Bus voltage ranking for unbalanced three-phase distribution networks and voltage stability enhancement

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ABSTRACT
Voltage instability problems have become important issues in unbalanced distribution networks. In this paper, a new bus positive sequence voltage index of $V_{\text{collapse}}/V_{\text{base-load}}$ is introduced to identify the weakest three-phase buses in unbalanced three-phase distribution networks. First, the proposed ranking index is validated based on grid losses and PV curves without and with compensation devices. Then, the index is utilized to place three-phase DG without and with SVC devices at the three-phase weakest buses of the modified IEEE unbalanced 13 node test feeder using the DlgSILENT Power Factory software. Finally, simulation results are presented to show the application of the proposed approach in improving voltage stability and increasing the maximum loading factor under unbalanced loading and/or network conditions.

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Introduction

Voltage instability has become a challenging problem in unbalanced distribution networks. The application of DG systems to the distribution network is increasing to reduce costs and power losses. It seems reasonable to expect that the connection of DG to the utility grid might improve the voltage profile and enhance the voltage stability of a distribution system while reducing active and reactive power losses [1-4]. These improvements will mainly depend on the configuration of the distribution network, type of distributed generation systems and load characteristics.

In balanced power systems, there are several techniques to detect the weakest bus for DG placement such as modal analysis [5, 6], sensitivity analysis [7], $V/V_0$ index [8, 9], bus voltage change index [10], and integrated bus voltage change index with reactive power margin [11]. These methods have the capability to identify which node is the weakest bus of a balanced system. In addition, providing adequate reactive power support at the appropriate location (the weakest bus) has been shown to solve voltage instability problems in some situations [12]. However, the problem becomes very complicated under unbalanced operating conditions.

Many three-phase continuation power flow (CPF) methods have been used to analyze voltage stability margins of unbalanced distribution networks [13-16]. The usual approach is to run power flow and generate PV curves by increasing the active power at selected loads.

Analyses of unbalanced networks indicate that there is at least one phase with clockwise direction (e.g., as the load levels increase on the PV curves, the voltage magnitudes decrease) with much lower voltage levels than the other two phases [16]. This considerably complicates the voltage stability analysis of unbalanced systems. Reference [14] shows that PV curves of phases ‘b’ and ‘c’ at bus 675 in the IEEE 13 node test feeder have anti-clockwise directions while phase ‘a’ has a clockwise direction. In addition, the PV curves for the unbalanced networks and/or unbalanced loads have shown different voltage stability margins on each phase. Therefore, it is very complicated to rank the buses and identify the weakest bus under different voltage level and voltage stability margin conditions on each phase. Recently, a system unbalanced voltage variance index has been proposed [1] for considering voltage profiles and grid losses to find the optimal location of DG. Therefore, bus ranking of unbalanced networks is also an essential task for the voltage stability analysis and enhancement of DG systems. This problem has not been addressed in the literature.

This paper presents a new voltage stability indexing approach to identify the weakest three-phase buses in unbalanced distribution networks. Symmetrical components are applied to the three-phase voltages resulting from three-phase power flow. A new index based on the positive sequence voltage ratio of $V_{\text{collapse}}/V_{\text{base-load}}$ is defined and used for bus ranking. Simulation results and extensive case studies without/with a voltage regulator, DGs and SVCs are presented for the modified IEEE unbalanced 13 node test feeder to show the validity of the proposed approach and its application for improving voltage stability and increasing the maximum loading factor. The maximum loading factor is defined as the ratio of the maximum of the selected loads which can be increased by keeping the power factor constant to the base load. The selected loads are increased until the power flow solution diverges.

The Proposed Bus Ranking Index

For Unbalanced Distribution Networks

In balanced networks, the bus ranking is important for the voltage stability enhancement. The purpose of bus ranking is to determine which node is the weakest bus for connecting DG and/or reactive power compensation devices. It has been shown that DG can be allocated at the first bus reaching the voltage limit to improve voltage profile and reduce grid losses [17]. In addition, the best location for reactive power compensation to improve voltage stability margin is the weakest bus in the network [8].

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Hence, it might be sufficient and reasonable to enhance voltage stability margins in unbalanced distribution networks by connecting DG and/or reactive power compensation devices at the suitable weak buses.

The approach taken in this study is utilizing the bus voltage ranking index to identify the weakest three-phase buses in unbalanced distribution networks. This section starts with the definition and derivation of the conventional voltage ranking index (VRI) \( VRI/V_{ao} \) using the two bus balanced network of Fig. 1 and continues to define a new bus voltage ranking index of \( V_{\text{collapse}}/V_{\text{base-load}} \) and extend its application to unbalanced networks using symmetrical components.

![Fig. 1. Equivalent circuit of a two bus balanced network](image)

The conventional VRI is defined for balanced three-phase networks [8, 9]:

\[
VRI_j^{\text{conventional}} = \frac{V_j}{V_o} = \frac{V_{j,\text{base-load}}}{V_{j,\text{no-load}}} \tag{1}
\]

where \( j \) is the bus number, \( V_{j,\text{base-load}} \) and \( V_{j,\text{no-load}} \) are the bus voltages for the base-load and no-load operating conditions, respectively.

Balanced three-phase load flow can be used to compute \( V_{j,\text{base-load}} \) by setting the complex power at bus \( j \) to zero:

\[
S_j = f(V_j) = P_j - jQ_j = (V_j \angle \delta_j) \left( \frac{V_i \angle \delta_i - V_j \angle \delta_j}{R_i + jX_i} \right) = 0 \tag{2}
\]

where \( V_i \angle \delta_i \) and \( V_j \angle \delta_j \) are the voltages at buses \( i \) and \( j \), respectively. Separating real and imaginary parts of (2):

\[
\begin{align*}
\text{Real}(S_j) &= 0 \\
\text{Imag}(S_j) &= 0 \Rightarrow \\
(P_{\text{real}}(\delta_j), V_j) &= (P_i \angle \delta_i + Q_i \angle \delta_i, V_j \angle \delta_j - (V_j^2 - V_i^2)/(R_i + jX_i)) = 0
\end{align*} \tag{3}
\]

where \( \delta_{ij} = \delta_i - \delta_j \). The voltage \( V_j \) is computed by squaring and adding the real and imaginary parts of (3):

\[
V_j^4 + 2b V_j^2 + (c - c) + (P_j^2 + Q_j^2) \left( R_j^2 + X_j^2 \right) = 0
\]

There are four solutions to (4).

\[
V_j = \pm \frac{1}{2} \left[ -b \pm \sqrt{(b^2 - 4c)} \right] \tag{5}
\]

where \( b = -(V_i^2 - 2P_i R_i - 2Q_i X_i) \) and \( c = (P_j^2 + Q_j^2) (R_j^2 + X_j^2) \). However, \(-b\) is always positive because the term \(-(2P_i R_i - 2Q_i X_i)\) is small as compared to \(V_i^2\) and also \(4c\) is small as compared to \((b^2)\); therefore, the unique positive and stable solution of (5) is

\[
V_j = V_j,\text{based-load} = \frac{1}{2} \left[ -b + \sqrt{(b^2 - 4c)} \right] \tag{6}
\]

Substituting (6) in (1) results in

\[
VRI_j^{\text{conventional}} = \frac{V_j}{V_o} = \sqrt{\frac{(0.5V_j^2 - P_j R_j - Q_j X_j) + A}{V_j}} \tag{7}
\]

where \( A = 0.25(V_j^2 - 2P_j R_j - 2Q_j X_j)(P_j^2 + Q_j^2)(R_j^2 + X_j^2) \).

The propose index in balanced network is defined as:

\[
VRI_j^{\text{balanced}} = \frac{V_{j,\text{collapse}}}{V_{j,\text{base-load}}} \tag{8}
\]

To compute the proposed VRI for balanced three-phase networks, \( V_{j,\text{collapse}} \) is computed based on the Newton-Raphson load flow by forcing (3) to zero. The Jacobian corresponding to (3) is defined as follows:

\[
J = \begin{bmatrix}
-V_j \sin \delta_{ij} & V_j \cos \delta_{ij} - 2V_j \\
V_j \cos \delta_{ij} & V_j \sin \delta_{ij}
\end{bmatrix} \tag{9}
\]

At the collapse point, Jacobian matrix is singular, therefore:

\[
det(J) = 0 \Rightarrow \frac{V_j \cos \delta_{ij}}{V_j} = \frac{1}{2} \Rightarrow V_{j,\text{collapse}} = \frac{0.5V_j}{\cos \delta_{ij}} \tag{10}
\]

Substituting (6) and (10) in (8) results in

\[
VRI_j^{\text{balanced}} = \frac{V_{j,\text{collapse}}}{V_{j,\text{base-load}}} = \frac{0.5V_j}{\cos \delta_{ij}} \tag{11}
\]

Note that

\[
VRI_j^{\text{balanced}} = \frac{\cos \delta_{ij}}{0.5} VRI_j^{\text{conventional}} \tag{12}
\]

Compared to the conventional index (1), the proposed index (12) is sensitive to both voltage magnitude (e.g., \( V/V_o \)) and voltage phase angle (\( \delta_{ij} \)). The angle is computed from (13):

\[
\begin{align*}
\delta_{ij} &= \tan^{-1} \left[ \frac{[P_j X_j - Q_j R_j]}{[P_j R_j + Q_j X_j] + (V_j)^2} \right] \tag{13}
\end{align*}
\]

To extend and generalize the proposed VRI for unbalanced networks, symmetrical components are applied to the three-phase voltages resulting from three-phase power flow. The new index is defined as the ratio of the positive sequence voltage at collapse point to the positive sequence voltage at the base-load:

\[
VRI_j^{\text{unbalanced}} = \frac{V_{j,\text{collapse}}^+}{V_{j,\text{base-load}}^+} \tag{14}
\]

where \( V_{j,\text{collapse}}^+ \) and \( V_{j,\text{base-load}}^+ \) are the positive sequence bus voltages at the point of voltage collapse and the base case load, respectively. This new index can be computed and used to reveal the weakest buses of the unbalanced networks with unbalanced loads and/or configurations. The node with the lowest index value is classified as the weakest bus.

![Fig. 2. PV curve based on positive sequence voltages](image)

**The Modified Unbalanced Three-Phase 13 Node Test System**

The modified unbalanced three-phase 13 node test feeder shown in Fig. 2 has been simulated using DlgSILENT PowerFactory software [18]. The system data is taken from [19] and modified to have balanced and unbalanced operations. This three-phase (un)balanced feeder consists of overhead lines, two
underground lines (through buses 684, 652 and 692, 675), balanced/unbalanced spot loads (Y-PQ, D-PQ, Y-I, D-I, Y-Z, D-Z), distributed loads (Y-PQ) between buses 632 and 671, a single-phase shunt capacitor (at buses 611), a three-phase shunt capacitor (at buses 675) and an in-line transformer (between buses 633 and 634). There is also a three-phase voltage regulator connected between buses 650 and RG60.

Simulations are performed on the modified balanced/unbalanced 13 node test feeder (Fig. 2) for the following case studies:

**Case 1:** without a voltage regulator (fixed transformer tap ratio set to 1.0).

**Case 2:** with a voltage regulator (variable transformer tap ratio).

**Case 3:** Case 2 with a DG (three-phase induction generator) injecting 358 kW active power (e.g., 10% of the total load) installed at the weakest three-phase node (bus 675).

**Case 4:** Case 2 with one DG (358 kW) and one SVC (0.36 MVar, acting as an unbalanced voltage controller) installed at the weakest three-phase node (bus 675).

**Case 5:** similar to Case 4 with the DG and SVC installed at bus 680.

![Figure 2. The modified balanced/unbalanced 13 node test feeder](image)

In the following sections, Eq. 14 will be utilized to locate the weakest three-phase buses for the placement of three-phase DGs with SVC to enhance voltage stability. At each compensation level, the proposed index (Eq. 14) is recalculated and the bus ranking is updated since the system configuration is changed. To show the validity of the proposed bus ranking and the effectiveness of the compensation devices (DG and SVC), grid losses, PV curves (based on positive sequence voltages) and voltage stability margins are calculated and compared for the aforementioned cases.

**Bus Ranking Based on the Proposed Index**

**Bus ranking without a voltage regulator**

Figures 3 and 4 show the bus rankings for Cases 1 and 2 based on Eq. 14 without and with a voltage regulator, respectively. According to these figures, the voltage regulator has no effect on the order of bus ranking.

Note that the four nodes with the lowest VRI are buses 675, 652, 611 and 684. Therefore, the most appropriate location for the installation of three-phase DG and SVC compensators is bus 675.

![Figure 3. Bus ranking for Case 1 (without any voltage regulators)](image)

![Figure 4. Bus ranking for Case 2 (with a voltage regulator)](image)

**Bus ranking with DG at the most suitable bus**

DG devices (e.g., induction generators) are to be connected at the weakest three-phase buses (e.g., weakest buses with the lowest VRI values) to improve the voltage stability. Simulation results of Fig. 5 (Case 3) indicate that the application of one DG (an induction generator) at bus 675 does not change the order of VRI values and therefore has no impact on the order of bus ranking.

![Figure 5. Bus ranking for Case 3 (with one DG at bus 675)](image)

**Validation of the Proposed Bus Ranking Index Based on Grid Losses**

Grid losses associated with the placement of DG units at each node (e.g., all possible locations of DG) are computed and compared with the losses generated with the DG unit connected at the weakest bus as identified by the proposed index of Eq. 14.

**Grid losses with one DG unit**

A three-phase induction generator is placed at different buses of the modified IEEE 13 node feeder (Fig. 2) and system active and reactive losses are plotted in Fig. 7. This figure confirms that bus 675 (resulting in the lowest grid losses) is the most suitable bus for DG placement, as was previously identified by Eq. 14.

![Figure 6. Bus ranking for Case 4 (with one DG and one SVC at bus 675)](image)
Grid losses with two DG units

According to Eq.14, with the addition of one DG (at bus 675, Fig. 5), the most suitable location for the connection of a second DG unit is still at bus 675. This is in agreement with the grid loss plots of Fig. 8 generated by connecting the first DG at bus 675 and placing a second DG at different buses of the modified IEEE 13 node feeder. These results further confirm the accuracy of the proposed bus ranking index.

Fig. 8. Reactive and active power losses associated with the first DG installed at bus 675 and the second DG connected at different buses of Fig. 2 (Case 3).

Grid losses with DG and SVC Devices

Figure 9 shows grid losses with a combination of DG and SVC units placed at bus 675 (Case 4) and adding a second DG unit at different buses. These results indicate the lowest reactive loss occurs at bus 671 and active power loss occurs at bus 675. Therefore, the calculation of grid losses is not the same as the result of VRI values (Fig. 6) with multiple DG and SVC compensation.

Fig. 9. Reactive and active power losses with DG and SVC installed at bus 675 and a second DG connected at different buses of Fig. 2 (Case 4)

Validation of the Proposed Bus Ranking Index Based on PV Curves

Fig. 10. PV curves of positive sequence voltage at each bus for Case 2

Improving Maximum Loading Factor with the Proposed Bus Ranking Index

Application of the proposed bus ranking index for the placement of DGs without/SVCs at the weakest three-phase will improve the maximum loading factors as demonstrated in Table 1. According to the tabulated results:

- A comparison of the maximum loading factors for Cases 1 and 2 indicates that the voltage stability margin is higher with a voltage regulator. Therefore, voltage regulators can help to improve the voltage stability margins of unbalanced distribution systems.
- After connecting DG at bus 675 (Case 3), the voltage stability margin has slightly decreased from 2.375 to 2.343.
- There is a significant improvement in the maximum loading factor when a combination of DG and SVC units is placed at the weakest three-phase bus. For example, after connecting DG and SVC (358 kW and 0.36 MVar) at buses 680 (Case 5) and 675 (Case 4) the maximum loading factor is improved (from 2.375 with no compensation) to 4.390 and 4.967, respectively.

Enhancement of Maximum Loading Factor by Proper Sizing of DG Units

The maximum loading factors of Table I are computed for DG compensation values of 358 kW. These factors can be improved by proper sizing of the compensation devices as shown in Fig. 12.

Fig. 12 shows the impact of increasing the number of DG units on the maximum loading factor. Each DG unit injects 358 kW of active power. According to this figure, the maximum loading factor can be improved from 4.967 (Case 4) to 5.223 if the level of DG compensation at the weakest three-phase node (bus 675) is increased from 358 kW to 5.012 MW.
A relatively simple procedure is used to properly place and size the compensation devices to further improve the maximum loading factor of the unbalanced distribution system. The approach is to place one compensation unit (e.g., a 358 kW DG with SVC) at the weakest bus and compute the corresponding maximum loading factor. The procedure is then repeated by relocating the weakest bus (based on Eq. 14 with all previous units in service) and placing more compensation devices.

With the above-mentioned approach for placement of DG (with a 0.36 MVar SVC used for voltage regulation) are shown in Fig. 13. The selected size of the unit DG is 358 kW. Based on Fig. 13, the maximum loading factor can be further improved to 6.119 with a total DG of 716 kW (consisting of 358 kW and 358 kW units at buses 675 and 634, respectively).

**Fig. 13. Simulation results for placement and sizing of DG units in the unbalanced IEEE 13 node test feeder (Fig. 2)**

**Conclusion**

This paper proposed a ranking index to identify the weakest three-phase buses of unbalanced distribution networks. The method of symmetrical components is applied to the problem of bus ranking in unbalanced networks. The validity of the new index is demonstrated for the modified IEEE unbalanced 13 node test feeder based on grid losses and PV curves. The proposed index is used to improve the maximum loading factor by placing DGs (without and with SVCs) at the weakest three-phase buses. Main conclusions regarding the stability of unbalanced distribution networks are as follows:

- The proposed ranking index can accurately identify the weakest buses under different operating conditions without/voltage regulators and DGs (without/SVCs).
- Voltage regulators have positive impacts on voltage stability margins under unbalanced conditions.
- After connecting induction generator at bus 675 (Case 3), the voltage stability margin has slightly decreased from 2.375 to 2.343. However, a combination of DG and SVC devices at the weakest three-phase bus will considerably increase the maximum loading factor and significantly improve the voltage stability.
- The order of bus ranking cannot be changed without reactive power compensation devices. The order of bus ranking is changed when SVC with voltage controller is installed at the weakest bus (bus 675) and at bus 680.
- Both the new VRI and PV curves based on positive sequence voltage can be properly utilized to identify the weakest bus under unbalanced conditions. However, the calculation of grid losses can only be used to rank the bus with only DGs placement.
- Proper sizing of one compensation device (DG with SVC) at the weakest bus will improve the maximum loading factor.
- A relatively simple procedure is implemented to further improve the maximum loading factor by proper placement and sizing of multiple compensation devices.

**References**


### TABLE I

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Description</th>
<th>Order of bus ranking (Eq. 1)</th>
<th>Maximum loading factor*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No regulation</td>
<td>675, 652, 611, 684, 680</td>
<td>2.199</td>
</tr>
<tr>
<td>2</td>
<td>With regulation</td>
<td>675, 652, 611, 684, 680</td>
<td>2.375</td>
</tr>
<tr>
<td>3</td>
<td>DG at bus 675</td>
<td>675, 652, 611, 684, 680</td>
<td>2.343</td>
</tr>
<tr>
<td>4</td>
<td>Combination of DG and SVC at bus 675</td>
<td>634, 633, 646, 645, 632</td>
<td>4.967</td>
</tr>
<tr>
<td>5</td>
<td>Combination of DG and SVC at bus 680</td>
<td>634, 633, 646, 645, 675</td>
<td>4.390</td>
</tr>
</tbody>
</table>

*) Computed by increasing the active power of all loads until the power flow solution diverges.

### Biographies

**Parachai Juanuwattanakul** (S’10) received his B.Eng (2nd Class Hons.) and M.Eng degrees in Electrical Engineering from Mahanakorn University of Technology, Bangkok, Thailand and Kasetsart University, Bangkok, Thailand in 1994 and 1998, respectively. He is presently working towards his PhD degree at the Electrical and Computer Engineering Department, Curtin University of Technology, Perth, Australia. His research interests include voltage stability, distributed generation and FACTS devices.

**Mohammad A.S. Masoum** (S’88–M’91–SM’05) received his B.S., M.S. and Ph.D. degrees in Electrical and Computer Engineering in 1983, 1985, and 1991, respectively, from the University of Colorado, Boulder, USA. His research interests include optimization, power quality and stability of power systems/electric machines and distributed generation. Dr. Masoum is the co-author of “Power Quality in Power Systems and Electrical Machines” (Elsevier, 2008) and “Power Conversion of Renewable Energy Systems” (Springer, 2011). Currently, he is an Associate Professor and the discipline leader for electrical power engineering at the Electrical and Computer Engineering Department, Curtin University, Perth, Australia and a senior member of IEEE.