Configurational modeling and analysis of multicomponent parallel system with imperfect failure detection, repair/replacement and common cause failure

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ABSTRACT

This paper investigates a two multi-component unit parallel system model with imperfect detection and common cause failure. A single repair facility is always available with the system but whenever a regular detector fails in detection of the failure cause, then the unit goes for replacement for which a single replacement facility is always available. Using regenerative point technique various measure of system effectiveness are obtained. The behaviour of MTSF and profit function have been studied in a particular case.

Keywords
System states,
Sojourn times,
Reliability analysis.

Introduction

Repair maintenance, inspection and replacement are the some important means for increasing the availability of the system. Under these tools, a number of authors including (2-5) have carried out the stochastic analysis of various redundant systems. Most of these studies are concentrated at the stochastic analysis of two unit redundant systems. A very few authors including (1) have analysed the multi-component redundant systems by using supplementary variable technique.

In the literature of reliability commonly, perfect pre repair inspection is considered. But there are some situations where pre repair inspection may be imperfect. In case of imperfect inspection the failed unit should be replaced.

Under the above facts in view, the purpose of present study is to analyse a two multicomponent unit parallel system with imperfect failure detection, repair/replacement and common cause failure.

2. Model Description and Assumptions

In the present model we assume:
(i) The system consists of two identical units arranged in parallel configuration. Each unit consists of ‘c’, (c ≥ 1) repairable independent components arranged in series network.
(ii) Each unit of the system has two modes normal (N) and total failure (F).
(iii) Upon failure of a unit it goes for the detection to determine which component failure is the cause of unit failure. For this purpose a detector is always available with the system. But the detector is not always successful in detection.
(iv) If detection is successful, then the failed component goes for repair otherwise the failed unit goes for replacement.
(v) A single repair facility as well as a single replacement facility is always available with the system.
(vi) During the repair the detector is busy in monitoring of the repair process.
(vii) Unit/component fails either due to its normal failure or due to common cause failure. Common cause failure is defined as any instance where multiple units or components fail due to a single cause. A common cause failure may occur due to voltage fluctuation, temperature, fire, operational and maintenance error, etc.
(viii) The repaired discipline is FCFS and the repaired unit is as good as new.
(ix) Detection time and failure time distributions are taken as negative exponential where as repair time distributions are taken as general.

Using regenerative point technique, the following economic measures of interest to system designers and managers have been obtained.
(1) Reliability of the system and mean time to system failure (MTSF)
(2) Expected up time of the system during (O, t) and in steady state.
(3) Expected busy period of the repairman during (O, t) and in steady state.
(4) The cost benefit analysis of the system is carried out by using the above characteristics.

3. Notations and symbols for the system states

\( E \) : set of regenerative states, i.e., \( E = \{ S_{0} - S_{9} \} \)
\( \alpha \) : failure rate of the operative unit
\( \gamma \) : detection rate of the failed unit
\( \eta \) : rate of replacement of the failed component
\( p/q \) : probability of success/failure of detection
\( \beta \) : common cause failure rate
\( G_{c}(\bullet) \) : c.d.f. of repair time of the failed cth component
\( H(\bullet) \) : c.d.f. of repair time due to common cause
\( *, \sim \) : symbols for Laplace and Laplace Stieltjes transform i.e.,
\( \tilde{Q}_{ij}(s) = \int e^{-st} Q_{ij}(t), \)
\( q_{ij}(s) = \int e^{-st} q_{ij}(t)dt \)

\( \odot \) : symbol for ordinary convolution i.e.,
\( A(t) B(t) = \int_{0}^{t} B(t-u) A(u) \, du \)
Symbols for the states of the system

- **No.** : unit is operative
- **F/F<sub>WD</sub>** : unit in failure mode and under detection/waiting for detection
- **F<sub>cr</sub>** : unit in failure mode and under repair due to the failure of c<sub>th</sub> component.
- **F/R/F<sub>WR</sub>** : unit in failure mode and under replacement/waiting for replacement

Using the above notations and symbols the possible states and the possible transitions among the states are shown in fig. 1.

4. Transition probabilities and sojourn times

The non-zero elements \( p_{ij} \) of the transition probability matrix (tpm) for the considered system model are as follows:

\[
\begin{align*}
\rho_{01} &= \frac{2\alpha}{\beta + 2\alpha}, \\
\rho_{09} &= \frac{\beta}{\beta + 2\alpha}, \\
\rho_{12c} &= \rho_c \frac{\gamma}{(\alpha + \beta + \gamma)}, \\
\rho_{13} &= \frac{\alpha}{(\alpha + \beta + \gamma)}, \\
\rho_{16} &= \frac{\alpha}{(\alpha + \beta + \gamma)}, \\
\rho_{19} &= \frac{\beta}{(\alpha + \beta + \gamma)}, \\
\rho_{2c,0} &= \tilde{G}_c(\alpha + \beta), \\
\rho_{2c,1}^{(4c)} &= \frac{\alpha}{(\alpha + \beta)}[1 - \tilde{G}(\alpha + \beta)], \\
\rho_{2c,9} &= \frac{\beta}{(\alpha + \beta)}[1 - \tilde{G}_c(\alpha + \beta)],
\end{align*}
\]

\[\Delta\text{The limit of integration when } O \text{ to } \infty \text{ are not mentioned.}\]

\[
\begin{align*}
\rho_{13} &= \frac{\alpha}{(\alpha + \beta + \gamma)}, \\
\rho_{16} &= \frac{\alpha}{(\alpha + \beta + \gamma)}, \\
\rho_{19} &= \frac{\beta}{(\alpha + \beta + \gamma)}, \\
\rho_{2c,0} &= \tilde{G}_c(\alpha + \beta), \\
\rho_{2c,1}^{(4c)} &= \frac{\alpha}{(\alpha + \beta)}[1 - \tilde{G}(\alpha + \beta)], \\
\rho_{2c,9} &= \frac{\beta}{(\alpha + \beta)}[1 - \tilde{G}_c(\alpha + \beta)].
\end{align*}
\]

Using the formula \( \psi_i = \int P(T_i > t)dt \) for the mean sojourn time in state \( S_i \in E \), its values for various states are:

\[
\begin{align*}
\psi_0 &= \frac{1}{(2\alpha + \beta)}, \\
\psi_1 &= \frac{1}{(\alpha + \beta + \gamma)}, \\
\psi_{2c} &= \frac{1}{(\alpha + \beta + \gamma)}, \\
\psi_3 &= \frac{1}{(\alpha + \beta + \eta)}, \\
\psi_{4c} &= 1 = \psi_9, \\
\psi_5 &= \frac{1}{(\eta + \gamma)}, \\
\psi_6 &= \frac{1}{\gamma}, \\
\psi_{7c} &= \frac{1}{\eta}, \\
\psi_8 &= \frac{1}{\eta}.
\end{align*}
\]

5. Reliability analysis

The reliability of the system when it starts operation from \( S_i \in E \) is given by

\[
R_i(t) = P[T_i > t]
\]

By probabilistic arguments, we have the following recursion relations:

\[
\begin{align*}
R_0(t) &= Z_0(t) + q_{01} \odot R_1(t) \\
R_1(t) &= Z_1(t) + \sum_q Z_{1,2c} R_2(t) + q_{13}(t) \odot R_3(t) \\
R_2c(t) &= Z_{2c}(t) + q_{2c,0}(t) \odot R_0(t) \\
R_3(t) &= Z_3(t) + q_{30} \odot R_0(t)
\end{align*}
\]

where,

\[
\begin{align*}
Z_0(t) &= e^{-(2\alpha + \beta)t}, \\
Z_1(t) &= e^{-(\alpha + \beta + \gamma)t}, \\
Z_{2c}(t) &= \tilde{G}(t)e^{-(\alpha + \beta)t}, \\
Z_3(t) &= e^{-(\alpha + \beta + \gamma)t}.
\end{align*}
\]

Taking Laplace transform of relations (1-4) and simplifying for \( R_0^*(s) \), we obtain,

\[
R_0^*(s) = \frac{Z_0 + q_{01}Z_1 + \sum_q Z_{1,2c} + q_{01}q_{30}Z_3}{1 - q_{01}q_{2c,0} - q_{01}q_{13}q_{30}}
\]

\[\text{The } \Sigma \text{ is extended from } o \text{ to } c, \text{ whenever used.}\]
For brevity, the argument ‘s’ is omitted from \( q_i^* (s) \) and 
\( Z_i^* (s) \).

By taking inverse Laplace transform of (5), we can obtain the expression for \( R(t) \). Using the usual formula, the MTSF is given by
\[
E(T_e) = \lim_{s \to 0} R_e^*(s) = \frac{\psi_0 + \tP_1\psi_1 + \tP_2\psi_2 + \tP_3\psi_3}{1 - \tP_1\tP_2\tP_3} = \frac{1}{1 - \tP_1\tP_2\tP_3}
\]
(6)

6. Availability analysis

From the theory of regenerative process, the pointwise availabilities \( A_i(t) \) (i = 0) of the system are seen to satisfy the following recursion relations
\[
A_0(t) = Z_0(t) \cup q_0(t) \cup A_0(t) \cup q_0(t) \cup A_0(t) \bigcup q_4c \cup A_0(t)
\]
\[
A_0(t) = Z_0(t) \cup q_0(t) \cup A_0(t) \cup q_3(t) \cup A_3(t) \cup q_0(t) \cup A_0(t) \bigcup q_4c \cup A_0(t)
\]
\[
A_0(t) = q_5(s) \cup A_0(t) \cup A_5(s) \cup A_0(t) \bigcup q_4c \cup A_0(t)
\]
\[
A_0(t) = A_5(s) \cup A_0(t) \bigcup q_4c \cup A_0(t)
\]
\[
A_0(t) = q_5(s) \cup A_0(t)
\]
(7-16)

where,
\[
N_2(s) = \left[ 1 - \sum_{2c,1}^{(4c)} \left( 1 - q_0(s) \sum_{5,7c}^{(4c)} \sum_{3}^{(4c)} - q_3(s) q_3(s) \right) \right]
\]
\[
- \left[ 1 - q_0(s) \sum_{5,7c}^{(4c)} \sum_{3}^{(4c)} - q_3(s) q_3(s) \right]
\]
\[
N_2(s) = \left[ 1 - \sum_{2c,1}^{(4c)} \left( 1 - q_0(s) \sum_{5,7c}^{(4c)} \sum_{3}^{(4c)} - q_3(s) q_3(s) \right) \right]
\]
\[
N_2(s) = \left[ 1 - \sum_{2c,1}^{(4c)} \left( 1 - q_0(s) \sum_{5,7c}^{(4c)} \sum_{3}^{(4c)} - q_3(s) q_3(s) \right) \right]
\]
(18)

\[
D_2(s) = \sum_{5,7c}^{(4c)} \sum_{3}^{(4c)} \left( 1 - q_0(s) \sum_{5,7c}^{(4c)} \sum_{3}^{(4c)} - q_3(s) q_3(s) \right)
\]
\[
D_2(s) = \sum_{5,7c}^{(4c)} \sum_{3}^{(4c)} \left( 1 - q_0(s) \sum_{5,7c}^{(4c)} \sum_{3}^{(4c)} - q_3(s) q_3(s) \right)
\]
\[
D_2(s) = \sum_{5,7c}^{(4c)} \sum_{3}^{(4c)} \left( 1 - q_0(s) \sum_{5,7c}^{(4c)} \sum_{3}^{(4c)} - q_3(s) q_3(s) \right)
\]
(19)

For brevity, the argument ‘s’ is omitted from \( q_i^* (s) \) and 
\( Z_i^* (s) \). Now, the steady state availability is given by
\[
A_0 = N_2 / D_2
\]
(20)

where,
8. Profit function analysis

The net expected profit incurred during \((t, o)\) is given by

\[
P(t) = \text{Expected total revenue during } (t, o) - \text{Expected total expenditure during } (t, o)
\]

where \(K_j\) be the revenue per unit up time by the system and \(K_j/K_i\) be the amount spent per unit of time in repair of the components failed due to common cause/cth component respectively be the amount spent per unit of time in replacement/detection or inspection of failed components.

Also, \(\mu_{up}(t) = \int_0^t A_0(u)du\) for different values of \(\lambda_1, 0.01, 0.02, 0.03\) when other parameters are kept fixed as \(C_0 = 5000, C_1 = 600, C_2 = 350, C_3 = 400\) and \(C_4 = 250\) while other parameters take some values as in graphical study of MTSF except the values of \(C_0\) and \(C_1\).

9. Particular case

When the repair time distributions for common cause failure and failed cth component are taken as exponential with parameters \(\beta_j\) and \(\beta_i\) respectively, the changes are as follows:

\[
P_{2,0} = \frac{\lambda_2}{\lambda_2 + \alpha \theta_2}, \quad P_{2,1} = \frac{\lambda_3}{\lambda_2 + \alpha \theta_2}, \quad P_{2,9} = \frac{\beta}{\lambda_2 + \alpha \theta_2}
\]

\[
P_{7c,2c} = \frac{\eta}{\lambda_2 + \eta}, \quad P_{7c,3} = \frac{\lambda_2}{\lambda_2 + \eta}
\]

10. Graphical analysis

For study of the system behaviour graphically, we plot curves for two important measures of system effectiveness of MTSF and profit function w.r.t. failure rate of the operative unit (\(\alpha\)).

Figure 2 shows the variation in MTSF w.r.t. \(\alpha\) for different values of \(\lambda_1\) when other parameters are kept fixed as \(\gamma = 0.01, \eta = 0.02, \beta = 0.001, \lambda_i = 0.025\) and \(p = 0.01\). From graph, it is obvious that the MTSF rapidly decreases, initially and uniformity decreases for large values of \(\alpha\). It is further observed that the values of MTSF increase as the value of repair rate \(\lambda_1\) increases.

Fig. 3 represents the change in profit function w.r.t. \(\alpha\) for different values of \(\gamma\) and \(\lambda_1\) when other parameters are kept fixed as \(C_0 = 5000, C_1 = 600, C_2 = 350, C_3 = 400\) and \(C_4 = 250\) while other parameters take some values as in graphical study of MTSF except the values of \(\gamma\) and \(\lambda_1\). It is clear from the graph that the profit function decreases as the value of failure rate \(\alpha\) increases while the value of profit function increases as the value of \(\gamma\) and \(\lambda_1\) increase.

References