Effect of chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface

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ABSTRACT
Effect of chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface is studied here. The governing equations involved in the present analysis are solved by the two-term perturbation method. The velocity, temperature, concentration, skin friction and Nusselt number are studied for different parameters like thermal Grashof number, mass Grashof number, Schmidt number, magnetic field parameter, Permeability parameter, Prandtl number, Eckert number and chemical reaction parameter.

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\[
\begin{align*}
\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} \left(M + \frac{1}{K}\right) u &= -G \theta - G_n C, \quad (8) \\
\frac{\partial^2 \theta}{\partial y^2} + P_r \frac{\partial \theta}{\partial y} &= -P_r E \left(\frac{\partial u}{\partial y}\right)^2, \quad (9) \\
\frac{\partial^2 C}{\partial y^2} + S_e \frac{\partial C}{\partial y} - S_e k_e C &= 0. \quad (10)
\end{align*}
\]

Also, the boundary condition (6) reduces to:
\[
\begin{align*}
u = 0, \quad \theta = 1, \quad C = 1 \text{ at } y = 0; \\
u \to 0, \quad \theta \to 0, \quad C \to 0 \text{ as } y \to \infty.
\end{align*}
\]

The dimensionless governing equations (8), (9) and (10), subject to the boundary conditions (11), are solved by the perturbation method. Expanding \( u, \theta \) and \( C \) in the power of the Eckert number \( E \) (assuming that \( E \) is very small). We can write:
\[
\begin{align*}
u &= \nu_0 + \nu \mathcal{E}^2, \\
\theta &= \theta_0 + \eta \mathcal{E}^2, \\
C &= C_0 + C \mathcal{E}^2
\end{align*}
\]

Substituting the equation (12) into equations (8)-(10), equating the coefficients at the terms with the same power of \( \mathcal{E}^2 \) and higher orders, we get the following equations:

**Zero order;**
\[
\begin{align*}
\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} - m u_0 &= -G_0 \theta_0 - G_n C_0, \quad (13) \\
\frac{\partial^2 \theta_0}{\partial y^2} + P_r \frac{\partial \theta_0}{\partial y} &= 0, \quad (14) \\
\frac{\partial^2 C_0}{\partial y^2} + S_e \frac{\partial C_0}{\partial y} - S_e k_e C_0 &= 0. \quad (15)
\end{align*}
\]

**First order;**
\[
\begin{align*}
\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial u_1}{\partial y} - m u_1 &= -G_1 \theta_0 - G_n C_1, \quad (16) \\
\frac{\partial^2 \theta_1}{\partial y^2} + P_r \frac{\partial \theta_1}{\partial y} &= -P_r \left(\frac{\partial u_0}{\partial y}\right)^2, \quad (17) \\
\frac{\partial^2 C_1}{\partial y^2} + S_e \frac{\partial C_1}{\partial y} - S_e k_e C_1 &= 0. \quad (18)
\end{align*}
\]

The corresponding boundary conditions are as follows:
\[
\begin{align*}
u_0 &= 0, \nu_1 = 0, \quad \theta_0 = 1, \theta_1 = 0, \quad C_0 = 1, C_1 = 0 \text{ at } y = 0; \\
u_0 \to 0, \nu_1 \to 0, \quad \theta_0 \to 0, \theta_1 \to 0, \quad C_0 \to 0, C_1 \to 0 \text{ as } y \to \infty.
\end{align*}
\]

Solving equations (13)-(18) under the boundary conditions (19) and then using (12), we get the solution, which is as under:
\[
\begin{align*}
\theta &= (1 + EA_1) e^{-\lambda y} - G \theta - G_n C, \\
\theta &= \frac{1}{2} \left(\theta_0 - \theta_1 \right) e^{-\lambda y} + G \theta - G_n C, \\
C &= \frac{1}{2} \left(C_0 - C_1 \right) e^{-\lambda y} - S_e k_e C
\end{align*}
\]

Skin friction:

The non-dimensional skin friction is given by:
\[ \tau = \left( \frac{\partial u}{\partial y} \right)_{y=0} = \mu G_r + P_r G_2 - \lambda \left( G + E A_3 \right) \]

\[ + E G_i \left[ A_6 G_r - 2 A_8 P_r - 2 A_3 + \frac{A_4}{\lambda + P_r} \right] \]

\[ - E G_i \left[ 2 \mu A_4 + \frac{A_8}{\lambda + P_r} \right] \]

\[ + \frac{2 \mu P_r G_2 E G_i}{(\mu + \lambda - P_r) \left( (\mu + \lambda)^2 - (\mu + \lambda) - m \right)} \].

Nusselt number:
The non-dimensional Nusselt number is given by:

\[ Nu = - \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = P_r \left( 1 + E A_1 \right) \]

\[ - P E \left( \lambda A_4 + \mu A_9 + P_r G_2^2 + 2 P_r G_1 G_2 \right) \]

\[ + 2 E P_r G_2 \left[ P_r + \frac{2 \lambda \mu}{\mu + \lambda - P_r} \right] \].

Results and Discussion:
The velocity profiles for different parameters \( M, G_r, G_m, K, k_0, P_r, S_c \) and \( E \) are shown by figures-1 to 7.

From figure-1, it is clear that the velocity increases when Eckert number \( E \) is increased (keeping other parameters \( M = 2, G_r = 5, G_m = 5, K = 1, k_0 = 1, P_r = 0.71, S_c = 2.01 \) constant). Velocity profile for different values of Eckert number \( E \) is shown in figure-2. It shows that the velocity increases with increasing mass Grashof number \( G_m \), thermal Grashof number \( G_r \) and Prandtl number \( P_r \) (keeping other parameters constant). Similar pattern is observed in figure-1 and figure-3, i.e. the velocity increases when chemical parameter \( k_0 \) is increased.

Figure-1: Velocity profile for different Eckert number \( E \)

Figure-2: Velocity profile for different Eckert number \( E \)

Figure-3: Velocity profile for different chemical parameter \( k_0 \)

Figure-4: Velocity profile for different magnetic field parameter \( M \)

Figure-5: Velocity profile for different thermal Grashof number \( G_r \)
But in figure-4 the velocity decreases when magnetic field parameter $M$ is increased.

In figure-5 it is observed that velocity increases when thermal Grashof number $r_G$ is increased. Similarly in figure-6 and figure-7, the velocity increases when the value of permeability parameter $K$ and mass Grashof number $m_G$ is increased respectively.

Temperature profile is shown in figures-8 to 12. From figure-8, it is clear that temperature decreases when Eckert number $E$ is increased (keeping other parameters $M = 2, G_r = 5, G_m = 5, K = 1, k_0 = 1, P_r = 0.71, S_r = 2.01$ constant). In figure-9, it is shown that temperature increases when magnetic field parameter $M$ is increased.

Figure-10 shows that temperature decreases with increasing the permeability parameter $K$. Similar pattern is observed in figure-11, the temperature decreases with increasing the thermal Grashof number $G_r$. 

Figure-6: Velocity profile for different permeability parameter $K$

Figure-7: Velocity profile for different mass Grashof number $G_m$

Figure-8: Temperature profile for different Eckert number $E$

Figure-9: Temperature profile for different magnetic field $M$

Figure-10: Temperature profile for different permeability parameter $K$

Figure-11: Temperature profile for different thermal Grashof number $G_r$
In this paper a theoretical analysis has been done to study the effect of chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface. Solutions for the model have been derived by using two-term perturbation method. Some conclusions of the study are as below:

• Velocity increases with the increase in \( E, k_0, G_r, K \) and \( G_m \) and decreases with increase in \( M \) .

• Temperatures of the fluid increase when \( M \) and \( k_0 \) are increased.

• Concentration of the fluid decreases when \( k_0 \) is increased.

• Skin friction increases when magnetic field parameter, Eckert number, thermal Grashof number and mass Grashof number are increased but decreases when chemical parameter, Prandtl number, Schmidt number and permeability parameter are increased.

• Nusselt number increases when magnetic field parameter, mass Grashof number and Prandtl number are increased but decreases when thermal Grashof number, Eckert number, chemical parameter, permeability parameter and Schmidt number are increased.

**Notation**

\( C \) - non-dimensional fluid concentration; \( C' \) - concentration, mol/m\(^3\); \( C_w \) - fluid concentration far away from the wall, mol/m\(^3\); \( C_p \) - specific heat at a constant pressure, J/(kg.deg); \( D \) - mass diffusivity, m\(^2\)/sec; \( E \) - Eckert number; \( G_m \) - mass Grashof number; \( G_r \) - thermal Grashof number; \( g \) - gravitational acceleration, m/sec\(^2\); \( K \) - non-dimensional permeability coefficient of a porous medium; \( k_0 \) - non-dimensional rate of a chemical reaction; \( K_r \) - rate of chemical reaction, sec\(^{-1}\); \( K_p \) - permeability of a porous medium, m\(^2\); \( M \) - magnetic field parameter; \( Nu \) - Nusselt number; \( P_r \) - Prandtl number; \( S_c \) - Schmidt number; \( T_w \) - fluid temperature far away from the wall, \(^0\)C; \( T' \) - temperature, \(^0\)C; \( u' \), \( v' \) velocity components, m/sec; \( u \) - non-dimensional velocity; \( v_0 \) - suction velocity, m/sec; \( x' \), \( y' \) - space coordinates, m; \( y \) - non-dimensional space coordinate; \( \alpha \) - thermal conductivity, W/(m.deg); \( \beta \) - coefficient of volume expansion, 1/deg; \( \beta' \) - coefficient of volume expansion with concentration, m\(^3\)/mol; \( \theta \) - non-dimensional temperature; \( \nu \) - kinematic viscosity, m\(^2\)/sec; \( \rho \) - fluid density, kg/m\(^3\); \( \tau \) - non-dimensional skin friction.

**Subscripts and Superscripts:**

\( W \) - wall; \( \infty \) - far away from the wall; 0 and 1 - zero and first orders.

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**References:**


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**Table 1: Skin friction and Nusselt number for different parameters**

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