Tuning of PID controllers for isothermal continuous stirred tank reactor  
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Second Order Plus Time Delay system with a Zero (SOPTDZ).

ABSTRACT  
Tuning proportional integral and derivative (PID) controllers for isothermal CSTR system is proposed. The PID controller is designed based on internal model control and stability analysis principles. The proposed controllers are applied to stable transfer function models of isothermal CSTR to avoid Van de Vusse reaction. Simulation results on non-linear model equations of isothermal CSTR carrying out Van de Vusse reaction is given to show the effectiveness of the proposed PID controllers. The performance under model uncertainty is also studied considering perturbation in one parameter at a time. The performance of proposed controllers is compared with the direct synthesis method (Chien et al., 2003).

In the present work, a simple method for designing PID controllers based on internal model control principles and stability analysis is proposed. The PID tuning parameters are given as function of process model parameters. Simulation results on isothermal CSTR and non-linear models are given to show the efficiency of the proposed controllers.  

Proposed methods  
Imc method  
The process transfer function for SOPTD with a positive zero is given by

\[ G_p(s) = \frac{K_p (1 - ps)e^{-\lambda s}}{a_1s^2 + a_2s + 1} \]  

(1)

Now, let \( G_p(s) = G_p^+(s)G_p^-(s) \)

(2)

Where \( G_p^+(s) \) is invertible part and \( G_p^-(s) \) is non-invertible part. Therefore, the invertible part of the process transfer function in Eq (14) is

\[ G_p^+(s) = \frac{K_p}{(a_1s^2 + a_2s + 1)} \]  

(3)

The IMC controller for this system is given as,

\[ Q = [G_p^+(s)]^{-1} \cdot f \]  

(4)

Where \( f \) is IMC filter and is given by \( f = \frac{1}{(1 + \lambda s)^n} \). The order (n) of denominator is selected in such a way to make controller realizable (the controller numerator order should be less than or equal to that of the order of denominator). Therefore IMC filter \( f \) is given by

\[ f = \frac{1}{(\lambda s + 1)^2} \]  

(5)
Using Eq (16) and Eq (18) in Eq (17), IMC controller is given by
\[ Q = \frac{(a_1s^3 + a_2s + 1)}{K_p(\lambda s + 1)^2} \] (6)

Where \( \lambda \) is IMC filter time constant which is a tuning parameter and \( \lambda \) is selected by trial and error procedure in such a way that the maximum magnitude of complementary sensitivity function is between 1 to 1.5 [16]. The equivalent PID controller is obtained by using Eq (7).
\[ G_c(s) = \frac{Q}{(1 - G_p(s)Q)} \] (7)

Substituting Eq (1) and Eq (6) in the Eq (7) and using Pade’s approximation for time delay, \( e^{-\lambda s} = \frac{1 - 0.5Ls}{1 + 0.5Ls} \)
\[ G_c(s) = \frac{(1+0.5L\delta)(a_1s^2+a_2s+1)}{K_p[(\lambda s+1)(1+0.5L\delta)-(1-0.5L\delta)]} \]

Eq (8) can be rearranged into the following form,
\[ G_c(s) = K_c[1 + \frac{1}{\tau_i s} + \tau_d s] \frac{(1+\alpha \delta)}{(\alpha \delta^2 + \alpha \delta s + 1)} \] (9)

This is a PID controller with a lead lag filter
Where, \( K_c = \frac{a_2}{K_p(2\lambda + L + p)} \) (10)
\[ \tau_i = a_2 \] (11)
\[ \tau_D = \frac{a_1}{a_2} \] (12)
\[ \alpha_0 = 0.5L \] (13)
\[ \alpha_1 = \frac{0.5L\lambda^2}{2(\lambda + L + p)} \] (14)
\[ \alpha_2 = \frac{\lambda^2 + L\lambda - 0.5Lp}{(2\lambda + L + p)} \] (15)

IMC-PID controller settings for stable SOPTD with a positive zero are calculated using the Eq (10) to Eq (15).

(II) Stability analysis method
The process transfer function for SOPTD with a positive zero is given by
\[ G_p(s) = \frac{K_p(1 - ps)e^{-\lambda s}}{a_1s^2 + a_2s + 1} \]
The transfer function of the controller is given by
\[ G_c(s) = K_c[1 + \frac{1}{\tau_i s} + \tau_d s] \]
\[ G_c(s)G_p(s) = \frac{[K_p(1 - ps)e^{-\lambda s}][K_c(1 + \frac{1}{\tau_i s} + \tau_d s)]}{a_1s^2 + a_2s + 1} \] (17)
let
\[ \tau_i = a_2 \] (18)

With these assumptions, the Eq (17) becomes
\[ G_c(s)G_p(s) = \frac{K_cK_p(1 - ps)e^{-\lambda s}}{\tau_i s} \] (20)

Phase angle criteria for the given system is as
\[ -\frac{\pi}{2} = -L\omega_c - \tan^{-1}(p\omega_c) + \pi = 0 \] (21)

Eq (21) is solved by using fsolve of MATLAB for \( \omega_c \)
Using amplitude criteria, \( K_{c,max} \) is given by
\[ K_{c,max} = \frac{\omega_c\tau_i}{K_p\sqrt{1 + p^2\omega^2}} \] (22)

The design value of \( K_c \) is calculated by using Eq (23). The PID parameters are given by
\[ K_c = \frac{K_{c,max}}{G_{m}} \] (23)
\[ \tau_i = a_2 \] (24)
\[ \tau_D = \frac{a_1}{a_2} \] (25)

Simulation results
In this section, proposed IMC-PID controller (PID controller designed by IMC method) and proposed SA-PID controller (PID controller designed by stability analysis method) are applied to various CSTR transfer function models and non-linear models to show the efficiency of the proposed controllers. The performance of the proposed controllers is compared with DS-PID controller (PID controller designed by direct synthesis method [12]).

Case study-1
Consider a continuous stirred tank reactor where in the following isothermal series-parallel (Van de Vusse) reactions takes place. Here the product B is the desired one.

\[ \begin{align*}
A & \xrightarrow{k_1} B \xrightarrow{k_3} C \\
2A & \xrightarrow{k_5} D \\
\end{align*} \]

The non-linear model equations for the species A and B are,
\[ \frac{dC_A}{dt} = \frac{F}{V} (C_{A_f} - C_A) - K_1C_A - K_3C_A^2 \] (26)
\[ \frac{dC_B}{dt} = -\frac{F}{V} C_B + K_1C_A - K_2C_B \] (27)

The above non-linear equations are linearised around stable operating point \( C_{A}=3 \) g mole/l and \( C_B=1.1170 \) g mole/l. The process parameters are given in Table 1. The transfer function relating the desired concentration \( (C_B) \) to the ratio flow rate and volume of the reactor along with a measurement time delay of 0.1 min is given by
\[ \frac{C_B(s)}{F/V(s)} = \frac{-1.117s + 3.1472}{s^2 + 4.6429s + 5.3821}e^{-0.1s} \] (28)

The IMC-PID controller, SA-PID controller and DS-PID [12] controller parameters are given in Table 2. The servo and regulatory problem of the process transfer function model with IMC-PID controller, SA-PID controller, and DS-PID [12]
controller is shown in Fig. 1. The performance of servo and regulatory problems is almost same for all the controllers. The performance in terms of ISE, IAE, and ITAE of the process transfer function model with IMC-PID controller, SA-PID controller and DS-PID [12] controller is given in Table 3. The simulation of non-linear model equations with IMC-PID controller, SA-PID controller and DS-PID [12] controller for servo problem (change in product concentration $C_B$ from 1.117 to 1.2 g mole/l) is shown in Fig 2. The regulatory problem of constant volume CSTR for the change in flow rate $(F/V)$ from 0.5714 to 0.6 min$^{-1}$ is shown in Fig 3. For servo problem, all the controllers give almost similar response. But for regulatory problem SA-PID controller is superior compared to the IMC-PID controller and DS-PID [12] controller. The performance comparison of the proposed controllers for nonlinear models in terms of ISE, IAE, and ITAE for both servo and regulatory problems are given in Table 4. The controllers are designed based on nominal model parameters and applied to the process with perturbed parameters. The proposed IMC-PID controller is robust for model uncertainty considering one parameter at a time compared to SA-PID controller and DS-PID [12] controller. Maximum magnitude of complimentary sensitivity function of these three controllers is given in Table 5.


References

Nomenclature

\( G_c \) Control transfer function
\( G_p \) Process transfer function
\( G_m \) Gain margin
\( K_c \) Controller gain
\( K_{c,max} \) Ultimate controller gain
\( \tau_i \) Integral time
\( K_p \) Process gain
\( L \) Time delay
\( P \) Numerator time constant
\( t \) Time
\( \tau \) Time constant
\( \tau_D \) Derivative time

\( \alpha_0, \alpha_1, \alpha_2 \) Filter time constants
\( a_1, a_2 \) Denominator constants
\( \omega \) Cross over frequency
\( \lambda \) IMC filter time constant

ISE  Integral square error
IAE  Integral absolute error
ITAE  Integral time absolute error

DS-PID Proportional Integral and Derivative Controller designed by Direct synthesis method (Chien et al., 2003)
SA-PID Proportional Integral and Derivative Controller designed by Stability analysis method
IMC-PID Proportional Integral and Derivative Controller designed by Internal model control.

Table 1. Parameter values of Isothermal CSTR.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case study-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F/V )</td>
<td>0.5714 min(^{-1})</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>5 min(^{-1})</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>5 min(^{-1})</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>1 min(^{-1})</td>
</tr>
<tr>
<td>( C_{Af} )</td>
<td>10 gmole/l</td>
</tr>
</tbody>
</table>

Table 2. PID settings for different methods.

<table>
<thead>
<tr>
<th>Controller</th>
<th>( K_c )</th>
<th>( \tau_i )</th>
<th>( \tau_D )</th>
<th>( \alpha_0 )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMC-PID</td>
<td>1.4685</td>
<td>0.8627</td>
<td>0.2154</td>
<td>0.05</td>
<td>0.0038</td>
<td>0.0850</td>
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<tr>
<td>SA-PID</td>
<td>1.4685</td>
<td>0.8627</td>
<td>0.2154</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>DS-PID</td>
<td>1.4588</td>
<td>0.8627</td>
<td>0.2154</td>
<td>---</td>
<td>---</td>
<td>0.0415</td>
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</tbody>
</table>

Table 3. Performance comparison in terms of ISE, IAE, ITAE for linear models.

<table>
<thead>
<tr>
<th>Controller</th>
<th>ISE</th>
<th>IAE</th>
<th>ITAE</th>
<th>ISE</th>
<th>IAE</th>
<th>ITAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMC-PID</td>
<td>1.0634</td>
<td>1.2171</td>
<td>0.8732</td>
<td>0.1808</td>
<td>0.6582</td>
<td>1.1067</td>
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<tr>
<td>SA-PID</td>
<td>1.0131</td>
<td>1.1622</td>
<td>0.7933</td>
<td>0.1714</td>
<td>0.6582</td>
<td>1.1267</td>
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<tr>
<td>DA-PID</td>
<td>1.0656</td>
<td>1.2316</td>
<td>0.8967</td>
<td>0.1829</td>
<td>0.6613</td>
<td>1.1136</td>
</tr>
</tbody>
</table>

Table 4. Performance comparison for non-linear model

<table>
<thead>
<tr>
<th>Performance</th>
<th>Controller</th>
<th>Change in Concentration of B(From:1.117 to 1.2 mole/l)</th>
<th>Change in F/V(From:0.5471 to 0.6 min(^{-1}))</th>
<th>Change in feed concentration of A(from:10 to 12 mole/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISE</td>
<td>IMC-PID</td>
<td>0.0079</td>
<td>1.456×10(^{-4})</td>
<td>0.0076</td>
</tr>
<tr>
<td>IAE</td>
<td>IMC-PID</td>
<td>0.1119</td>
<td>0.0188</td>
<td>0.1103</td>
</tr>
<tr>
<td>ITAE</td>
<td>IMC-PID</td>
<td>0.087</td>
<td>0.0320</td>
<td>0.1274</td>
</tr>
<tr>
<td>Servo problem</td>
<td>SA-PID</td>
<td>0.0922</td>
<td>0.0325</td>
<td>0.1309</td>
</tr>
<tr>
<td>Regulatory Problem</td>
<td>DS-PID</td>
<td>0.0881</td>
<td>0.0322</td>
<td>0.1280</td>
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</tbody>
</table>
Table 5. Maximum magnitude of complimentary sensitivity function

<table>
<thead>
<tr>
<th>Transfer function model</th>
<th>Controller</th>
<th>Complimentary sensitivity function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case study-1</td>
<td>IMC-PID</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>SA-PID</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>DS-PID</td>
<td>1</td>
</tr>
</tbody>
</table>