A summation formula allied with hypergeometric function
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ABSTRACT
The main objective of the present paper is to establish a summation formula based on half argument in connection with Hypergeometric function.

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Keywords
Contiguous relation, Recurrence Relation, Gauss second summation theorem.

Introduction
Generalized Gaussian Hypergeometric function of one variable
\[ {}_nF_{n-1}(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_{n-1}; z) = \sum_{k=0}^{\infty} \frac{(a_1)_{k} \cdots (a_n)_{k}}{(b_1)_{k+1} \cdots (b_{n-1})_{k} k!} z^k \]
(1)
or
\[ {}_nF_{n-1}(a_1; b_1; z) = {}_1F_0((a_1); (b_1); z) = \sum_{k=0}^{\infty} \frac{(a_1)_{k}}{(b_1)_{k+1} k!} z^k \]
(2)
where the parameters \( b_1 \), \( b_2 \), \ldots, \( b_{n-1} \) are neither zero nor negative integers and \( A \), \( B \) are non-negative integers.

Contiguous Relations
\[ {}_nF_{n-1}((a+1)_k; (a,b; c); z) = {}_nF_{n-1}((a, b+1; c); z) \]
(3)
Gauss second summation theorem is defined as
\[ {}_2F_1\left( \frac{a+b+1}{2}; \frac{a+b+2}{2}; z \right) = \frac{2z^{\frac{1}{2}}}{\Gamma(\frac{a+b+1}{2}) \Gamma(\frac{a+b+2}{2})} {}_2F_1\left( \frac{a+b}{2}; \frac{a+b+1}{2}; z \right) \]
(4)
Recurrence relation
\[ \Gamma(\zeta+1) = \zeta \Gamma(\zeta) \]
(5)
Main summation formula:
\[ {}_2F_1\left( \frac{a+b+1}{2}; \frac{a+b+2}{2}; z \right) = \frac{2z^{\frac{1}{2}}}{\Gamma(\frac{a+b+1}{2}) \Gamma(\frac{a+b+2}{2})} {}_2F_1\left( \frac{a+b}{2}; \frac{a+b+1}{2}; z \right) \]
Derivation of main formula:

Replacing \( c = \frac{a+b+c}{2} \) and \( z = \frac{1}{2} \) in equation (3), we get

\[
\binom{a-b}{2} \binom{a+1}{b} = a \binom{a+1}{b} - b \binom{a}{b+1}
\]

Now preceding the same parallel method of Ref [4], the main formula is derived.

References