mu_B(x, y, q) \leq \mu_A(x, y, q)$ for all $x$ and $y$ in $R_1$ and $R_2$, respectively. Therefore, $A \cap B$ is a Q-intuitionistic L-fuzzy subnearring of $R_1 \times R_2$.
Then, \((x, e)\) and \((e, y)\) are in \(R \times R\). We get, \(\mu_\theta(x - y, q) = \mu_\theta(x - y, q) \wedge \mu_\theta(x, q \wedge \mu_\theta(x - y, q) = \mu_\theta(x - y, q) \wedge \mu_\theta(x, q) \wedge \mu_\theta(x - y, q) = \mu_\theta(x - y, q) \wedge \mu_\theta(x, q) \wedge \mu_\theta(x - y, q)
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Thus, \(\mu_\theta(x - y, q) \geq \mu_\theta(x, q) \wedge \mu_\theta(y, q), \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). Also, \(\mu_\theta(x, q) \geq \mu_\theta(x, q) \wedge \mu_\theta(y, q) \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). And, \(\mu_\theta(x, y, q) \geq \mu_\theta(x, y, q) \wedge \mu_\theta(y, q) \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\).

Therefore, \(\mu_\theta(x, y, q) \geq \mu_\theta(x, q) \wedge \mu_\theta(y, q), \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). Also, \(\mu_\theta(x, y, q) = \mu_\theta(x, y, q) \wedge \mu_\theta(y, q) = \mu_\theta(x, y, q) \wedge \mu_\theta(y, q) \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). Therefore, \(\mu_\theta(x, y, q) \geq \mu_\theta(x, q) \wedge \mu_\theta(y, q) \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). And, \(\mu_\theta(x, y, q) \geq \mu_\theta(x, y, q) \wedge \mu_\theta(y, q) \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\).

Thus (i) is proved. (iii) is clear.

2.6 Theorem: Let A be a \(Q\)-intuitionistic \(L\)-fuzzy subset of a \(Q\)-nearing relation and \(B\) be the strongest \(Q\)-intuitionistic \(L\)-fuzzy relation of \(R\). Then \(A\) is a \(Q\)-intuitionistic \(L\)-fuzzy subnearing of \(R\) if and only if \(A\) is a \(Q\)-intuitionistic \(L\)-fuzzy subnearing of \(R\).

Proof: Suppose that \(A\) is a \(Q\)-intuitionistic \(L\)-fuzzy subnearing of a \(Q\)-nearing relation \(R\). Then for any \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). We have, \(\mu_\theta(x - y, q) = \mu_\theta(x - y, q) \wedge \mu_\theta(x, q) \wedge \mu_\theta(x - y, q) = \mu_\theta(x - y, q) \wedge \mu_\theta(x, q) \wedge \mu_\theta(x - y, q) = \mu_\theta(x - y, q) \wedge \mu_\theta(x, q) \wedge \mu_\theta(x - y, q)
\]

Thus, \(\mu_\theta(x - y, q) \geq \mu_\theta(x, q) \wedge \mu_\theta(y, q), \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). Also, \(\mu_\theta(x, q) \geq \mu_\theta(x, q) \wedge \mu_\theta(y, q) \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). And, \(\mu_\theta(x, y, q) \geq \mu_\theta(x, y, q) \wedge \mu_\theta(y, q) \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\).

Therefore, \(\mu_\theta(x - y, q) \geq \mu_\theta(x, q) \wedge \mu_\theta(y, q), \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). Also, \(\mu_\theta(x, y, q) = \mu_\theta(x, y, q) \wedge \mu_\theta(y, q) = \mu_\theta(x, y, q) \wedge \mu_\theta(y, q) \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). Therefore, \(\mu_\theta(x, y, q) \geq \mu_\theta(x, q) \wedge \mu_\theta(y, q) \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\). And, \(\mu_\theta(x, y, q) \geq \mu_\theta(x, y, q) \wedge \mu_\theta(y, q) \) for all \(x\) and \(y\) in \(R\) and \(q\) in \(Q\).

Thus (i) is proved. (iii) is clear.

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